14-04-2020

P.No 1

Department of BE(E)

Course Details

Course Title:- Electro Magnetic field theory

Module:- 4th

Instructor :- Engr Dr Rafiq mansoor

Total Marks :- 30

Student Detail

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Q.-1 Solve the following short questions.

(a) Transform the vector $B = yi(x+z)j$ located at point $(-2, 6, 3)$ into cylindrical coordinates.

Sol. $B = yi(x+z)j$

The Given points are $(-2, 6, 3)$ then,

$$B = yi(xj + zj)$$

$$B = yxij + yzij$$

Now

$$\rho = \sqrt{x^2 + y^2}$$

$$\rho = \sqrt{(-2)^2 + (6)^2}$$

$$\rho = \sqrt{40}$$

$$\boxed{\rho = 6.32}$$

Now

$$\phi = \tan^{-1} y/x$$

$$\phi = \tan^{-1}\left(\frac{6}{-2}\right)$$

$$\phi = \tan^{-1}(-3)$$

$$\boxed{\phi = -71.56}$$

Now

$$z = z \quad \text{so}$$

$$\boxed{z = 3}$$

$$B = 6.32, -71.56, 3.$$

(b) Convert the point (3, 4, 5) from Cartesian to spherical coordinates.

Sol. $P(3, 4, 5)$ where $x = 3$, $y = 4$, $z = 5$

Now

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$r = \sqrt{3^2 + 4^2 + 5^2}$$

$$r = \sqrt{50}$$

$$\boxed{r = 7.07}$$

Now

$$\theta = \cos^{-1}\left(\frac{z}{r}\right)$$

$$\theta = \cos^{-1}\left(\frac{5}{\sqrt{50}}\right)$$

$$\boxed{\theta = 45^\circ}$$

Now

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\phi = \tan^{-1}\left(\frac{4}{3}\right)$$

$$\phi = \tan^{-1}(1.333)$$

$$\boxed{\phi = 53.1^\circ}$$

$= \alpha = \alpha = \alpha = \alpha = \alpha = \alpha = \alpha = \alpha$

(c) Find the spherical coordinates of $A(2, 3, -1)$

Sol.. As we know that

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$r = \sqrt{2^2 + 3^2 + (-1)^2}$$

$$r = \sqrt{14}$$

$$\boxed{r = 3.74}$$

Now

$$\theta = \cos^{-1}\left(\frac{z}{r}\right)$$

$$\theta = \cos^{-1}\left(\frac{-1}{3.74}\right)$$

$$\theta = \cos^{-1}(-0.26)$$

$$\boxed{\theta = 105^\circ}$$

Now

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\phi = \tan^{-1}\left(\frac{3}{2}\right)$$

$$\phi = \tan^{-1}(1.5)$$

$$\boxed{\phi = 56.31^\circ}$$

(d) Find the Cartesian coordinates of $B(4, 25, 120)$.

Sol. The given B points are given in spherical (r, θ, ϕ). So we have to find (x, y, z).

Now

$$x = r \sin\theta \cdot \cos\phi$$
$$x = 4 \sin(25) \cdot \cos(120)$$
$$x = 4(0.42)(-0.5)$$
$$\boxed{x = -0.84}$$

Now

$$y = r \sin\theta \cdot \sin\phi$$
$$y = 4 \sin(25) \cdot \sin(120)$$
$$y = 4(0.42)(0.86)$$
$$\boxed{y = 1.45}$$

Now

$$z = r \cos\theta$$
$$z = 4 \cos(25)$$
$$z = 4(0.90)$$
$$\boxed{z = 3.62}$$

(e) Find the force between two charges when they are brought in contact and separated by 4cm apart, charges are 2nC and -1nC (in μN).

Sol: Given data

$q_1 = 2nC$, $q_2 = -1nC$, $r = 4$ cm

$F = ?$

where

$$F = K \frac{q_1 q_2}{r^2}$$

$$K = \frac{1}{4\pi\epsilon_0}$$

$$F = \frac{2\times10^{-9} \times -1\times10^{-9}}{4(3.14)\times 8.85\times10^{-12}\times(4\times10^{-2})^2}$$

$$F = -1.124\times10^{-5}\ N$$

$$F = -11.24\ \mu N$$

When a charges are brought in contact and then separated, charges on each sphere is, $(q_1+q_2)/2$ which is equal to 0.5nC.

On calulating the force with $q_1 = q_2 = 0.5nC$, So

$$\boxed{F = 1.404\ \mu N}$$

(f) Find the electric field intensity of two charges $-2C$ and $-1C$ separated by a distance 1m in air.

Given data

$$q_1 = -2C \quad , \quad q_2 = -1C$$

$$d = 1\,m$$

Required

$$E = ?$$

Sol.

$$E_1 = \frac{Kq_1}{d^2} \qquad K = 9 \times 10^9$$

$$E_1 = \frac{9 \times 10^9 \times -2}{(1)^2}$$

$$\boxed{E_1 = -18 \times 10^9 \ V/m}$$

Now

$$E_2 = \frac{K\,q_2}{d^2}$$

$$E_2 = \frac{9 \times 10^9 \times (-1)}{(1)^2}$$

$$\boxed{E_2 = -9 \times 10^9 \ V/m}$$

(8) Determine the charge that produce an electric field strength of 40 V/cm at a distance of 30 cm in vacuum (in 10^{-8} C).

Sol. Given data

$E = 40$ V/cm , $d = r = 30$ cm

$E = 4000$ V/m $\quad = 0.3$ m

$Q = ?$

where

$$E = \frac{KQ}{r^2}$$

$$\frac{Er^2}{K} = Q$$

Now putting values

$$Q = \frac{(4000) \times (0.3)^2}{9 \times 10^9}$$

$$\boxed{Q = 4 \times 10^{-8} \text{ C}}$$

(h) A charge of 2×10^{-7} is acted upon by a force of 0.1N. Determine the distance to the other charge of 4.5×10^{-7}C, both the charges are in vacuum.

Sol: Given data

$q_1 = 2\times10^{-7}C$, $q_2 = 4.5\times10^{-7}C$, $F = 0.1N$

Required $d = r = ?$

Now By formula

$$F = K\frac{q_1 q_2}{r^2}$$

$$r^2 = K\frac{q_1 q_2}{F}$$ → Now put values.

$$r^2 = \frac{9\times10^9 \times (2\times10^{-7})(4.5\times10^{-7})}{0.1}$$

$$r^2 = 8.1\times10^{-3}\ m$$

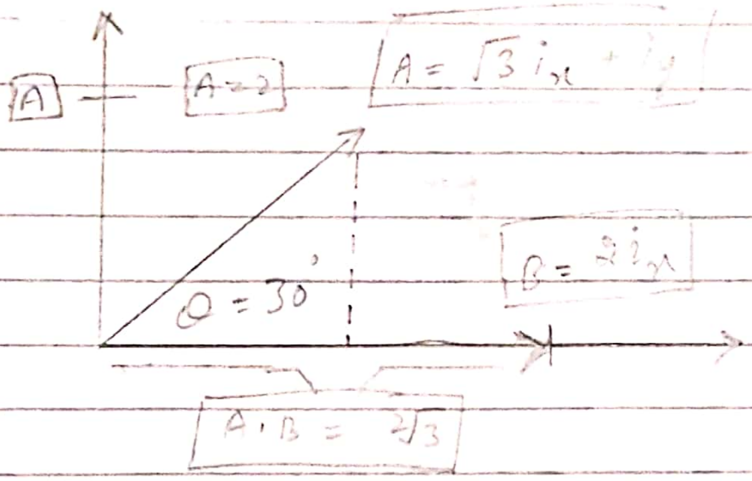
$$r^2 = 0.0081\ m$$

Now Take under-root on b. sides

$$\sqrt{r^2} = \sqrt{0.0081}\ m$$

$$\boxed{r = 0.09\ m}$$

Q:-2(a) Find the angle between the vector shown in figure.



Sol:- As

$$A \cdot B = |A||B| \cos\theta \rightarrow (*)$$

$$A \cdot B = 2\sqrt{3}$$

$$|A| = \sqrt{2^2} \qquad \& \qquad |B| = \sqrt{2^2}$$

$$|A| = 2 \qquad\qquad |B| = 2$$

put values in (*)

So (*) becomes

$$2\sqrt{3} = 2 \times 2 \cos\theta$$

$$2\sqrt{3} = 4 \cos\theta$$

$$\frac{1\cancel{2}\sqrt{3}}{2\cancel{4}} = \cos\theta$$

$$\frac{\sqrt{3}}{2} = \cos\theta$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\theta = \cos^{-1}\left(\frac{1.73}{2}\right)$$

$$\theta = \cos^{-1}(0.866)$$

$$\theta = 30^\circ$$

(b) Find the gradient of each of the following functions where a and b are constant

(i) $f = ax^2 + by^3 z$

Sol.

$$f = ax^2 + by^3z$$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(ax^2 + by^3z)$$

$$\frac{\partial f}{\partial x} = 2ax$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(ax^2 + by^3z)$$

$$\frac{\partial f}{\partial y} = 3by^2z$$

$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial z}(ax^2 + by^3z)$$

$$\frac{\partial f}{\partial z} = by^3$$

$$\nabla f(x, y, z) = (2ax + 3by^2z,\ by^3)$$

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(iii) $f = ar^2 \sin\phi + brz\cos 2\phi$.

Sol: Gradient for Spherical

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}.$$

So

$$\Rightarrow \nabla f = \frac{\partial}{\partial r}(ar^2\sin\phi + brz\cos 2\phi)\hat{r} + \frac{1}{r}\frac{\partial}{\partial \theta}(ar^2\sin\phi + brz\cos 2\phi)\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(ar^2\sin\phi + brz\cos 2\phi)\hat{\phi}.$$

Now take partial derivative.

$$\Rightarrow \nabla f = (2ar\sin\phi + bz\cos 2\phi)\hat{r} + \frac{1}{r}(0) + \frac{1}{r\sin\theta}(ar^2\cos\phi - 2brz\sin 2\phi)\hat{\phi}.$$

So

$$\Rightarrow \nabla f = (2ar\sin\phi + bz\cos 2\phi)\hat{r} + \frac{1}{r\sin\theta}(ar^2\cos\phi - 2brz\sin 2\phi)\hat{\phi}.$$

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Now gradient for cylindrical

$$\nabla f = \frac{\partial f}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z}.$$

$$\Rightarrow \nabla f = \frac{\partial}{\partial \rho}(ar^2 \sin\phi + brz\cos 2\phi)\hat{\rho} +$$

$$\frac{1}{\rho}\frac{\partial}{\partial \phi}(ar^2 \sin\phi + brz\cos 2\phi)\hat{\phi} +$$

$$\frac{\partial}{\partial z}(ar^2 \sin\phi + brz\cos 2\phi)\hat{z}.$$

Now Take partial derivatives:
Then the first term becomes Zero.

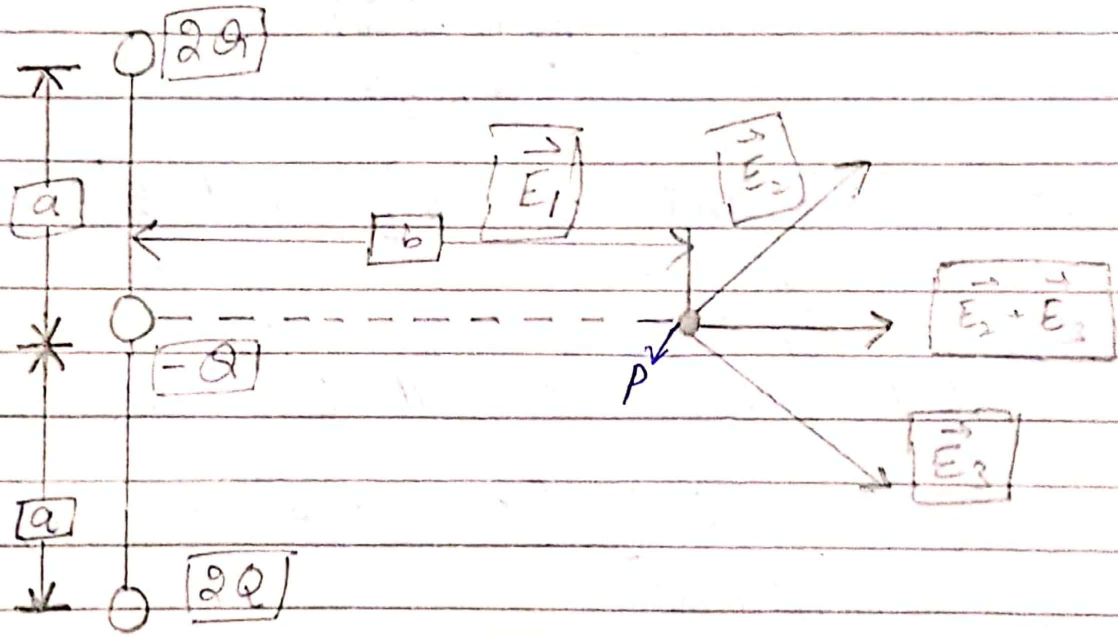
$$\Rightarrow \nabla f = \frac{1}{\rho}(ar^2\cos\phi - 2brz\sin 2\phi)\hat{\phi} +$$

$$(br\cos 2\phi)\hat{z}.$$

So

$$\nabla f = \frac{1}{\rho}(ar^2\cos\phi - 2brz\sin 2\phi)\hat{\phi} + (br\cos 2\phi)\hat{z}.$$

Q:-3 Three pointer charges are placed on the y-axis as shown. Find the electric field at point P on the x-axis.



Sol:- The distance between charge 2Q and point "P" is

$$r^2 = b^2 + a^2$$

so

$$r = \sqrt{b^2 + a^2}$$

Let assume that charge 2Q make angle (α) and $(-\alpha)$ with x-axis.

Magnitude of $|\vec{E_1}| = |\vec{E_2}| = \frac{Kq}{r^2}$

$$= \frac{K(2Q)}{r^2}$$

$$= \frac{K(2Q)}{b^2+a^2}$$

So Resultant of $\vec{E_1}$ and $\vec{E_2}$ is

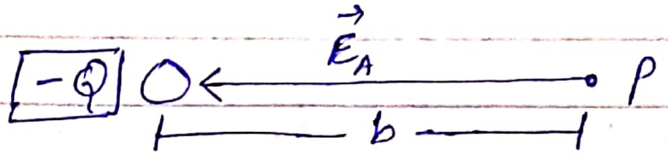
$\vec{E}_{1+2} = \vec{E_1} + \vec{E_2} = \vec{E}_{1x} + \vec{E}_{2x}$
(Y- component will be cancel).

$$= \frac{K(2Q)}{b^2+a^2}(\cos(\alpha) + \cos(-\alpha))$$

$$= \frac{K(2Q)}{b^2+a^2}(2\cos(\alpha)) \quad \because \cos(\alpha) = \cos(-\alpha)$$

$$\vec{E}_{1+2} = \frac{4KQ\cos(\alpha)}{b^2+a^2} \longrightarrow (i)$$

→ Now electric field at point "p" due to charge "−Q".
→ As charge is Negative Electric field at point will to directed towards charg "−Q".



$$\vec{E}_A = \frac{-K(Q)}{b^2}$$

Net electric field at point 'p' will be

$$\vec{E}_{net} = \vec{E}_A + (\vec{E}_1 + \vec{E}_2)$$

$$= \frac{-K(Q)}{b^2} + \frac{4KQ\cos\alpha}{b^2 + a^2}$$

$$= \frac{-KQ(a^2+b^2) + 4KQb^2\cos\alpha}{b^2(a^2+b^2)}$$

$$= \frac{KQ}{b^2(a^2+b^2)}\left[4b^2\cos\alpha - (a^2+b^2)\right]$$

where $K = 9\times10^9 \ Nm^2/c^2$.

$$\vec{E}_{Net} = \frac{9\times10^9 Q}{b^2(a^2+b^2)}\left[4b^2\cos\alpha - (a^2+b^2)\right]$$

Now

$$\alpha = \tan^{-1}\left(\frac{a}{b}\right)$$

So

$$\vec{E}_{Net} = \frac{9\times10^{9}Q}{b^2(a^2+b^2)}\left[4b^2\cos\left[\tan^{-1}\left(\frac{a}{b}\right)\right]-(a^2+b^2)\right]$$

THE END.