(1)

(Question # 01)

Advanced Mechanics of Materials
Student ID # 11345

Solution = The moment at Section A is

$$M = 11000 \times 11 = 121000 \text{ Nm}$$

& the torque on the shaft is

$$T = 11000 \times 0.11 = 1210 \text{ Nm}$$

The normal stress due to M at A is

$$\sigma = \frac{64Md}{2\pi d^4} = \frac{32M}{\pi d^3}$$

maximum Shear stress due to T at A is

$$\tau = \frac{32Td}{2\pi d^4} = \frac{16T}{\pi d^3}$$

The Shear Stress due to Shear force F is zero at A.

$$\sigma_{1,3} = \tfrac{1}{2}\sigma \pm \frac{1}{2}(\sigma^2 + 4\tau^2)^{1/2}$$

(i) Maximum Shear Stress Theory

$$\tau_{max} = \tfrac{1}{2}(\sigma_1 - \sigma_3)$$

$$= \tfrac{1}{2}(\sigma^2 + 4\tau^2)^{1/2}$$

$$= \frac{1}{2}\,\frac{32}{\pi d^3}(M^2 + T^2)^{1/2}$$

$$= \frac{16}{\pi d^3}(121000^2 + 1210^2)^{1/2} = \frac{391072}{d^3} \text{ Pa}$$

with a factor of Safety N = 11 the value of τ_{max} becomes

$$N\tau_{max} = \frac{45264}{d^3} \text{ Pa}$$

This should not exceed the maximum shear stress value at yielding in uniaxial tension test. Thus

$$\frac{1}{d^3}(45264) = \frac{\sigma_y}{2} = \frac{207}{2} \times 10^6$$

$$d^3 = 4812 \times 10^{-6} m^3$$

$$d = 41.25 \times 10^{-2} = 41 cm$$

(ii) Octahedral Shear stress. Theory

$$\tau_{oct} = \frac{1}{3}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]^{1/2}$$

with $\sigma_2 = 0$

$$\tau_{oct} = \frac{1}{3}\left[2\sigma_1^2+2\sigma_3^2-2\sigma_1\sigma_3\right]^{1/2}$$

$$\tau_{oct} = \frac{\sqrt{2}}{3}(\sigma^2+3\tau^2)^{1/2}$$

$$= \frac{\sqrt{2}}{3\pi d^3}\left[(32M)^2+3(16T)^2\right]^{1/2}$$

$$= \frac{16\sqrt{2}}{3\pi d^3}\left(4M^2+3T^2\right)^{1/2}$$

$$= \frac{16\sqrt{2}}{3\pi d^3}\left[4(121000)^2+3(1210)^2\right]^{1/2}$$

$$= \frac{\sqrt{2}}{3\pi d^3} \times 393418 = \frac{\sqrt{2}}{3}\sigma_y$$

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Equating this to octahedral shear stress at yielding of an uniaxial tension bar & using a factor of safety = 11

$$\frac{\sqrt{11}}{3Ad^3} \times 11 \times 343418 = \frac{\sqrt{17}}{3}\sigma_y$$

or $11 \times 343418 = \pi d^3 \sigma_y = \pi d^3 \times 207 \times 10^6$

$d^3 = 3.77 \times 10^{-3}$

$$\boxed{d = 37.7\,cm}$$

Advanced Mechanics of materials

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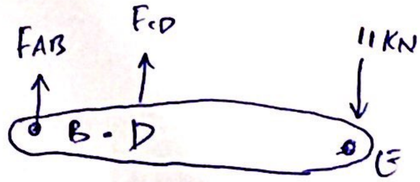
(Question # 02)

Solution, Apply a free body analysis to the Bar BDE to find the forces exerted by links AB & DC

→ Evaluate the deformation of links AB & DC or the displacements of B & D.

→ work out the geometry to find the deflection at E given the deflection at B & D.

(Free Body:)



Bar BDE

$\Sigma M_B = 0$

$0 = -(11KN \times 0.6) + F_{CD} \times 0.2m$

$\boxed{F_{CD} = +33 \quad \text{Tension}}$

$\Sigma M_D = 0$

$0 = -(11KN \times 0.4) - F_{AB} \times 0.2m$

$\boxed{F_{AB} = -22 \quad \text{compression}}$

Displacement of B:

$$\delta_B = \frac{PL}{AE}$$

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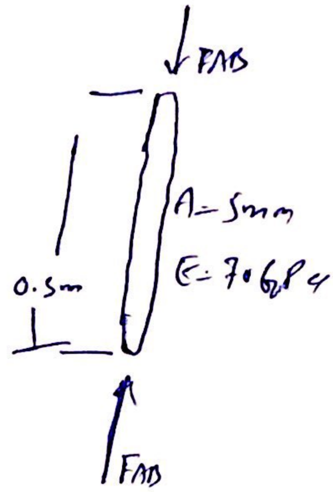
$$= \frac{(-22 \times 10^3 N)(0.3m)}{(5 \times 10^{-6} m^2)(70 \times 10^9 Pa)}$$

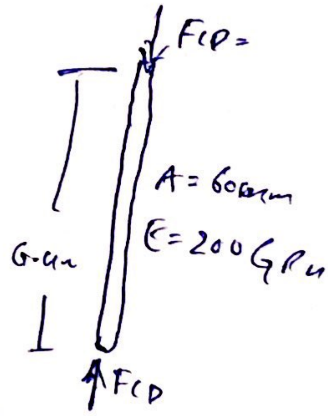


$$= -0.0188 \times 10^{-6} m$$

$$\delta_B = \boxed{0.188 mm}$$

Displacement of D: $\frac{(33 \times 10^3 N)(0.4m)}{(60 \times 10^{-6} m^2)(200 \times 10^9)}$

$$\delta_D = \boxed{1.1 \times 10^{-6}} m$$



Question # 3

Solution:

→ Apply a Static equilibrium analysis on the two Shafts to find a relationship b/w T_{CD} & T_0

→ Apply a Kinematic analysis to relate the angular rotations of the gears.

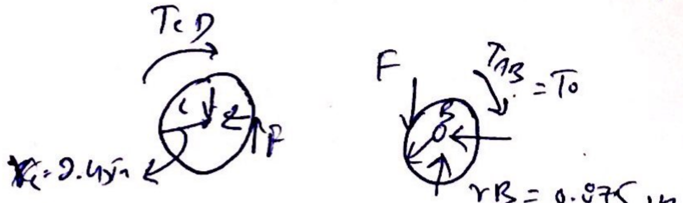
→ Find the maximum allowable torque on each Shaft – Choose the Smallest.

→ Find the corresponding angle of twist for each Shaft & the net angular rotation of end 4.

$G = 11 \times 10^6$ psi

$\rho = 10$ si;

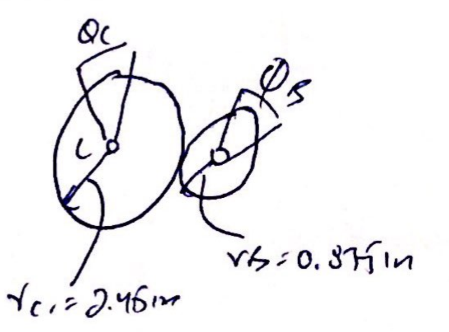
Dimension $d = 11 + 10 = 21$

→ 

$\Sigma M_B = 0 = F(0.875\,in) - T_0$

$\Sigma M_C = 0 = F(2.45\,in) - T_{CD}$

$\boxed{T_{CD} = 2.8\, T_0}$

→ 

$r_B \phi_B = r_C \phi_C$

$\phi_B = \frac{r_C}{r_B} \phi_C = \frac{2.45}{0.875} \phi_C$

$\boxed{\phi_B = 2.8 \phi_C}$

$\tau_{max} = \frac{T_{AB} c}{J_{AB}}$ $\quad 8000 psi = \frac{T_0 (0.375 in)}{\frac{\pi}{2} (0.375 in)^4}$

$T_0 = 663 lb \cdot in$

$\tau_{max} = \frac{T_{CD} c}{J_{CD}}$ $\quad 8000 psi = \frac{2.8 T_0 (0.5 in)}{\frac{\pi}{2} (0.5 in)^4}$

$T_0 = 561 lb \cdot in$ $\quad \boxed{T_0 = 561 lb \cdot in}$

$$\phi_{A/B} = \frac{T_{AB}L}{J_{AB}G} = \frac{(581\,lb.in)(24in)}{\frac{\pi}{2}(0.375in)^4(11.2\times10^6 psi)}$$

$= 0.387\,rad = 2.22^\circ$

$$\phi_{C/D} = \frac{T_{CD}L}{J_{CD}G} = \frac{2.8(581\,lb.in)(24in)}{\frac{\pi}{2}(0.5in)^4(11.2\times10^6 psi)}$$

$= 0.514\,rad = 2.95^\circ$

$\phi_B = 2.8\,\phi_C = 2.8\,(2.95^\circ) = 8.26$

$\phi_A = \phi_B + \phi_{A/B} = 8.26 + 2.22$

$\boxed{\phi_A = 10.48^\circ}$

Question = 04

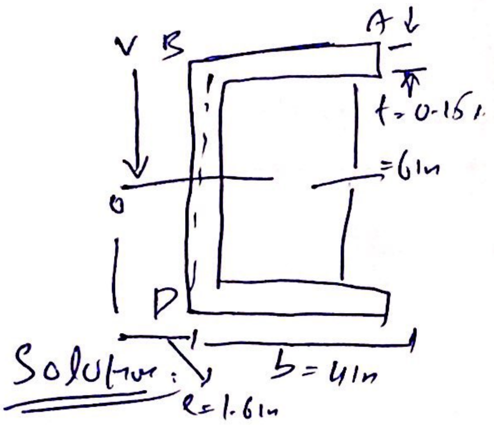
Given Data

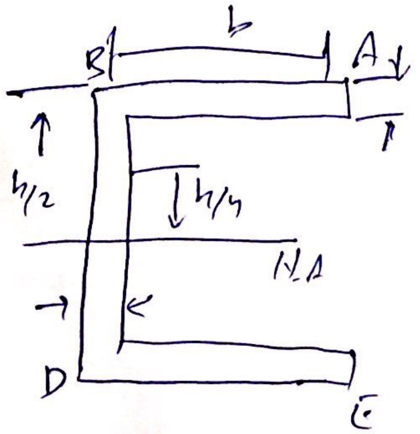
$b = 4$ inch

$h = 6$ inch

$t = 0.15$ in

$V = 11 + 3 = 14$ kips





Solution:

Shear stress distribution for $V = 14$ kips

$$\tau = \frac{q}{t} = \frac{VQ}{It}$$

Shearing stresses in the flanges

$$\tau = \frac{VQ}{It} = \frac{V}{It}(st)\frac{h}{2} = \frac{Vh}{2I}s.$$

$$\tau_B = \frac{Vhb}{2\left(\frac{1}{12}th^2\right)(6b+h)} = \frac{6Vb}{th(6b+h)}$$

$$= \frac{14(2.5\text{ kips})(4\text{ in})}{(0.15\text{ in})(6\text{ in})(6\times4 + 6\text{ in})} = \frac{140}{129.6}$$

τ_B $\boxed{\cancel{\ \ }\text{ ksi}} = \boxed{1.080\text{ ksi}}$

Shearing stresses in the web

$$\tau_{max} = \frac{VQ}{It} = \frac{V(\frac{1}{8}ht)(4b+h)}{\frac{1}{12}th^2(6b+h)t}$$

$$= \frac{3V(4b+h)}{2th(6b+h)}$$

$$= \frac{3(14)(4\times4+6)}{2(0.15)(6in)(6\times6in+6in)}$$

$$= \frac{4032}{388.8}$$

$$\boxed{\tau_{max} = 10.37\,ksi}$$

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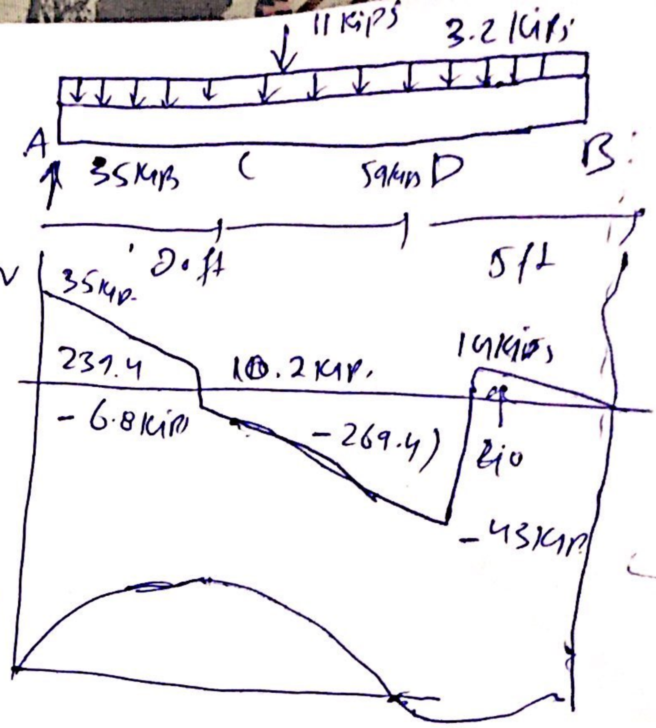
(Question # 05)



Solution:

- Determine reaction at A & D

$\Sigma M_A = 0 \Rightarrow R_D = 59$ kips

$\Sigma M_D = 0 \Rightarrow R_A = 35$ kips

- Determine maximum shear & bending moment from shear & bending moment.

$|M|_{max} = 293$ kip.in with $V = 10.2$ kips

$|V|_{max} = 43$ kips

- Calculate required section modulus & select appropriate beam section

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{24 \text{ kip in}}{24 \text{ ksi}} = \boxed{119.7 \text{ in}^3}$$

Select W21 x 62 beam section

- Find maximum shearing stress

Assuming uniform shearing stress in web,

$$\tau_{max} = \frac{V_{max}}{A_{web}} = \frac{43\,kips}{8.40\,in^2} = 5.12\,ksi < 14.5\,ksi$$

- Find maximum normal stress

$$\sigma_a = \frac{M_{max}}{S} = \frac{2873.60\,kip\cdot in}{127\,in^3} = 22.6\,ksi$$

$$\sigma_b = \sigma_a \frac{y_b}{c} = (22.6\,ksi)\frac{9.88}{10.5} = 21.3\,ksi$$

$$\tau_b = \frac{V}{A_{web}} = \frac{V}{A_{web}} = \frac{12.2\,kips}{8.40\,in^2} = 1.45\,ksi$$

$$\sigma_{max} = \frac{21.3\,ksi}{2} + \sqrt{\left(\frac{21.3\,ksi}{2}\right)^2 + (1.45\,ksi)^2}$$

$$\boxed{= 21.4\,ksi < 24\,ksi}$$