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DEPARMENT: SOFTWARE ENGG.

PAPER: DIFFERENTIAL EQUATION

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Question : 1 ;

Define 2nd order linear homogenous/non-homogenous D.E along with example.

(A) HOMOGENOUS D.E :-

The one which is all the terms involving the unknown function are collected together on side of equation, the other side is 0.

*) EXAMPLE :-

$y''. 2y_1 + y = 0$ is homogenous —— (1)

But $y''. 2y^1 + y = x$ — (1) is non-homogenous. equation (2) can be converted in homogenous equation by replacing hand side by 0.

$$a(x)y'' + b(x)y' + c(x)y = 0.$$

*) NON-HOMOGENOUS DIFFERENTIAL EQUATION :-

The non-homogenous differential equation has type form of.

$$y'' + Py' + qy = f(a).$$

Where p and q are constant for each equation, we can relate homogenous & complementary equation.

The general solution of non-homogenous equation is sum of general solution related to particular solution.

$$y(x) = y_0(x) + y_1(x)$$

*) EXAMPLE :-

① $y'' + y = \sin 2x$

② $y'' + y' - 6y = 36x$

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PART : B :

(1) $4y'' - 6y' + 7y = 0$

Sol:-

$$4y'' - 6y' + 7y = 0$$

A second order linear homogenous ODE has form of $ay'' + by' + cy = 0$

for equation $ay'' + by' + cy = 0$,

assume e^{rt}

$$y = e^{rt}$$

$$4((e^{rt}))'' - 6((e^{rt}))' + 7e^{rt} = 0.$$

$$e^{rt}(4y^2 - 6y + 7) = 0.$$

$e^{rt}(4y^2 - 6y + 7) = 0$, $r = \frac{3}{4} + i\frac{\sqrt{19}}{4}$, $r = \frac{3}{4} - i\frac{\sqrt{19}}{4}$.

$$r = \frac{3}{4} + i\frac{\sqrt{19}}{4}, \quad r = \frac{3}{4} - i\frac{\sqrt{19}}{4}.$$

For Two complex root

$$r \neq 1/2$$

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$$e^{3/4}\left(c_1 \cos \frac{\sqrt{19}}{4} x\right) + c_2 \sin\left(\frac{\sqrt{19}}{4} r\right)$$

Solution is

$$y = e^{3t/4}\left(c_1 \cos \frac{\sqrt{19}}{4} t\right) + c_2 \sin\left(\frac{\sqrt{19}}{4}\right)$$

(iii) $y'' - 4y' - 12y = 3e^{5t}$

Sol:-

$$y'' - 4y' - 12y = 0$$

characteristics roots are.

$$r^2 - 4r - 12 = (r-6)(r+2) = 0.$$

$$\Rightarrow \quad r_1 = -2, \quad r_2 = 6.$$

$$y_e(t) = c_1 e^{-2t} + c_2 e^{6t}$$

$$y_p(t) = Ae^{5t}$$

$$25Ae^{5t} - 4(5Ae^{5t}) - 12(Ae^{5t})$$

$$= 3e^{5t} - 7Ae^{5t} = 3e^{5t}.$$

$$-7A = 3, \quad A = -3/7.$$

Solution:

$$y_p(t) = -\frac{3}{7}e^{5t}.$$

Q2 part 1

The characteristic solution is.

$16r^2 - 40r + 25 = (4r-5)^2 = 0, r_1, 2 = \frac{5}{4}.$

The general solution.

$16r^2 - 40r + 25 = (4r-5)^2 = 0, r_1, 2 = 5/4.$

The general derivation is

$y(t) = c_1 e^{5t/4} + c_2 t e^{5t/4}$

$y'(6) = \frac{5}{4} c_1 e^{5t/4} + c_2 e^{5t/4} + e^{5t/4}$

$3 = y(0) = c_1.$

$-\frac{9}{4} = y'(0) = \frac{5}{4} c_1 + c_2.$

$c_1 = 3, \quad c_2 = -6.$

$y(t) = 3e^{5t/4} - 6te^{5t/4}.$ Ans.

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(ii) $y'' + 14y' + 49y = 0, \quad y(-4) = -1, \; y'(-4) = 5.$

Sol:-

$r^2 + 14r + 49 = (r+7)^2 = 0, \quad r_1, r_2 = -7.$

$y(6) = c_1 e^{-7t} + c_2 t e^{-7t}.$

$y'(6) = -7c_1 e^{-7t} + c_2 e^{-7t} - 7c_2 e^{-7t}$

$-1 = y(-4)\; c_1 e^{28} - 4c_2 e^{28}$

$5 = y_1(-4) = 7c_1 e^{28} + c_2 e^{28} + 28c_2 e^{28} =$

$-7c_1 e^{28} + 29c_2 e^{28}.$

$c_1 = -9e^{-28}$

(iii) $y'' - 4y' + 9y = 0, \quad y(0) = 0, y'(0) = -8.$

Sol:-

$r^2 - 4 + 9 = (r-2)^2 = 0, \; r_1, r_2 = 2.$

$y(6) = c_1 e^{2t} + c_2 t e^{2t}.$

$y'(6) = 2c_1 e^{2t} + c_2 e^{2t} + 2c_2 t e^{2t}.$

$12 = y(0) = c_1$

$-3 = y_1(0) = 2c_1 + c_2$

$c_1 = 12, \quad c_2 = -27.$

$y(t) = 12e^{t} - 27te^{2t}.$

(iv) $y'' - 8y' + 17y = 0$, $y(0) = -4$ $y(0) = -1$.

$r_1 = r_2 = 4$.

$y(x) = c_1 e^{4x} + c_2 e^{4x} x$.

$y(t) = c_1 e^{4t} + c_2 t e^{4t}$, $\frac{d}{dt}\left[te^{4t}\right]$.

$= 4te^{4t} + e^{4t}$.

$y'(t) = 4c_1 e^{4t} + 4c_2 t e^{4t} + c_2 e^{4t}$.

$-2 = c_1 e^{4}(0) + c_2(0)e^{4}(0)$ $-2 = c_1$, $c_1 = -2$

$-\frac{22}{3} + 4e c_1 e^{4}(0) + 4c_1(0)e^{4}(0) + c_2 e^{4}(0)$

$-\frac{22}{3} = 4(-2) + c_2$, $c_2 = 1/3$.

Solution is

$y(x) = \frac{1}{3} e^{4x} (x - 6)$.

Ans.

Question : 3 :-

Define Laplace form? with example.

*) LAPLACE FORM:-

It's a technique for solving D.E, Here D.E or first domain is transformed in algebric equation of frequency domain form.

$$\mathcal{L}[g(t) = 1.$$

$$\mathcal{L}[4(t) = \frac{1}{3}$$

and so on.

$$F(s) = \int_0^{8} F(t) \cdot e^{-8t} dt.$$

*) Example:-

1. $$\int_0^{8} e^{ct} \cdot dt$$

2. $$\mathcal{L}\{(\sin(at)\}.$$

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(A)

(1) $f(t) = 6(e^{-5t}) + e^{3t} + 5(t^3) - 9.$

Sol:-

$$F(s) = 6\frac{1}{s-(-5)} + \frac{1}{s-3} + 5\frac{3!}{s^{3+1}} - 9\frac{1}{s}$$

$$= \frac{6}{s+5} + \frac{1}{s-3} + \frac{30}{s^4} - \frac{9}{s}.$$

(2) $g(t) = 4\cos(4t) - 9\sin(4t) + 2\cos(10t).$

Sol:-

$$G(s) = 4\frac{s}{s^2+(4)^2} - 9\frac{4}{s^2+(4)^2} + 2\frac{s}{s^2+(10)}$$

$$= \frac{4s}{s^2+16} - \frac{36}{s^2+16} + \frac{2s}{s^2+10}.$$

(3) $h(t) = e^{3t} + \cos(6t) - e^{\wedge 3t}\cos 6t.$

$$H(s) = 3\frac{2}{s^2-(2)^2} + \frac{3(2)}{s^2+(2)^2}$$

$$= \frac{6}{s^2-4} + \frac{6}{s^2+4}$$

Question : 4 :-

(1) $y'' - 10y' + 9y = 5t \qquad y(0) = 1, \; y'(0) = 2$.

Solution :-

first convert into D.E.

$$\mathcal{L}\{y''\} - 10\mathcal{L}\{y''\} + 9\mathcal{L}\{y'\} = \mathcal{L}\{5t\}.$$

$$s^2 y(s) - 8y(0) - y'(0) - 10(sy(s) - y(10)) + 9y(3) = \frac{5}{s^2}$$

$$(8^2 - 10s + 9)\, Y(s) + 5 - 12 = \frac{5}{s^2}$$

$$Y(s) = \frac{5}{s^2(s-9)(s-1)} + \frac{12-5}{(s-9)(0-1)}$$

Combining terms

$$Y(s) = \frac{5 + 12s^2 - s^2}{s^2(s-9)(s-1)}$$

$$Y(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-9} + \frac{D}{s-1}.$$

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$$5 + 12s^2 - s^3 = As(s-9)(s-1) + B(s-9)(s-1)$$
$$+ C^2 s(s-1) + Ds^2(s-9).$$

$s = 0$ $\quad 5 = 9B$

$s = 1$ $\quad 16 = -8D.$

$s = 9$ $\quad 24B = 648C$

$s = 2$ $\quad 45 = -14A + \frac{4345}{81}.$

$$B = \frac{5}{9} = \frac{50}{81/s} + \frac{3/9}{s^2} + \frac{\frac{31}{81}}{s-9} - \frac{2}{s-1}.$$

Solution is.

$$y(t) = \frac{50}{81} + \frac{5}{9}t + \frac{31e^{9t}}{81} - 2et.$$

(ii) $y'' - 6y' + 15y = 2\sin(3t)$, $y(0) = -1$, $y'(0) = 4$.

Sol:-

$$s^2 Y(s) - s y(0) - 6\left(sY(s) - y(0)\right) +$$

$$15Y(s) = 2\,\frac{3}{s^2+9}.$$

$$(s^2 - 6s + 15)\,Y(s) + s - 2 = \frac{6}{s^2+9}.$$

$$Y(s) = \frac{-s^3 + 2s^2 - 9s + 24}{(s^2+9)(s^2-6s+15)}.$$

$$Y(s) = \frac{As+B}{s^2+9} + \frac{Cs+D}{s^2-6s+15}.$$

$$-s^3 + 2s^2 - 9s + 24 = (As+D)(s^2-6s+15)$$
$$+ (Cs+D)(s^2+9)$$

$$= (A+C)s^3 + -(6A+B+D)s^2 + (15A-6B+9C)$$
$$+ 15B + 9D.$$

s^3 : $A + C = -1$.

s^2 : $-6A + B + D = 2$

s^1 : $15A - 6B + 9C = -9$

s^0 : $15B + 9D = 24$

$A = 1/10$

$B = -11/10$

$C = -1/10$

$D = 5/2$

$$Y_s = \frac{1}{10}\left(\frac{s+1}{s^2+9} + \frac{-11s+25}{s^2-6s+15}\right)$$

$$= \frac{1}{10}\left(\frac{s+1}{s+9} + \frac{-11(s-3+3)+25}{(s-3)^2+6}\right)$$

$$= \frac{1}{10}\left(\frac{s}{s^2+9} + \frac{13/3}{s^2+9} \quad \frac{-11(s-3)}{(s-3)^2+6} - \frac{\frac{8\sqrt{6}}{\sqrt{6}}}{(s-3)^2+3}\right.$$

Solution is .

$$y(6) = \frac{1}{10}\Big(\cos 3t + \frac{1}{3}\sin 3t - 11e^{3t}\cos(\sqrt{6}t) - \frac{8}{\sqrt{6}}\, \cancel{\cos}\, e^{3t}\sin(\sqrt{6}t)\Big).$$

Answer.