Department of Electrical Engineering
Assignment
Date:13/04/2020

Course Details

Course Title: Digital Signal Processing **Module:** 6th
Instructor: Pir Meher Ali Sha **Total Marks:** 30

Student Details

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Q1.	(a)	Consider the following analog signal $x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$ i. Determine the minimum sampling rate required to avoid aliasing. ii. Suppose that the signal is sampled at the rate F_s = 100Hz.What is the discrete-time signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time signal. iii. What is the analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation?	Marks 5 CLO 1
	(b)	Consider a discrete time signal which is given by $x(n) = \begin{cases} 0.5n, & n \geq 0 \\ 0, & n < 0 \end{cases}$ This is signal is sampled at the rate F_s= 200Hz. i. Draw the sampled signal. ii. The samples of the signals are intended to carry 3 bits per sample. Determine the quantization level and quantization resolution to quantized the sampled signal achieved in part i . iii. Perform the process of truncation and rounding off on all the values of the sampled signal and find the quantization error for each of the sampled data. Express your answer in tabular form.	Marks 5 CLO 1
Q2.	(a)	Determine the response of the system to the following input signal with given impulse response $x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\}$, $h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$	Marks 5 CLO 2

Section Break(Next Page)

	(b)	Compute the convolution y(n) of the following signal $x(n)=\begin{cases} \alpha^{n+1}, & -3 \le n \le 5 \\ 0, & \text{elsewhere} \end{cases}$ $h(n)=\begin{cases} 2^{n}, & 0 \le n \le 4 \\ 0, & \text{elsewhere} \end{cases}$	Marks 5 CLO 2
Q3.		Determine the z- transform of the following signals and also sketch its Region of Convergence (ROC). i) $X(n)\begin{cases} \left(\frac{1}{4}\right)^{n}, & n \ge 0 \\ \left(\frac{1}{3}\right)^{-n}, & n<0 \end{cases}$ ii) $X(n)=\begin{cases} \left(\frac{1}{2}\right)^{n}\cdot 3^{n}, & n \ge 0 \\ 0, & \text{elsewhere} \end{cases}$	Marks 10 CLO 2

Attempt all the questions:-

"QUESTION 1"

PART A :-

Q Consider the following signal:

$$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

(i) Determine the minimum sampling rate required to avoid aliasing?

Solution :-

In this signal we have two frequencies. F_1 and F_2.

So,
First we find F_1 and F_2.

$3\cos 100\pi t$

$$\omega = 100\pi$$

As we know that $\omega = 2\pi F$

$$2\pi F_1 = 100\pi$$

$$F_1 = \frac{\cancel{100}^{50}\cancel{\pi}}{\cancel{2}\cancel{\pi}}$$

$$\boxed{F_1 = 50\ Hz}$$

$4\sin 200\pi t$

$$\omega = 200\pi$$

$$2\pi F_2 = 200\pi$$

$$F_2 = \frac{\cancel{2}00\cancel{\pi}}{\cancel{2\pi}}$$

$$\boxed{F_2 = 100\ Hz}$$

As we know that "Nyquist Sampling rate" is the lowest Sampling rate which we can used without aliasing.

To avoid aliasing the sampling rate for an analog signal must be at least two times the bandwidth of the signal.

So,

Nyquist formula is $F_s = 2F_m$.

F_m = maximum frequency.

Here we have maximum frequency = 100 Hz.

So,

$$F_s = 2F_m$$

$$F_s = 2 \times 100$$

$$\boxed{F_s = 200\ Hz}$$

So, it is proved that $F_s = 200\ Hz$ is a frequency which we required to avoid aliasing.

(ii) Suppose that the signal is sampled at the rate $F_s = 100\ Hz$. What is the discrete time signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time signal?

Solution :-

$$x(n) = 3\cos\frac{\cancel{100}\pi n}{\cancel{100}}$$

$$\boxed{= 3\cos \pi n}$$

$$x(n) = 4\sin \frac{200\pi}{100} n$$

$$\boxed{= 4\sin 2\pi n}$$

So,

the discrete time signal is.

$$\boxed{x_a(t) = 3\cos \pi n + 4\sin 2\pi n}$$

Effect :-

As $F_s = 100\ Hz$

Folding frequency formula is $F_1 - F_s$

$$F_o = F_1 - F_s$$
$$= 50 - 100$$
$$\boxed{= 50\ Hz}$$

$$F_o = F_2 - 2F_s$$
$$= 100 - 2(100)$$
$$= 100 - 200$$
$$\boxed{= 100\ Hz}$$

So the folding frequency of Sampled signal is equal to the orginal signal frequencies. So there is no effect on newly generated signal.

(iii) What is the analog signal $Y_a(t)$ we can reconstruct from the samples if we use ideal interpolation?

Sample Signal :-

$$x_a(t) = 3\cos \pi n + 4\sin 2\pi n$$

Samples = F_s = 100 Hz

So,

when we convert it in to time domain

The equation is.

$$x_a(t) = 3\cos\pi t + 4\sin 2\pi t$$

Multiplying sample frequency F_s with this signal.

$$x_a(t) = 3\cos\pi \times F_s t + 4\sin 2\pi \times F_s t$$
$$= 3\cos\pi(100)t + 4\sin 2\pi(100)t$$
$$\boxed{x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t}$$

So the reconstruct signal is same as a original signal.

So,

$$\boxed{y_a(t) = 3\cos 100\pi t + 4\sin 200\pi t}$$

PART B :-

Q Consider a discrete time signal which is given by.

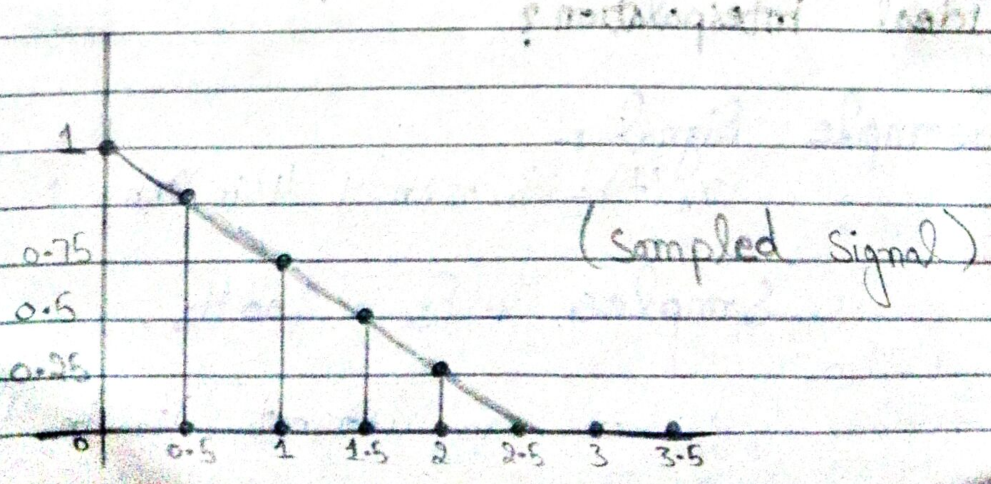
$$x(n) = \begin{cases} 0.5^n, & n \geq 0 \\ 0 & n < 0 \end{cases}$$

This is signal is sampled at the rate $F_s = 2Hz$.

(i) Draw the sampled signal.

As $F_s = 2Hz$

$t = 0.5$ sec



(ii) The sample of a signal are intended to carry 3 bits per sample. Determine the quantization level and quantization resolution is quantized the sample signal achieved in part (i).

Solution :-

a, Quantization level formula $\Rightarrow L = 2^n$

So here $n = 3$ bits

$$L = 2^n$$
$$L = 2^{(3)}$$
$$\boxed{L = 8}$$

No of levels $= 8$

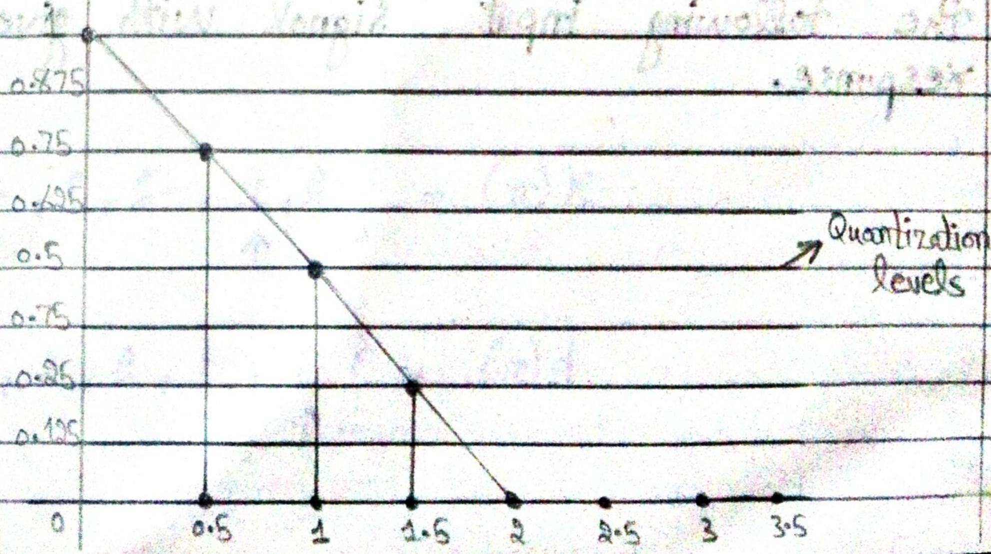
b, Quantization resolution formula $\Rightarrow \Delta = \dfrac{x_{max} - x_{min}}{L-1}$

So,

When we draw a graph maximum point is 1 and minimum is 0.

So it means $x_{max} = 1$
$x_{min} = 0$

$$\Delta = \frac{1-0}{8-1} = \frac{1}{7} = \boxed{0.1429}$$



iii) Perform the process of truncation and rounding off number of the sampled signal and find the quantization error for each of the sampled data. Express the answer in tabular form.

n	$x(n)$	$x_q(n)$ (Truncation)	$x_q(n)$ (Rounding off)	$x_q(n) - x(n)$ (error)
0	1	1.0	1.0	0.0
1	0.875	0.8	0.9	-0.1
2	0.75	0.7	0.8	-0.1
3	0.625	0.6	0.6	0.0
4	0.5	0.5	0.5	0.0
5	0.375	0.3	0.4	-0.1
6	0.25	0.2	0.3	-0.1
7	0.125	0.1	0.1	0.0

"QUESTION 2"

PART A :-

Q Determine the response of the system to the following input signal with given impulse response.

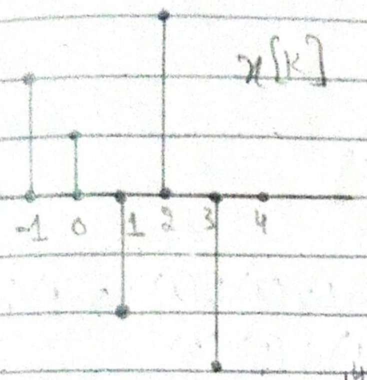
$$x(n) = \{2, \underset{\uparrow}{1}, -2, 3, -4\}$$

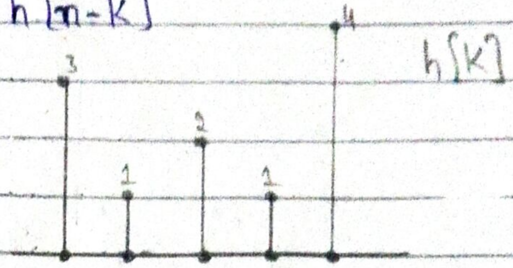
$$h(n) = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$$

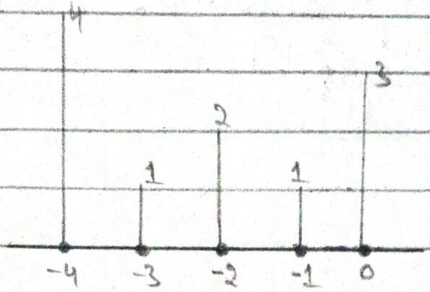
Solution :-

As we know

$$Y[n] = \sum_{K=-\infty}^{\infty} x[K]\, h[n-K]$$







For Y=0:-

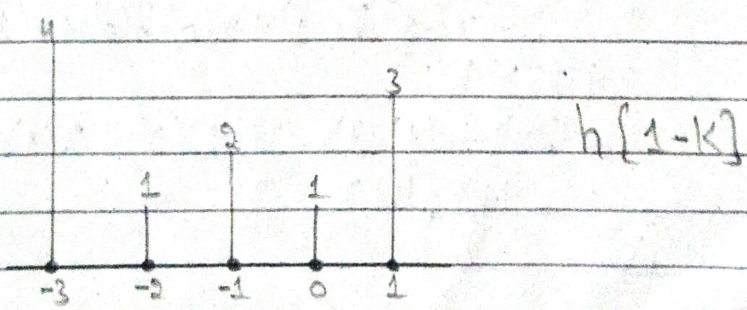
$$Y[0] = \sum_{K=1}^{0} x[-1]\, h[-1] + x[0]\, h[0]$$

$$= 2\times1 + (1)3$$

$$= 2+3$$

$$\boxed{Y[0] = 5}$$

For Y = 1 :-



$$Y[1] = \sum_{K=-1}^{1} x[n]\, h[1-K]$$

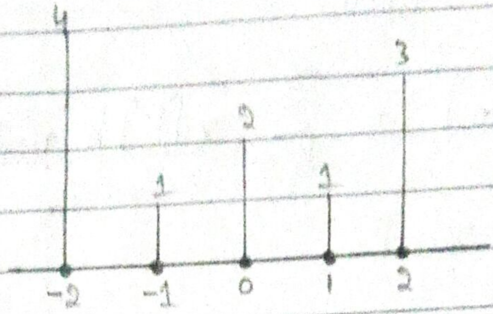
$$= x(-1)h(-1) + x(0)h(0) + x(1)h(1)$$

$$= (2)(2) + (1)(1) + (3)(-2)$$

$$= 4+1-6$$

$$\boxed{Y[1] = -1}$$

For $y = 2$:-



$h[2-K]$

$$y[2] = \sum_{K=-1} x[n]\, h[2-K]$$

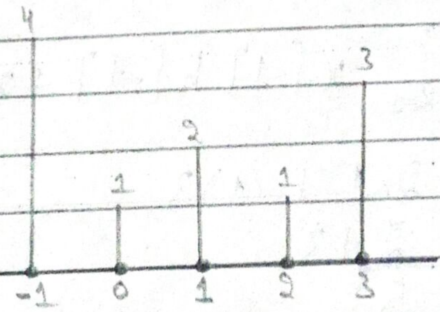
$$Y[2] = x(-1)h(-1) + x(0) + h(0) + x(1)h(1) + x(2)h(2)$$

$$Y[2] = (2)(1) + (1)(2) + (-2)(1) + (3)(3)$$

$$Y[2] = 2+2-2+9$$

$$\boxed{Y[2] = 11}$$

For $y = 3$:-



$h[3-K]$

$$Y[3] = \sum_{K=-1}^{3} x[n]\, h[3-K]$$

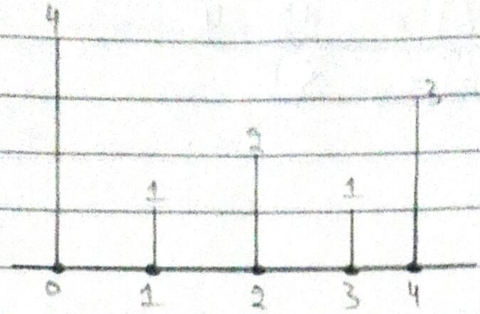
$$= x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2)$$
$$+ x(3)h(3)$$

$$= 2\times 4 + (1)(1) - 1(-2)(2) + 3(1) + (-4)(3)$$

$$= 4+1-4+3-12$$

$$\boxed{Y[3] = -8}$$

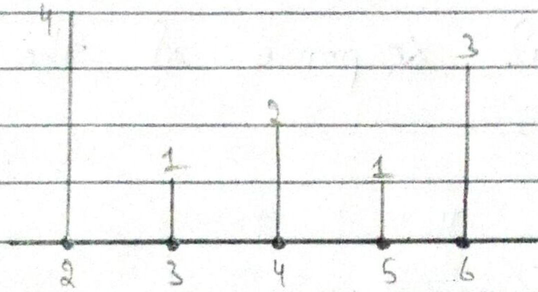
For $y = 4$:-



$$y(4) = \sum_{K=1}^{4} x[n]\, h[4-K]$$

$$= x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$$
$$= 1 \times 4 + (-2)(1) + 3(2) + (-4)(1)$$
$$= 4 - 2 + 6 - 4$$
$$\boxed{Y[4] = 4}$$

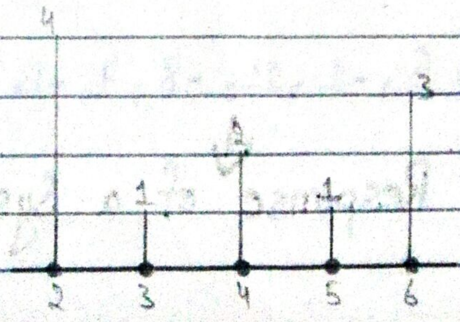
For $y = 5$:-



$$Y[5] = \sum_{K=1}^{5} x[n]\, h[5-K]$$

$$= x(1)h(1) + x(2)h(2) + x(3)h(3)$$
$$= (-2)(4) + (3)(1) + (-4)(2)$$
$$= -8 + 3 - 8$$
$$\boxed{Y[5] = -13}$$

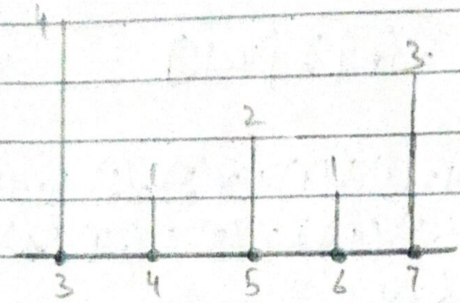
For $y = 6$:-



$$Y[6] = \sum_{K=2}^{K=3} x(2)h(2) + x(3)h(3)$$

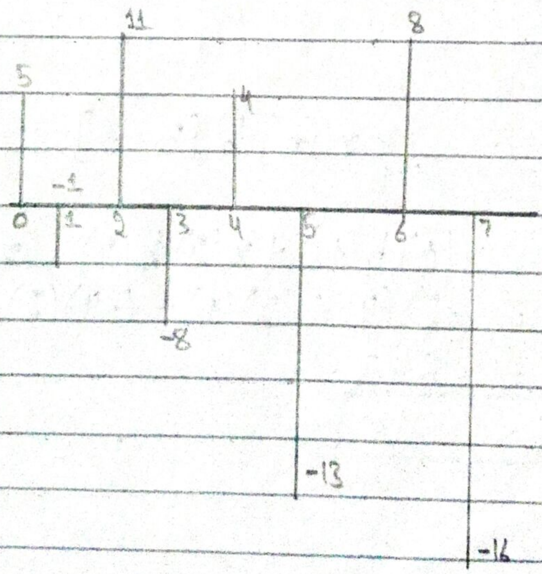
$$Y[6] = (3)(4) + (1)(-4)$$
$$Y[6] = 12 - 4$$
$$\boxed{Y[6] = 8}$$

For $y = 7$:-



$$Y[7] = x[3]h[3]$$
$$= 4 \times (-4)$$
$$\boxed{Y[7] = -16}$$

So the final response of the System is.



$$Y[n] = [5, -1, 11, -8, 4, -13, 8, -16]$$

Response of a System.

PART B :-

Q Compute the convolution $Y[n]$ of the following signal.

$$x(n) = \begin{cases} \alpha^{n+1} & , -3 \leq n \leq 5 \\ 0 & , \text{elswhere} \end{cases}$$

$$h(x) = \begin{cases} 2^n & , 0 \leq n \leq 4 \\ 0 & , \text{elsewhere} \end{cases}$$

Solution :-

Minimum limits $= -3$
Maximum limit $= 5$

Formula:-

$$Y[n] = \sum_{n=-3}^{5} x(K)\, h(n-K)$$

$$Y[n] = \sum_{n=-3}^{5} \alpha^{K+1} \cdot 2^{n-K}$$

$$= \sum_{n=-3}^{5} \alpha^{K} \cdot \alpha \cdot 2^{n} \cdot 2^{-K}$$

$$= \alpha 2^{n} \sum \alpha^{K} \cdot 2^{-K}$$

$$= 2^{n} \alpha \sum_{K=-3}^{5} (2^{-1}\alpha)^{K}$$

$$= 2^{n} \alpha \sum_{K=3}^{5} \left(\frac{\alpha}{2}\right)^{K}$$

So For $\alpha = 1$:-

$$\sum_{K=m}^{N} a^{K} = N - m + 1$$

For $\alpha \neq 1$

$$\sum_{K=m}^{N} a^{K} = a^{m} + a^{m+1} \text{------} a^{N}.$$

Also

$$\sum_{k=m}^{N} a^k = \begin{cases} \frac{a^m - a^{N+1}}{1-a} & ; \quad a \neq 1 \\ N-m+1 & ; \quad a = 1 \end{cases}$$

$m = -3$

$N = 5$

$a = \frac{\alpha}{2}$

So,

$$Y[n] = \alpha 2^n \sum_{k=-3}^{5} \left(\frac{\alpha}{2}\right)^k$$

Now for finding values of [Y] we apply formula.

$$Y[n] = 2^n \alpha^n [\alpha^{-3} + \alpha^{-2} + \alpha^{-1} + \alpha^0 + \alpha^1 + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5]$$

$$y[n] = 2^n [\alpha^{-2} + \alpha^{-1} + \alpha^0 + \alpha^1 + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5 + \alpha^6]$$

Now

$$Y[-3] = \frac{1}{8} [\alpha^{-2} + \alpha^{-1} + \alpha^0 + \alpha^1 + \alpha^2 - \cdots - \alpha^6]$$

$$Y[-2] = \frac{1}{4} [\alpha^{-2} + \alpha^{-1} + \alpha^0 + \alpha^1 + \alpha^2 + \cdots - \alpha^6]$$

$$Y[-1] = \frac{1}{2} [\alpha^{-2} + \alpha^{-1} + \alpha^0 + \alpha^1 + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5 + \alpha^6]$$

$$Y[0] = [\alpha^{-2} + \alpha^{-1} + \cdots - \alpha^6]$$

$$Y[1] = 2[\alpha^{-2} + \alpha^{-1} + \alpha^0 + \alpha^1 + \cdots - \alpha^6]$$

$$Y[2] = 4[\alpha^{-2} + \alpha^{-1} + \alpha^0 + \alpha^1 + \cdots - \alpha^6]$$

$$Y[3] = 8[\alpha^{-2} + \alpha^{-1} + \alpha^0 + \cdots - \alpha^6]$$

$$Y[4] = 16[\alpha^{-2} + \alpha^{-1} + \alpha^0 + \alpha^1 + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5 + \alpha^6]$$

"QUESTION 3"

Q Determine the Z-transform of the following signals and sketch its region of convergence (ROC)

(i)
$$x(n) = \begin{cases} (1/4)^n & , n \geq 0 \\ (1/3)^n & , n \leq 0 \end{cases}$$

Solution:-

Formula of Z-transform.

$$X(z) = \sum_{n=-\alpha}^{\alpha} x(n) z^{-n}$$

putting values.

$$= \sum_{-\alpha}^{\alpha} \left(\frac{1}{4}\right)^n x(n)$$

$$= \sum_{-\alpha}^{0} \left(\frac{1}{3}\right)^{-n} z^{-n} + \sum_{0}^{\alpha} \left(\frac{1}{4}\right)^n z^{-n}$$

$$= \sum_{-\alpha}^{0} \left(\frac{1}{3} z\right)^n + \sum_{0}^{\alpha} \left(\frac{1}{4} z^{-1}\right)^n$$

$$= \frac{1}{1-\frac{1}{3}z} - 1 + \frac{1}{1-\frac{1}{4}z^{-1}}$$

By L.C.M
$$= \frac{\left(1-\frac{1}{4}z^{-1}\right) - \left(1-\frac{1}{3}z\right)\left(1-\frac{1}{4}z^{-1}\right) + \left(1-\frac{1}{3}z\right)}{\left(1-\frac{1}{3}z\right)\left(1-\frac{1}{4}z^{-1}\right)}$$

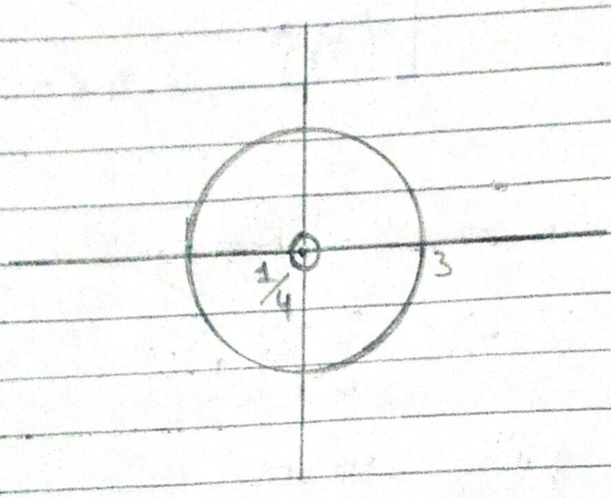
$$= \frac{1-\frac{1}{4}z^{-1}\left(1-\frac{1}{4}z^{-1} - \frac{1}{3}z + \frac{1}{12}\right) + 1 - \frac{1}{3}z}{\left(1-\frac{1}{3}z\right)\left(1-\frac{1}{4}z^{-1}\right)}$$

$$= \frac{1-\frac{1}{4} - 1 + \frac{1}{4}z^{-1} + \frac{1}{3}z - \frac{1}{12} + 1 - \frac{1}{3}z}{\left(1-\frac{1}{3}z\right)\left(1-\frac{1}{4}z^{-1}\right)}$$

$$X(z) = \frac{11/12}{\left(1-\frac{1}{3}z\right)\left(1-\frac{1}{4}z^{-1}\right)}$$

So, ROC is $\frac{1}{4} < |z| < 3$.

ROC :-



(ii)

$$x(n) = \begin{cases} \left(\frac{1}{2}\right)^n - 3 & , \; n \geq 0 \\ 0 & , \; \text{elsewhere} \end{cases}$$

Solution :-

Formula of Z-transform.

$$X(z) = \sum_{n=\infty}^{\infty} x(n) z^{-n}$$

$$X(z) = \sum_{0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n} - \sum_{0}^{\infty} 3^n z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2} z^{-1}\right)^n - \sum_{n=0}^{\infty} \left(3z^{-1}\right)^n$$

$$X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-3z^{-1}}$$

Now by L.C.M.

$$= \frac{(1-3z^{-1})-(1-\frac{1}{2}z^{-1})}{(1-\frac{1}{2}z^{-1})(1-3z^{-1})}$$

$$= \frac{\cancel{1}-3z^{-1}-\cancel{1}+\frac{1}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-3z^{-1})}$$

$$= \frac{-3z^{-1}+\frac{1}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-3z^{-1})}$$

$$= \frac{z^{-1}(\frac{1}{2}-3)}{(1-\frac{1}{2}z^{-1})(1-3z^{-1})}$$

So,

$$X(z) = \frac{-\frac{5}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-3z^{-1})}$$

So, ROC is $|z| > 3$, $|z| > \frac{1}{2}$.

So overall ROC is $|z| > \frac{1}{2}$.

ROC :-