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subject linear Aljebra

final Paper

semester 3rd

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Q. No. 1

Determine if the following system is consistent or not:

$$x_1 - 4x_2 + x_3 = 0$$
$$2x_2 - 8x_3 = 8$$
$$5x_1 - 5x_3 = 10$$

Sol:- $x_1 - 4x^2 + x_3 = 0$
$$2x_2 - 8x_3 = 8$$
$$5x_1 - 5x_3 = 10$$

$$AX = D$$

$$A = \begin{bmatrix} 1 & -4 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & -5 & 10 \end{bmatrix} R_3 - 5R_1$$

$$= \begin{bmatrix} 1 & -4 & 1 & 0 \\ 0 & 2 & -8 & 0 \\ 0 & 45 & -10 & 10 \end{bmatrix} R_3/5$$

$$= \begin{bmatrix} 1 & -4 & 1 & 0 \\ 0 & 2 & -8 & 0 \\ 0 & 9 & -2 & 2 \end{bmatrix} \frac{R_2}{2}$$

$$= \begin{bmatrix} 1 & -4 & 1 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 9 & -2 & 2 \end{bmatrix} R_3 - 9R_2$$

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$$\begin{bmatrix} 1 & -4 & 1 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 0 & 34 & 2 \end{bmatrix} \quad \frac{R_3}{34}$$

$$\begin{bmatrix} 1 & -4 & 1 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 0 & 1 & 17 \end{bmatrix}$$

So,

$$x = 0$$
$$y = 0$$
$$z = 17$$

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Q2

$$\begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 4x-11 \\ 5 & -2 & 7 \end{bmatrix}$$

Sol:

$$B = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 9 \\ 5 & -2 & 7 \end{bmatrix}$$

$$B_{11} = (-)^{1+1} \begin{vmatrix} -1 & 9 \\ -2 & 7 \end{vmatrix} = (-7)-(-2)(9) = 11$$

$$B_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 9 \\ 5 & 7 \end{vmatrix} = (2)(7)-(5)(9)$$
$$= 14 - 45 = -31$$

$$B_{13} = (-1)^{1+3} \begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix} = (2)(-2)-(5)(-1)$$
$$= 1$$

$$B_{21} = (-1)^{2+1} \begin{vmatrix} 4 & 5 \\ -2 & 7 \end{vmatrix} = (4)(7)-(-2)(5)$$
$$= 38$$

$$B_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 5 \\ 5 & 7 \end{vmatrix} = (3)(7)-(5)(5)$$
$$= -4$$

$$B_{23} = (-1)^{2+3} \begin{vmatrix} 3 & 4 \\ 5 & -2 \end{vmatrix} = (3)(-2)-(5)(4)$$
$$= -26$$

$$B_{31} = (-1)^{3+1} \begin{vmatrix} 4 & 5 \\ -1 & 9 \end{vmatrix} = (4)(9) - (-1)(5)$$
$$= 41$$

$$B_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 5 \\ 2 & 9 \end{vmatrix} = (3)(9) - (2)(5)$$
$$= 17$$

$$B_{33} = (-1)^{3+3} \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = (3)(-1) - (2)(4)$$
$$= -11$$

co factor of B =

$$B = \begin{bmatrix} 11 & -31 & 1 \\ 38 & -4 & -26 \\ 41 & 17 & -11 \end{bmatrix}$$

Now take transpose of B

$$B^t = \begin{bmatrix} 11 & 38 & 41 \\ -31 & -4 & 17 \\ 1 & -26 & -11 \end{bmatrix}$$

Ans

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Q. No. 3;
Solve the following ~~equati~~ systems of linear equations by Guass-Jordan Method.

$$2x + 2y + 4z = 18$$
$$x + 3y + 2z = 13$$
$$3x + 2y - 3z = 14$$

Sol:- In matrix form;

$$\left[\begin{array}{ccc:c} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 2 & 9 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{array}\right] ; \frac{R_1}{2}$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 3 & 2 & -3 & 14 \end{array}\right] ; R_2 - R_1$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 0 & -1 & -9 & -13 \end{array}\right] ; R_3 - 3R_1$$

$$\sim \left[\begin{array}{ccc:c} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & -1 & -9 & -13 \end{array}\right] ; \frac{R_2}{2}$$

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$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & -1 & -9 & -13 \end{array}\right] ;\ R_1 - R_2$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -9 & -11 \end{array}\right] ;\ R_3 + R_2$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 11/9 \end{array}\right] ;\ -\frac{R_3}{9}$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 0 & 41/9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 11/9 \end{array}\right] ;\ R_1 - 2R_3$$

So, $x = 41/9$

$y = 2$

$z = 11/9$

Q. No. 4.

Show that is Diagnoisable.

$$A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

Solution:-

Matrix A is diagnoisable if $A = CDC^{-1}$

$\det(A - \lambda I_3) = 0$

$$A - \lambda I_3 = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 4-\lambda & 2 & -2 \\ -5 & 3-\lambda & 2 \\ -2 & 4 & 1-\lambda \end{bmatrix}$$

$\Rightarrow 4-\lambda((3-\lambda)(1-\lambda)-8) - 2(-5(1-\lambda)+4)$
$-2(-20+2(3-\lambda)) = 0$

$\Rightarrow 4-\lambda[3-3\lambda-\lambda+\lambda^2-8] - 2[-5+5\lambda+4]$
$-2[-20+6-2\lambda] = 0$

$\Rightarrow 4-\lambda[\lambda^2-4\lambda-5] - 2[5\lambda-1] - 2$
$[-14-2\lambda] = 0$

$$\Rightarrow 4x^2 + 16x - 20 - x^3 + 4x^2 + 5x - 10x + 2 + 28 + 4x = 0$$

$$\Rightarrow -x^3 + 8x^2 + 15x + 10 = 0$$

$x = 9.65$

$x = -0.82$

$x = -0.829$

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Q.No.5.

$$3x_1 + 5x_2 - 4x_3 = 0$$
$$-3x_1 - 25x_2 + 4x_3 = 0$$
$$6x_1 + x_2 - 8x_3 = 0$$

Sol:-

In matrix form;

$$\begin{bmatrix} 3 & 5 & -4 & 0 \\ -3 & -25 & 4 & 0 \\ 6 & 1 & -8 & 0 \end{bmatrix}$$

We will solve the solution of these equations by Guassian Elimination;

∴ Reduce matrix to Row Echelon form;

i-e,

$$\begin{bmatrix} 3 & 5 & -4 & 0 \\ -3 & -25 & 4 & 0 \\ 6 & 1 & -8 & 0 \end{bmatrix}$$

$R_1 \leftrightarrow R_3$

$$\sim \begin{bmatrix} 6 & 1 & -8 & 0 \\ -3 & -25 & 4 & 0 \\ 3 & 5 & -4 & 0 \end{bmatrix}$$

$R_2 + \frac{1}{2} \cdot R_1 \rightarrow R_2$

$$\sim \begin{bmatrix} 6 & 1 & -8 & 0 \\ 0 & -49/2 & 0 & 0 \\ 3 & 5 & -4 & 0 \end{bmatrix}$$

$R_3 - \frac{1}{2} \cdot R_1 \rightarrow R_3$

$$\sim \begin{bmatrix} 6 & 1 & -8 & 0 \\ 0 & -49/2 & 0 & 0 \\ 0 & 9/2 & 0 & 0 \end{bmatrix}$$

$R_3 + 9/49 \cdot R_2 \rightarrow R_3$

$$\sim \begin{bmatrix} 6 & 1 & -8 & 0 \\ 0 & -49/2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Now,

Reduce Matrix to Reduced Row Echelon form;

$$\begin{bmatrix} 6 & 1 & -8 & 0 \\ 0 & -49/2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$-\frac{2}{49} \cdot R_2 \rightarrow R_2$

$$\sim \begin{bmatrix} 6 & 1 & -8 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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$R_1 - 1 \cdot R_2 \rightarrow R_1$

$$\sim \begin{bmatrix} 6 & 0 & -8 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\frac{1}{6} \cdot R_1 \rightarrow R_1$

$$\sim \begin{bmatrix} 1 & 0 & -4/3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow$ zero Row in Reduced Matrix indicates infinite solutions

i-e:

$$x_1 - \frac{4}{3} x_3 = 0 \quad \Rightarrow x_1 = 0$$

$$\boxed{\begin{aligned} x_2 &= 0 \\ x_3 &= 0 \end{aligned}}$$ substitute

Also;

If we see, the Rank of matrix is 2 and the number of unknowns is system are 3.

So; $2 < 3$. According to condition, the system has non-trivial solution.

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Q#6
$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Ans 6

Maximum Posible Rank for Matrix A is 3 if

$|A| = 0$

$$|A| = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Rank = No of non-zero rows

$$A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

$R_3 - R_1$

$1 - 1 = 0$

$3 - 3 = 0$

$4 - 4 = 0$

$0 - 3 = 3$

$$= \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 4 & 3 \\ 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -3 \end{bmatrix} \quad R_2/3$$

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$$= \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

$R_2 - R_1$

$1-1=0$

$3-3=0$

$4-4=0$

$3-3=0$

Rank = 2