Mid – Term Examinationsummer2020
Date:20/8/2020

Course Code: MTH 101 **Course Title:** Linear Algebra
Prerequisite: NA **Instructor:** HIMAYTULLAH
Module: 1 **Program:** BEE **Total Marks:** 30 **Time Allowed:**

Note: Attempt all questions PLO: program learning outcome C:Cognitive

Q1.	(a)	Let $A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 3 & -1 \\ -2 & 2 \end{bmatrix}$. **Identify** the (3,2) entry of AB.	Marks 5 PLO1 C1
	(b)	**Label** the quadratic polynomial that interpolate the points (1,3), (2,4), (3,4)	Marks 5 PLO1 C1
Q2	(a)	If A and B are $n \times n$ matrices where $\lvert A \rvert = 2$ and $\lvert B \rvert = -3$,calculate $\lvert A^{-1}B^{T} \rvert$.	Marks 5 PLO2 C2
	(b)	**Estimate** the linear system of equation $x + y + 2z = 1$ $x - 2y + z = -5$ $3x + y + z = 3$	Marks 5 PLO2 C2
Q3		Find A^{-1} where $A = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix}$.	Marks 10 PLO2 C2

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Subject : Linear Algebra

Mid Term

Question 1

Part (a)

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 4 \\ 3 & -1 \\ -2 & 2 \end{bmatrix}$$

Sol: A.B

So,

$$\text{row}_3(A) \cdot \text{col}_2(B)$$

$$= \begin{bmatrix} 0 & 1 & -2 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$$

$$= (0)(4) + (1)(-1) + (-2)(2)$$

$$= 0 - 1 - 4$$

$$\boxed{= -5}$$

Part (b)

Interpolate ~~the~~ points

$(1,3), (2,4), (3,4)$

Sol:

As

$$a_2 x_1^2 + a_1 x_1 + a_0 = y_1$$

$$a_2 x_2^2 + a_1 x_2 + a_0 = y_2$$

$$a_2 x_3^2 + a_1 x_3 + a_0 = y_3$$

Now,

$(x_1, y_1) = (1,3)$, $(x_2, y_2) = (2,4)$

$(x_3, y_3) = (3,4)$

put these values in above

$$a_2(1)^2 + a_1(1) + a_0 = 3$$

$$a_2(2)^2 + a_1(2) + a_0 = 4$$

$$a_3(3)^2 + a_1(3) + a_0 = 4$$

$$a_2 + a_1 + a_0 = 3$$

$$4a_2 + 2a_1 + a_0 = 4$$

$$9a_2 + 3a_1 + a_0 = 4$$

$$A_b = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 3 \\ 4 & 2 & 1 & 4 \\ 9 & 3 & 1 & 4 \end{array}\right]$$

$$\sim R \left[\begin{array}{ccc:c} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -8 \\ 0 & -6 & -8 & -23 \end{array}\right] \begin{array}{l} R_2 - 4R_1 \\ R_3 - 9R_1 \end{array}$$

$$\sim R \left[\begin{array}{ccc:c} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -8 \\ 0 & 0 & 1 & 1 \end{array}\right] R_3 - 3R_2$$

So,

$$a_2 + a_1 + a_0 = 3 \quad \text{—— (1)}$$

$$-2a_1 - 3a_0 = -8 \quad \text{—— (2)}$$

~~Correct~~ (5)

$$\boxed{a_0 = 1} \quad \text{—— Put in (2)}$$

$$-2a_1 - 3(1) = -8$$

$$-2a_1 - 3 = -8$$

$$-2a_1 = -8 + 3$$

$$a_1 = \frac{-5}{-2}$$

$$\boxed{a_1 = \frac{5}{2}}$$ put in (1)

$$a_2 + \frac{5}{2} + 1 = 3$$

$$a_2 + \frac{5+2}{2} = 3$$

$$a_2 + \frac{7}{2} = 3$$

$$a_2 = 3 - \frac{7}{2}$$

$$a_2 = \frac{6-7}{2}$$

$$\boxed{a_2 = -\frac{1}{2}}$$

Question 2

Part (a)

Sol

$$|A| = 2, \quad |B| = -3$$

So,

$$|A^{-1} B^t| = |A^{-1}| \cdot |B^t|$$

$$\Rightarrow A^{-1} = \frac{1}{|A|} \cdot |B|$$

$$|B^t| = |B|$$

$$|A^{-1} B^t| = \frac{1}{|A|} |B|$$

$$= \frac{1}{2} \cdot 3$$

$$\boxed{|A^{-1} B^t| = \frac{3}{2}}$$

Question 2(b) (6)

$$x+y+2z=1$$
$$x-2y+z=-5$$
$$3x+y+z=3$$

Sol:

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & -2 & 1 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ 3 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 1 & -2 & 1 & -5 \\ 3 & 1 & 1 & 3 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 3 & 1 & 6 \\ 0 & -7 & 2 & -12 \end{array}\right] \quad R_1 - R_2, \quad 3R_2 - R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 3 & 1 & 6 \\ 0 & 0 & 1 & 6 \end{array}\right] \quad 7R_2 + 3R_3$$

$$\sim \begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 6 \end{bmatrix} \quad \frac{1}{3} R_2$$

$x + y + 2z = 1$ — ①

$y + z = 2$ — ②

$\boxed{z = 6}$ — put in ②

$y + 6 = 2$

$y = 2 - 6$

$\boxed{y = -4}$ — put in ①

$x + (-4) + 2(6) = 1$

~~$x + (-8)$~~ -

$x - 4 + 12 = 1$

$x = 1 + 5 - 12$

$\boxed{x = -6}$

Question 3

(8)

Find A^{-1}

$$A = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix}$$

Sol:

$$|A| = \begin{vmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{vmatrix}$$

$$= 3\begin{vmatrix} 6 & 2 \\ 0 & -3 \end{vmatrix} + 2\begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} + 1\begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix}$$

$$= 3(-18) + 2(-15-2) + 1(0-6)$$

$$= 3(-18) + 2(-17) + 1(-6)$$

$$= -54 - 34 - 6$$

$$= -94$$

$$\boxed{|A| = -94}$$

Now, cofactor of A

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 6 & 2 \\ 0 & -3 \end{vmatrix} = -18$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} = 17$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix} = -6$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 1 \\ 0 & -3 \end{vmatrix} = -6$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 1 \\ 1 & -3 \end{vmatrix} = -10$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 3 & -2 \\ 1 & 0 \end{vmatrix} = -2$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 1 \\ 6 & 2 \end{vmatrix} = -10$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix} = -1$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 3 & -2 \\ 5 & 6 \end{vmatrix} = 28$$

$$\text{adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^t$$

$$\text{adj } A = \begin{bmatrix} -18 & 17 & -6 \\ -6 & -10 & -2 \\ -10 & -1 & 28 \end{bmatrix}^t$$

$$\text{adj } A = \begin{bmatrix} -18 & -6 & -10 \\ 17 & -10 & -1 \\ -6 & -2 & 28 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$A^{-1} = \frac{1}{-94} \begin{bmatrix} 18 & 6 & 10 \\ -17 & 10 & 1 \\ 6 & 2 & -28 \end{bmatrix}$$