Saad Bin Tariq **ID# 5534**

Numerical Analysis **Assignment# O1**

QNO:-1

Consider the tri-diagonal matrix

$$A = \begin{pmatrix} 4 & 2 & 0 \\ 2 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

Solution:-

$$A = \begin{bmatrix} 4 & 2 & 0 \\ 2 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$
$$\quad A_1 \quad A_2 \quad A_3$$

So

$$A = QR$$

Where:- $Q = \dfrac{Q_1}{(|Q_1|)} \cdot \dfrac{Q_2}{(|Q_2|)} \cdot \dfrac{Q_3}{(|Q_3|)}$

Now:-

$$Q_1 = A_1 \Rightarrow Q_1 = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}$$

$$Q_2 = A_2 - Proj\ Q_1 (Q_1)$$

$$Q_2 = A_2 - \frac{A_2 \cdot A_1}{A_1 \cdot A_1} (A_1)$$

$$= \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} - \frac{\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}}{\begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}} \cdot \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} - \frac{12}{20} \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} - \begin{pmatrix} \frac{12}{5} \\ \frac{12}{10} \\ 0 \end{pmatrix} = \begin{pmatrix} -2/5 \\ 4/5 \\ 1 \end{pmatrix}$$

$$Q_3 = A_3 - \text{Proj } Q_1(Q_1) - \text{proj } Q_2 - Q_2$$

$$= \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - \begin{pmatrix} 12/5 \\ 12/10 \\ 0 \end{pmatrix} - \frac{A_3 \cdot A_2}{A_2 \cdot A_3} (A_2)$$

$$= \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} - \begin{bmatrix} 12/5 \\ 12/10 \\ 0 \end{bmatrix} - \frac{\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}}{\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}} \cdot \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 12/5 \\ 12/10 \\ 0 \end{bmatrix} - \cancel{3}^{1}/\cancel{9}^{3} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 12/5 \\ 12/10 \\ 0 \end{bmatrix} - \begin{bmatrix} 2/3 \\ 2/3 \\ 1/3 \end{bmatrix}$$

SAAD BIN TARIQ 5534 Pg# No. 3

$$Q_3 = \begin{bmatrix} -46/15 \\ -13/15 \\ 0 \end{bmatrix}$$

$$Q_1 = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}, \quad Q_2 \begin{pmatrix} -2/5 \\ 4/5 \\ 1 \end{pmatrix} \qquad Q_3 = \begin{pmatrix} -46/15 \\ -13/15 \\ 0 \end{pmatrix}$$

Find the length of Q_1, Q_2 & Q_3

$$\|Q_1\| = \sqrt{(4)^2 + (2)^2 + (0)^2}$$

$$= 4.47$$

$$\|Q_2\| = \sqrt{\left(-2/5\right)^2 + \left(4/5\right)^2 + (1)^2}$$

$$= 1.34$$

$$\|Q_3\| = \sqrt{\left(\frac{-46}{15}\right)^2 + \left(\frac{-13}{15}\right)^2 + (0)^2}$$

$$= 3.186$$

$$Q = \frac{1}{4.47}\begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix} \cdot \frac{1}{1.34}\begin{bmatrix} -2/5 \\ 4/5 \\ 1 \end{bmatrix} \frac{1}{3.186}\begin{bmatrix} -46/15 \\ -13/15 \\ 0 \end{bmatrix}$$

SAAD BIN TARIQ 5534 Pg # No 4

$$= \begin{bmatrix} \frac{4}{4.47} \\ \frac{2}{4.47} \\ \frac{0}{4.47} \end{bmatrix} \cdot \begin{bmatrix} \frac{-2/5}{1.34} \\ \frac{4/5}{1.34} \\ \frac{1}{1.34} \end{bmatrix} \begin{bmatrix} \frac{-46/15}{3.186} \\ \frac{-13/15}{3.186} \\ \frac{0}{3.186} \end{bmatrix}$$

$$\begin{bmatrix} 0.89 & -0.29 & -0.96 \\ 0.44 & 0.59 & -0.27 \\ 0 & 0.74 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0.89 & 0.44 & 0 \\ -0.29 & 0.59 & 0.74 \\ -0.96 & -0.27 & 0 \end{bmatrix}^T$$

$R = AQ^T$

$$R = \begin{bmatrix} 4 & 2 & 0 \\ 2 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0.89 & 0.44 & 0 \\ -0.29 & 0.59 & 0.74 \\ -0.96 & -0.27 & 0 \end{bmatrix}$$

$$R = \begin{bmatrix} 2.94 & 2.94 & 1.48 \\ 0.22 & 1.79 & 1.48 \\ -1.25 & 0.32 & 1.48 \end{bmatrix}$$

QNo: 2

Ans:-

Point $\frac{\pi}{4}$ for the function $f(x) = \sin(x)$

The formula for the tylor Series of degree n at at a point for any function is as fellow

$$f(x) = f(0) + \frac{f(a)}{1!}(x-0)^1 + \frac{f^2(a)}{2!}(x-a)^2 + \frac{f^3(a)}{3!}(x-a)^3 + \frac{f^4(a)}{4!}(x-a)^4 + \frac{f^4(a)}{5!}(x-a)^5 \cdots\cdots \frac{f^n(a)}{n!}(x-a)^n$$

Differentiating the function $\sin(x)$ 5 time and values of each derivatives at $\pi/4$

$f(x) = \sin(x)$	$f(\pi/4) = \frac{1}{\sqrt{2}}$
$f'(x) = \cos(x)$	$f'(\pi/4) = \frac{1}{\sqrt{2}}$
$f^2(x) = -\sin(x)$	$f^2(\pi/4) = \frac{-1}{\sqrt{2}}$
$f^3(x) = -\cos(x)$	$f^3(\pi/4) = -1/\sqrt{2}$
$f^4(x) = \sin(x)$	$f^4(\pi/4) - 1/\sqrt{2}$
$f^5(x) = \cos(x)$	$f^5(\pi/4) = 1/\sqrt{2}.$

$$f_5(x) = f(\pi/4) + \frac{f'(\pi/4)}{1!}(x - \pi/4) + \frac{f^2(\pi/4)}{2!}(x-\pi/4)^2$$

SAAD BIN TARIQ 5534 Pg#No16

$$+\frac{f_3(\pi/4)}{3!}(x-\pi/4)^3+\frac{f^4(\pi/4)}{4!}(x-\pi/4)^4+\frac{f^5(\pi/4)}{5!}(x-\pi/4)^5$$

So the taylor polynomial of degree is at point $\pi/4$ for $\sin(x)$ is

$$T_5(x)=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}\times 1!}(x-\pi/4)+\frac{-1}{\sqrt{2}\times 2!}(x-\pi/4)^2$$
$$+\frac{-1}{\sqrt{2}\times 3!}(x-\pi/4)^3+\frac{1}{\sqrt{2}\times 4!}(x-\pi/4)^4+\frac{1}{\sqrt{2}\times 5!}(x-\pi/4)^5$$

Simplifies

$$T_5(x)=\frac{1}{\sqrt{2}}+\frac{(x-\pi/4)}{\sqrt{2}}-\frac{(x-\pi/4)^2}{2\sqrt{2}}-\frac{(x-\pi/4)^3}{6\sqrt{2}}$$
$$+\frac{(x-\pi/4)^4}{24\sqrt{2}}+\frac{(x-\pi/4)}{120\sqrt{2}}$$