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Section / A

Subject / Hydraulic Engineering

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Exam / Final-term

Date / 25/06/2020

QNo1 \Rightarrow A prototype gate valve which will control the flow in a pipe system conveying paraffine is to be studied in a model. List the significant variables on which the pressure drop across the valve would depend. Perform dimensional analysis to obtain the relevant non-dimensional groups.

A $1/5$ scale model is built to determine the pressure drop across the valve with water as the working fluid.

- a) For a particular opening, when the velocity of paraffin in the prototype is 3.0 ms^{-1} . What should be the velocity of water in the model for dynamic similarity?
- b) What is the ratio of the quantities of flow in prototype and model?
- c) Find the pressure drop in the prototype if it is 6 kPa in the model.

(The density and viscosity of paraffin are 800 kg m^{-3} and $0.002 \text{ kg m}^{-1} \text{ s}^{-1}$ respectively. Take the kinematic viscosity of water as $1.0 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$)

The pressure drop Δp is expected to depend upon the gate opening h , the overall depth d , the velocity v , density ρ and viscosity μ .

Solution \Rightarrow

The pressure drop Δp is expected to depend upon the gate opening h , the overall depth d , the velocity v , density ρ and viscosity μ .

List the relevant variables.

$$\Delta p, h, d, v, \rho, \mu.$$

Write down dimensions:

$$\Delta p \quad \text{ML}^{-1}\text{T}^{-2}$$

$$h \quad \text{L}$$

$$d \quad \text{L}$$

$$v \quad \text{LT}^{-1}$$

$$\rho \quad \text{ML}^{-3}$$

$$\mu \quad \text{ML}^{-1}\text{T}^{-1}$$

Number of variables: $n = 6$

Number of independent dimensions: $m = 3$ (M, L and T)

Number of non-dimensional groups: $n - m = 3$

Chose $m (= 3)$ scaling variable

geometric (d): kinematic / time dependent (V); dynamic / mass-dependent (ρ):

Form dimensionless groups by non-dimensionalizing the remaining variable: Δp , h and μ

$$\Pi_1 = \Delta p d^e V^b \rho^c$$

$$M^0 L^0 T^0 = (ML^{-1}T^{-2}) (L)^a (LT^{-1})^b (ML^{-3})^c = M^{1+c} L^{-1+c+b-3c} T^{-2-b}$$

$$M: 0 = 1 + c \Rightarrow c = -1$$

$$T: 0 = -2 - b \Rightarrow b = -2$$

$$L: 0 = -1 + a + b - 3c \Rightarrow a = 1 + 3c - b = 0$$

$$\Pi_2 = \frac{h}{d} \quad (\text{by inspection since } h \text{ is a length})$$

$$\Pi_3 = \mu d^a V^b \rho^c \quad (\text{probably obvious by now, but here goes anyway})$$

$$M^0 L^0 T^0 = (ML^{-1}T^{-1})(L)^a (LT^{-1})^b (ML^3)^c$$

$$M: 0 = -1 + c \Rightarrow c = 1$$

$$T: 0 = -1 - b + 0 \Rightarrow b = -1$$

$$L: 0 = -1 + a + b - 3c \Rightarrow a = 1 + 3c - b = -1$$

$$\Pi_3 = \mu d^{-1} V^{-1} \rho^1 = \frac{\mu}{\rho V d}$$

Recognition of the Reynolds number suggests that we replace

$$\Pi_3 \text{ by } \pi_3 = (\Pi_3)^{-1} = \frac{\rho V d}{\mu}$$

Hence, dimensional analysis yields:

$$\pi_1 = f(\pi_2, \pi_3)$$

i.e.

$$\frac{\Delta P}{\rho V^3} = f\left(\frac{h}{d}, \frac{\rho V d}{\mu}\right)$$

a) Dynamic similarity requires that all non dimensional groups

be the same in model and prototype = i.e.

$$\pi_1 = \left[\frac{\Delta P}{\rho V^3}\right]_p = \left[\frac{\Delta P}{\rho V^3}\right]_m$$

$$\pi_2 = \left[\frac{h}{d}\right]_p = \left[\frac{h}{d}\right]_m \quad (\text{Automatic similar shape is geometric similarity})$$

$$\Pi_3 = \left(\frac{\rho v d}{\mu} \right)_p = \left(\frac{\rho v d}{\mu} \right)_m$$

From the last, we have a velocity ratio:

$$\frac{v_p}{v_m} = \frac{(\mu/\rho)_p}{(\mu/\rho)_m} \frac{d_m}{d_p} = \frac{0.002/800}{1.0 \times 10^{-6}} \times \frac{1}{5} = 0.5$$

Hence

$$v_m = \frac{v_p}{0.5} = \frac{3.0}{0.5} = 6.0 \text{ ms}^{-1}$$

(b) The ratio of the quantities of flow is

$$\frac{Q_p}{Q_m} = \frac{(\text{Velocity} \times \text{area})_p}{(\text{Velocity} \times \text{area})_m} = \frac{v_p}{v_m} \left(\frac{d_p}{d_m} \right)^2 = 0.5 \times 5^2 = 12.5$$

(c) Finally, for the pressure drop:

$$\Pi_1 = \left(\frac{\Delta P}{\rho v^2} \right)_p = \left(\frac{\Delta P}{\rho v^2} \right)_m \Rightarrow \frac{(\Delta P)_p}{\rho v_p^2} = \frac{(\Delta P)_m}{\rho v_m^2}$$

$$\Rightarrow \frac{(\Delta P)_p}{(\Delta P)_m} = \frac{\rho_p}{\rho_m} \left(\frac{v_p}{v_m} \right)^2 = \frac{800}{1000} \times 0.5^2 = 0.2$$

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Hence

$$\Delta p_p = 0.2 \times \Delta p_m = 0.2 \times 60 = 12.0 \text{ kPa}$$

QNO2 \Rightarrow Design a partial profile of gravity dam with the following data:

- 1 \Rightarrow Maximum Depth of water in the reservoir is $78 = R$
- 2 \Rightarrow Specific gravity of dam material is $G = 2.8$
- 3 \Rightarrow Allowable Compressive Strength for the dam masonry 787 Tm^2
- 4 \Rightarrow Height of wave is $H_w = 1.2 \text{ m}$
- 5 \Rightarrow G and H_w is $u = 0.7$

No uplift pressure: $C_u = 0$

Solution

$$1) H_{\text{limiting}} = \frac{G_{all}}{\gamma_w(G - C_u + 1)} = \frac{787 \times 1000}{1000(2.8 - 0 + 1)} = 231.47$$

$$H_{\text{limiting}} = 207.40 \text{ m} \quad 207.40 > H_w = 78 \text{ m}$$

So it is low gravity Dam.

2) Top width 'a'

$$\text{Free board} = 1.5 \text{ h wave} = 1.5 \times 1.2$$

$$\text{FB} = 1.8 \text{ m}$$

$$\text{FB} = 2.4 \text{ m}$$

$$\text{height of dam} = H_D = H_w + F.B = 78 + 2.6$$

~~$$H_D = 78 + 2.6$$~~

$$H_D = 80.4 \text{ m}$$

Now

$$a = 14\% \text{ of } H_D$$

$$a = 0.14 \times 80.4$$

$$a = 11.26 \text{ m}$$

3) Base width 'b' (will out of set)

(i) For No sliding criteria

$$b' = \frac{H_w}{uq} = \frac{78}{0.7 \times 2.8} = 39.8 = b'$$

$$b' \approx 40 \text{ m}$$

(ii) For No tension criteria:

$$b' = \frac{H_w}{\sqrt{q}} = \frac{78}{\sqrt{2.8}}$$

$$b' = 46.61$$

$$b' \approx 47 \text{ m}$$

(4) Depth of vertical portion on u/s side

$$h' = 2a\sqrt{g-cu}$$

$$h' = 2 \times 11.26 \sqrt{2.8-0}$$

$$h' = 37.58 \text{ m}$$

$$h' = 38 \text{ m}$$

5) Upstream off set:

$$\text{Upstream off set} = \frac{9}{16} = \frac{11.26}{16} = 0.70 \text{ m}$$

(6) Depth below the water level to the end of inclined portion in u/s

$$u/s = 3.14a\sqrt{g}$$

$$u/s = 3.14 \times 11.26 \sqrt{2.8}$$

$$u/s = 59.16 \text{ m}$$

7) Total width of the base of the dam.

$$b = b' = \frac{q}{16} = 47 + \frac{11.26}{16}$$

$$b = 47 + 0.70$$

$$b = 47.70 \text{ m}$$

8) $\tan Q = \frac{b'}{H} = \frac{47}{78}$

$$Q = \tan^{-1}(0.602)$$

$$Q = 31.05^\circ$$

9) Depth of vertical portion on D/s (from WL on U/s side)

$$\tan Q = \frac{q}{d'} = \frac{11.26}{d'}$$

$$\tan Q \times d' = 11.26$$

$$\tan(31.05) \times d' = 11.26$$

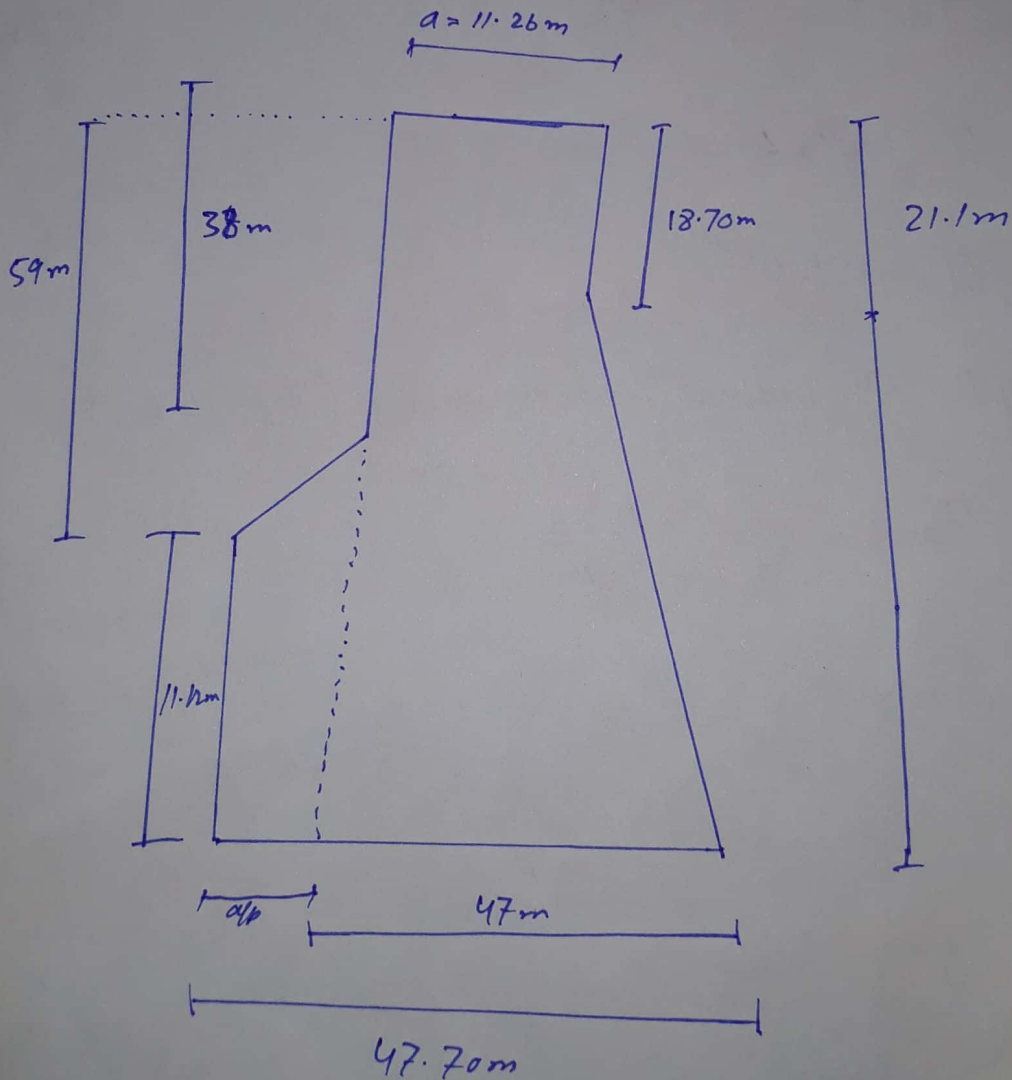
$$d' = \frac{11.26}{0.602}$$

$$d' = 18.70 \text{ m}$$

depth of vertical portion:

$$d = d' + F.B = 18.70 + 2.4$$

$$d = 21.1m$$



Q No 3 ⇒ Using any hydraulic model and explain the concept of Dimensional analysis and Similitude. E

Ans Dimensional Analysis and Similitude Analysis:

1 ▷ Background

- * Although many particle engineering problem involving Hydraulic engineering can be solved by Equation and Analytical procedure, but yet a large number of problems rely on experimental data for their solution.
- * In fact very few problems can be solved by only analytical procedure.
- * In general, solution is obtained through the use of a combination of analysis and experimental data.
- * An obvious goal of any experiment is to make the results widely applicable.

- * To achieve this goal, the concept of Similitude is often used so that measurements made on one system (laboratory) can be used to describe the behavior of other system (out side of laboratory)

Dimensional Analysis

It certain physical phenomenon is governed by

$$f(x_1, x_2, \dots, x_n) = 0 \quad \text{Where some/all of the variable } (x) \text{ are dimensional}$$

Then the above phenomena can be represented as

$$\psi(\pi_1, \pi_2, \dots, \pi_m) = 0 \quad \text{Where all the variable } (\pi) \text{ are non-dimensional}$$

The nature of f and ψ may be obtained from experiment.

Buckingham Pi Theorem

$$f(x_1, x_2, \dots, x_n) = 0 \Rightarrow \psi(\pi_1, \pi_2, \dots, \pi_m) = 0$$

Where some/all x
are dimensional

where all π are
non-dimensional

Where $m < n$, $m = n - k$

Minimum number of fundamental dimensions
involved k

Example

$$f(v, g, h) = 0$$

$$n = 3 \quad k = 2 \quad m = n - k = 3 - 2 = 1$$

Pi Theorem: Repeating and non-repeating variables

$$(x_1, x_2, \dots, x_n) \Rightarrow (x_{r1}, x_{r2}, \dots, x_{rn1}, x_{nr2}, \dots, x_{nrm})$$

Construction of Pi-terms

Selection of repeating variable:-

- * They must be dimensionally independent.
- * Together they must include all fundamental dimensions.

$$\pi_1 = x_{n1} (x_{r1})^{a_{11}} (x_{r2})^{a_{12}} (x_{r3})^{a_{13}} \dots (x_{rk})^{a_{1k}}$$

$$\pi_2 = x_{n2} (x_{r1})^{a_{21}} (x_{r2})^{a_{22}} (x_{r3})^{a_{23}} \dots (x_{rk})^{a_{2k}}$$

$$\pi_m = x_{nm} (x_{r1})^{a_{m1}} (x_{r2})^{a_{m2}} (x_{r3})^{a_{m3}} \dots (x_{rk})^{a_{mk}}$$

Advantages of dimensional analysis

$$f(v, g, h) = 0 \implies Fr = W(Fr/Re)$$

$$Fr = \frac{v}{\sqrt{gh}} \quad \frac{Fr}{Re} = \frac{v}{\sqrt{gh^3}}$$

- o Less number of experiments are necessary as opposed to the dimensional system.
- o Experiments become inexpensive
- o Data reduction becomes easier, single plot is sufficient to show the results.

Conclusion

One figure is enough, as opposed to many figures of 7 dimensional system.

- * Tank size is irrelevant as long as our assumptions aren't violated.

The plot, developed from experiment in a smaller (model) tank may be used for similar bigger (prototype) tank.

Similitude

For the present case study

$$Fr = \omega \left(\frac{v}{\sqrt{gh^3}} \right)$$

Since the relation holds for similar model and prototype tanks

$$\text{if } \left(\frac{v}{\sqrt{gh^3}} \right)_{\text{model}} = \left(\frac{v}{\sqrt{gh^3}} \right)_{\text{prototype}}$$

$$\text{then } (Fr)_{\text{model}} = (Fr)_{\text{prototype}}$$

Model Studies (similitude)

Certain fluid mechanical phenomenon is governed by

$f(\pi_1, \pi_2, \dots, \pi_n)$ where π_i are non-dimensional

When the model is similar to the prototype

$$(\pi_j)_{\text{model}} = (\pi_j)_{\text{prototype}}, \quad j = 1, 2, \dots, n$$

Complete similarity requires -

Geometric similarity + kinematic similarity + Dynamic similarity.

Q No 4 ⇒ What will be the effect of sediment particle diameter, particle density, particle concentration, particle shape, viscosity of water, turbulence of water flowing in reservoir on fall velocity? Explain:

Ans

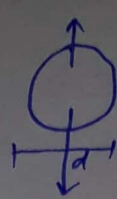
Fall velocity

⇒ When a grain falls down in still water it obtains a constant velocity when the upward fluid drag force on the grain is equal to the weight of the grain. This constant velocity is defined as the fall velocity of the grain.

This is also called settling velocity

Fall velocity depends on

- i) Particle diameter
- ii) Particle density
- iii) Particle concentration
- iv) Particle shape
- v) Viscosity of water (Temperature)
- vi) Turbulence.



F_D - Drag Force

$\downarrow W_s$ = Fall velocity

Submerged weight

The force balance between the drag force and the submerged weight gives

$$F_D = \text{Submerged weight}$$

$$\frac{1}{2} \rho C_D \frac{\pi d^2}{4} w_s^2 = (\rho_s - \rho) g \frac{\pi d^3}{6}$$

$$A = \frac{\pi d^2}{4} = \text{Projected Area}$$

C_D = Drag Coefficient

$$w_s = \text{Fall velocity of Sediment} = \frac{4gd}{3C_D} \left(\frac{\rho_s - \rho}{\rho} \right)$$

ρ = Density of water

ρ_s = Density of Sediment particle.

Particle diameter

\Rightarrow The diameter of the particle is

directly proportional to the fluid velocity, so it

will ~~become~~ ~~tend~~ become greater the size of particle

So, it will tends to the particle of small size thus there will be more gravitational force on particle of gravity size so it will fall quickly due to its weight.

Particle density

⇒ Particle density effect the settling fall velocity. As Air density increase with decreasing altitude at about 1% per 80 meter. (260ft) For every 150 meter of fall the net terminal speed decrease 1%.

Particle Concentration

⇒ When the suspended concentration of sediment increase, the settling velocity of each particle decrease due to the indification of the flow induced by previous particles.

Particle Shape \Rightarrow

Particle having regular shape tends to be affected more than irregular shape since regular shape particles have even surface which offers very little or no friction while particles with smaller surface area are more likely to be affected due to their less resistance.

Viscosity of water \Rightarrow

Fluid velocity through porous media is approximated as inversely proportional to the kinematic viscosity. A decrease in viscosity therefore increases the velocity of a compound through porous media.