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QUESTION

Given that

$$\Rightarrow p = \frac{2}{3} \quad n = 8$$

$$\Rightarrow q = 1 - p$$

$$\Rightarrow = 1 - \frac{2}{3}$$

$$\Rightarrow q = \frac{1}{3}$$

\Rightarrow Let "x" denotes the number of games won by it. Then

$$P(x=4)$$

$$\Rightarrow \binom{8}{4} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^4$$

$$\Rightarrow \frac{1120}{6561}$$

$$\Rightarrow 0.1707$$

$$P(x \geq 4)$$

$$\Rightarrow 1 - P(x < 4)$$

$$\Rightarrow 1 - \sum_{x=0}^3 \binom{8}{x} \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{8-x}$$

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$$\Rightarrow 1 - \left[\binom{8}{0} \left(\frac{1}{3}\right)^8 + 8 \binom{8}{1} \left(\frac{2}{3}\right) \left(\frac{1}{3}\right)^7 + 28 \binom{8}{2} \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^6 + 56 \binom{8}{3} \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^5 + 70 \binom{8}{4} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^4 + 56 \binom{8}{5} \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^3 + 28 \binom{8}{6} \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^2 + 8 \binom{8}{7} \left(\frac{2}{3}\right)^7 \left(\frac{1}{3}\right) + \binom{8}{8} \left(\frac{2}{3}\right)^8 \right]$$

$$\Rightarrow 1 - \frac{1}{6561} [1 + 16 + 112 + 448 + 1120 + 1792 + 1792 + 1120 + 16 + 1]$$

$$\Rightarrow 1 - \frac{577}{6561}$$

$$\Rightarrow \frac{6561 - 577}{6561}$$

$$\Rightarrow \frac{5984}{6561}$$

$$\Rightarrow 0.9121$$

$P(3 \leq X \leq 6)$

$$\Rightarrow \sum_{x=3}^6 \binom{8}{x} \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{8-x}$$

$$\Rightarrow \binom{8}{3} \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^5 + \binom{8}{4} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^4 +$$

$$\binom{8}{5} \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^3 + \binom{8}{6} \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^2$$

$$\Rightarrow \frac{8}{\left(\frac{1}{3}\right)^8} [56 + 140 + 224 + 224]$$

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$$\Rightarrow \frac{8 \times 644}{6561}$$

$$\Rightarrow \frac{5152}{6561}$$

$$\Rightarrow 0.7852$$

QUESTION

Binomial distribution

A Binomial distribution can be thought of as simply the probability of a success or failure outcome in an experiment or survey that is repeated multiple times

$$P(X=x) = \binom{n}{x} p^x q^{n-x}$$

Binomial frequency dist

If the binomial probability distribution is multiplied by N the number of experiments or sets. The resulting distribution is known as the binomial frequency dist

$$N \binom{n}{x} p^x q^{n-x}$$

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QUESTION

Since the C_i 's form a partition of the sample space we can apply the law of total probability for $A \cap B$

$$P(A \cap B) = \sum_{i=1}^M P(A \cap B | C_i) P(C_i)$$

$$P(A \cap B) = \sum_{i=1}^M P(A | C_i) P(B | C_i) P(C_i)$$

$\therefore A$ and B are conditionally independent.

$$P(A \cap B) = \sum_{i=1}^M P(A | C_i) P(B) P(C_i)$$

$\therefore B$ is independent of all C_i 's

$$P(A \cap B) = P(B) \sum_{i=1}^M P(A | C_i) P(C_i)$$

$$P(A \cap B) = P(B) P(A)$$

\therefore Law of total probability
Hence A and B are independent

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QUESTION

Mean and variance of Binomial variables.

The probability function for a binomial random variable is

$$b(x; n, p) = \binom{n}{x} p^x (1-p)^{n-x}$$

This is the probability of having success in a series of independent trials when the probability of success is any one of the trials is p . if x is a random variable with the probability distribution

$$\begin{aligned} E(x) &= \sum_{x=0}^n \binom{n}{x} p^x (1-p)^{n-x} \\ &= \sum_{x=0}^n \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} \\ &= \sum_{x=1}^n \frac{n!}{(x-1)!(n-x)!} p^x (1-p)^{n-x} \end{aligned}$$

Since $x=0$ term Let $y = x-1$

and $m = n-1$ Subbing $n = y+1$ and $n = m+1$ into the last sum

$$\begin{aligned} E(x) &= \sum_{y=0}^m \frac{(m+1)!}{y!(m-y)!} p^{y+1} (1-p)^{m-y} \\ &= (m+1)p \sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y} \end{aligned}$$

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$$= xP \sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

By Binomial theorem

$$(a+b)^m = \sum_{y=0}^m \frac{m!}{y!(m-y)!} a^y b^{m-y}$$

Set $a = p$ and $b = 1-p$

$$\begin{aligned} & \sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y} \\ &= \sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y} \\ &= \sum_{y=0}^m \frac{m!}{y!(m-y)!} a^y b^{m-y} \end{aligned}$$

$$= (a+b)^m$$

$$= (p+1-p)^m$$

$$= 1$$

$$E(x) = np$$

Similarly.

$$y = x-2 \quad \text{and} \quad m = n-2$$

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$$E(x(x-1)) = \sum_{x=0}^n x(x-1) \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^n x(x-1) \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= \sum_{x=2}^n \frac{n!}{(x-2)!(n-x)!} p^x (1-p)^{n-x}$$

$$= n(n-1)p^2 \sum_{m=2}^n \frac{(n-2)!}{(m-2)!(n-m)!} p^{m-2} (1-p)^{n-m}$$

$$= n(n-1)p^2 \sum_{y=0}^m \frac{m!}{y!(m-y)!} (p^y (1-p)^{m-y})$$

$$= n(n-1)p^2 (p + (1-p))^m$$

$$= n(n-1)p^2$$

So the variance of x is
 $E(x^2) - \bar{E}(x^2) = E(x(x-1)) + E(x)$

$$E(x^2) = (n(n-1)p^2 + np) - (np)$$

$$= np(1-p)$$

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QUESTION

When we are rolling two dice there are 36 different combinations. Counting those up

There are 15 possibilities less than 7: (1,1) (1,2) (1,3) (1,4) (1,5)

(2,1) (2,2) (2,3) (2,4) (3,1) (3,2)

(3,3) (4,1) (4,2) (5,1) Probability

of getting less than 7 is $\frac{15}{36} = \frac{5}{12}$

There are 6 possible combinations of getting a 7: (1,6) (2,5) (3,4)

(4,3) (5,2) (6,1) which gives a

prob of

$$\frac{6}{36} = \frac{1}{6}$$

This means that 21 possibilities account for getting less than or equal to 7. This is the same

as the prob of getting less

than 7. So the prob must be $\frac{5}{12}$

as well. Calculating this we must

assume each combination is equal

and therefore the dice are

fair or else the calculation doesn't

work.

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QUESTION
For Data Set A

$$CV = \frac{5}{M} \times 100$$

$$CV = \frac{3}{45} \times 100$$

$$CV = 6.7$$

For Data Set B

$$CV = \frac{11}{60} \times 100$$

$$CV = 18.3$$

For Data Set C

$$CV = \frac{5}{50} \times 100$$

$$CV = 10$$

For Data Set D

$$CV = \frac{15}{25} \times 100$$

$$CV = 60$$

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QUESTION

$B = \{ \text{The sum is even} \}$

$C = \{ \text{The sum is greater than 8} \}$

$D = \{ \text{The two dice had the same outcomes} \}$

$A = \{ (1,6), (2,5), (3,4), (5,2), (6,1), (4,3) \}$

$B = \{ (1,1), (1,3), (1,5), (1,7), (2,2), (2,4), (2,6), (2,8), (3,1), (3,3), (3,5), (3,7), (4,2), (4,4), (4,6), (4,8), (5,1), (5,3), (5,5), (5,7), (6,2), (6,4), (6,6), (6,8), (7,1), (7,3), (7,5), (7,7), (8,2), (8,4), (8,6), (8,8) \}$

$C = \{ (1,8), (2,7), (2,8), (3,6), (3,7), (3,8), (4,5), (4,6), (4,7), (4,8), (5,4), (5,5), (5,6), (5,7), (5,8), (6,3), (6,4), (6,5), (6,6), (6,7), (6,8), (7,2), (7,3), (7,4), (7,5), (7,6), (7,7), (7,8), (8,1), (8,2), (8,3), (8,4), (8,5), (8,6), (8,7), (8,8) \}$

$D = \{ (1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (7,7), (8,8) \}$

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$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)$
 $(1,7), (1,8), (2,1), (2,2), (2,3)$
 $(2,4), (2,5), (2,6), (2,7), (2,8)$
 $(3,1), (3,2), (3,3), (3,4), (3,5)$
 $(3,6), (3,7), (3,8), (4,1), (4,2)$
 $(4,3), (4,4), (4,5), (4,6), (4,7), (4,8)$
 $(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)$
 $(5,7), (5,8), (6,1), (6,2), (6,3)$
 $(6,4), (6,5), (6,6), (6,7), (6,8)$
 $(7,1), (7,2), (7,3), (7,4), (7,5)$
 $(7,6), (7,7), (7,8), (8,1), (8,2), (8,3)$
 $(8,4), (8,5), (8,6), (8,7), (8,8)\}$

det

$A = \{ \text{The sum is 7} \}$

$A \cap B = \{ \}$

$A \cap C = \{ \}$

$A \cap D = \{ \}$

$P(A) = 6/64, P(B) = 32/64$

$P(C) = 36/64, P(D) = 8/64$

(12)

$$P(A \cap B) = 0, P(A \cap C) = 0, P(A \cap D) = 0$$

Hence

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = 0 \times \frac{3^2}{6}$$

$$P(A/B) = 0$$

$$P(A/C) = \frac{P(A \cap C)}{P(C)} = 0 \times \frac{3^6}{64}$$

$$P(A/C) = 0$$

$$P(A/D) = \frac{P(A \cap D)}{P(D)} = 0 \times \frac{8}{64}$$

$$P(A/D) = 0$$