

Q6 :- Differentiate between Bi-nominal frequency distribution and Bi-nominal distribution with the help of formulas?

ANSWER :-

Bi-nominal frequency distribution :-

If the bi-nominal probability distribution is multiplied by  $N$ , the number of experiments or sets, the resulting distribution is known as the bi-nominal frequency distribution.

Formula :-

$$N \binom{n}{x} p^x q^{n-x}$$

Bi-nominal distribution :-

Beginning with the value of  $P(x=0)$ , probabilities for other values of " $x$ " the number of successes can be computed more easily by the recurrence formula i.e.

Formula :-

$$P(x=x) = \frac{n-x+1}{n} \frac{p}{q} P(x=x-1),$$

$$\text{for } x = 1, 2, 3, \dots$$



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Q11- A man throws two fair dice, what is the conditional probability that the sum of the two dice will be 7, given that

- 1 - The sum is even
- 2 - The sum is greater than 8
- 3 - The two dice had the same outcome.

Solution :-

The sample space  $S$  for this experiment is

$$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

Let

- $$A = \{\text{the sum is } 7\}$$
- $$B = \{\text{the sum is even}\}$$
- $$C = \{\text{the sum is greater than } 8\}$$
- $$D = \{\text{the two dice had the same outcome}\}$$

Then

- $$A = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$$
- $$B = \{(1,1), (1,3), (1,5), (2,2), (2,4), (2,6), (3,1), (3,3), (3,5), (4,2), (4,4), (4,6), (5,1), (5,3), (5,5), (6,2), (6,4), (6,6)\}$$
- $$C = \{(3,6), (4,5), (4,6), (5,4), (5,5), (5,6), (6,3), (6,4), (6,5), (6,6)\}$$
- $$D = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$$

$$A \cap B = \{\emptyset\}$$

$$A \cap C = \{\emptyset\}$$

$$A \cap D = \{\emptyset\}$$

$$P(A) = \frac{6}{36}$$

$$P(B) = \frac{18}{36}$$

$$P(C) = \frac{10}{36}$$

$$P(D) = \frac{6}{36}$$

$$P(A \cap B) = 0$$

$$P(A \cap C) = 0$$

$$P(A \cap D) = 0$$

Hence

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{36} \times \frac{18}{36} = 0$$

$$P(A/C) = \frac{P(A \cap C)}{P(C)} = \frac{0}{36} \times \frac{10}{36} = 0$$

$$P(A/D) = \frac{P(A \cap D)}{P(D)} = \frac{0}{36} \times \frac{6}{36} = 0$$



Q2:- Show that in a single throw of two dice, the probability of throwing more than 7 is equal to that of throwing less than 7, and hence find the probability of throwing exactly 7. State clearly what assumption you are making.

Solution:-

When we are rolling two dice, there are 36 different combinations.

Counting these up there are 15 possibilities less than 7.

(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2), (5,1).

The probability of getting less than a 7 is  $\frac{15}{36} = \frac{5}{12}$

There are 6 possible combinations of getting a 7, which gives a probability of  $\frac{1}{6}$ . This means that 21 possibilities account for getting less than or equal to 7, so there are 15 remaining possibilities of getting more than 7. This is the same as the probability of getting less than 7, so the probability must be  $\frac{5}{12}$  as well. In calculating this, we must assume that each combination is equally likely to roll as any other and therefore the dice are fair, or else the calculation don't work.



- Q3: A and B play a game in which A's probability of winning is  $\frac{2}{3}$ . In a series of 8 games, what is the probability that A will win
- 1 - Exactly 4 games
  - 2 - At least 4 games
  - 3 - From 3 to 8 games

Solution -

$$\begin{aligned} \text{Given that } p &= \frac{2}{3} & n &= 8 \\ q &= 1 - p \\ &= 1 - \frac{2}{3} \\ q &= \frac{1}{3} \end{aligned}$$

Let "x" denotes the number of games won by A. Then

$$\begin{aligned} \text{i) } P(X=4) &= \binom{8}{4} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^4 \\ &= \frac{1120}{6561} = 0.1707 \end{aligned}$$

$$\text{ii) } P(X \geq 4)$$

$$\begin{aligned} &= 1 - P(X < 4) \\ &= 1 - \sum_{x=0}^3 \binom{8}{x} \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{8-x} \\ &= 1 - \left[ \left(\frac{1}{3}\right)^8 + 8 \left(\frac{2}{3}\right) \left(\frac{1}{3}\right)^7 + 28 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^6 + 56 \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^5 \right] \\ &= 1 - \frac{1}{6561} [1 + 16 + 112 + 448] \\ &= 1 - \frac{577}{6561} \end{aligned}$$



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$$= \frac{6561 - 577}{6561}$$

$$= \frac{5984}{6561}$$

$$= 0.9121$$

3)  $P(3 \leq x \leq 6)$

$$\sum_{x=3}^6 \binom{8}{x} \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{8-x}$$

$x=3$

$$= \binom{8}{3} \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^5 + \binom{8}{4} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^4 + \binom{8}{5} \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^3 + \binom{8}{6} \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^2$$

$$= \frac{8}{(3)^8} [56 + 140 + 224 + 224]$$

$$= \frac{8 \times 644}{6561} = \frac{5152}{6561} = 0.7852$$

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Q7 :- Solution :-

Formula of Coefficient of Variation =

$$C.V = \frac{\sigma}{\bar{x}} \times 100$$

$$\bar{x} = \left( A + \frac{\sum f dx}{N} \right)$$

A = Assumed mean

$$dx = x - A$$

$$N = \sum f$$

| items<br>(x) | frequency<br>(f) | DEVIATIONS<br>dx = x - A | f dx            | f dx <sup>2</sup>     |
|--------------|------------------|--------------------------|-----------------|-----------------------|
| 45           | 3                | 5                        | 15              | 75                    |
| 60           | 11               | -10                      | -110            | 1100                  |
| 50 = A       | 5                | 0                        | 0               | 0                     |
| 25           | 15               | 25                       | 375             | 9375                  |
|              | $\sum f = 34$    |                          | $\sum dx = 280$ | $\sum f dx^2 = 10500$ |

$$\bar{x} = \left( A + \frac{\sum f dx}{N} \right)$$

$$= 50 + \frac{280}{34} = 58.23$$

$$\sigma = \sqrt{\frac{\sum f dx^2}{N} - \left( \frac{\sum f dx}{N} \right)^2}$$



So

$$\sigma = \sqrt{\frac{10550}{34} - \left(\frac{280}{34}\right)^2}$$

$$= \sqrt{310.29 - (8.23)^2}$$

$$= \sqrt{310.29 - 67.73}$$

$$= \sqrt{242.56} = 15.57$$

Now put in formula

$$C.V = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{15.57}{58.23} \times 100 = 26.73$$

Data is not accurate



Q5 & Solution &

Mean of Binomial Distribution

$$X \rightarrow f(x) = {}^n C_x p^x q^{n-x}$$

$$E(X) = \sum_{x=0}^n x f(x) \quad x = 0, 1, 2, \dots, n$$

$$f(x) = {}^n C_x p^x q^{n-x}$$

$$E(X) = \sum_{x=0}^n x {}^n C_x p^x q^{n-x} \quad \therefore \begin{matrix} 2! & 3! \\ 3 \cdot (3-1)! \end{matrix}$$

$$= \sum_{x=0}^n x \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

$$= \sum_{x=0}^n x \frac{n!}{x(x-1)!(n-x)!} p^x q^{n-x}$$

$$= \sum_{x=0}^n \frac{n!}{(x-1)!(n-x)!} p^x q^{n-x} \quad \therefore \frac{n!}{x!(n-x)!} \cdot \frac{n}{n-x+1}$$

$$\frac{(n-1) - (x-1)}{n-x - x+1} = \frac{n-x}{n-x}$$

$$= \sum_{x=0}^n \frac{n(n-1)!}{(x-1)!(n-x)!} p^x q^{n-x}$$

$$= n \sum_{x=0}^n \frac{(n-1)!}{(x-1)!(n-x)!} p^x q^{n-x} \cdot \frac{p}{p}$$



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$$= np \sum_{x=0}^n \frac{(n-1)!}{(x-1)!(n-x)!} p^x q^{n-x} \cdot p^{-1}$$

$$= np \sum_{x=0}^n \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1} q^{n-x}$$

$${}^n C_{x-1} p^{x-1} q^{n-x} ; x=0, 1, \dots, n$$

$$= np \sum_{x=1}^n \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1} q^{n-x}$$

$$= np \sum_{x=1}^n {}^{n-1} C_{x-1} p^{x-1} q^{n-x}$$

$$\sum f(x) \rightarrow \text{binomial P.d.f}$$
$$= np$$

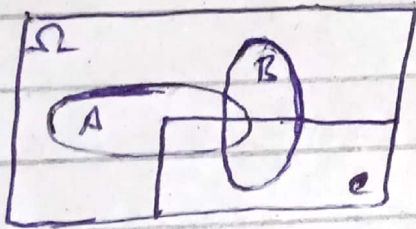
$$= E(X) = np \quad \mu = np$$

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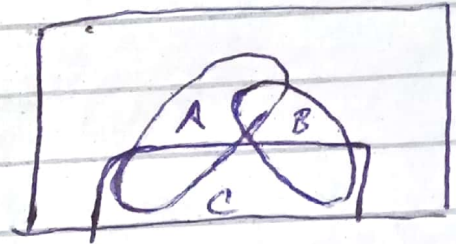
Q4 :- Solution :-

Conditional independence, given  $C$  is defined as independence under the probability law  $P(\cdot|C)$



$$P(A \cap B | C) = P(A | C) P(B | C)$$

if we told that  $C$  occurred are  $A$  and  $B$  independent?



Then

No,  $A$  and  $B$  is not independent

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