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**Department: BE(E)**

**Subject: Electromagnetic Field (EMF)**

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**Assignment:**

**Solve problem 4.1, 4.2, 4.3, 4.5, and 4.7 of course book.**



4.1:-

The value of  $E$  at  $P(P=2, \phi=4, z=3)$  is given by  $E = 100\hat{p} - 200\hat{\phi} + 300\hat{z}$  V/m. Determine the incremental work require to move a  $20 \mu\text{C}$  charge a distance of  $6 \mu\text{m}$ :-

(a):-

In the direction of  $\hat{p}$ :-

The incremental work is given by  $dw = -qE dl$ , where in this case  $dl = dp \hat{p} = 6 \times 10^{-6}$

$$dw = -(20 \times 10^{-6} \text{ C}) (100 \text{ V/m}) (6 \times 10^{-6} \text{ m}) = -12 \times 10^{-9} \text{ J} = \boxed{-12 \text{ nJ}}$$

(b):-

In the direction of  $2\hat{\phi}$ :-

In this case  $dl = 2d\phi \hat{\phi} = 6 \times 10^{-6} \hat{\phi}$  and so  $dw = -(20 \times 10^{-6}) (-200) (6 \times 10^{-6}) = 2.4 \times 10^{-8} \text{ J}$

$$= \boxed{24 \text{ nJ}}$$

(c):-

In the direction of  $\hat{z}$ :-

Here  $dl = dz \hat{z} = 6 \times 10^{-6} \hat{z}$  if  $\hat{z}_0$   
 $dw = -(20 \times 10^{-6}) (300) (6 \times 10^{-6}) = -3.8 \times 10^{-8} \text{ J}$

$$= \boxed{36 \text{ nJ}}$$

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(d):

In the direction of  $E$ :

$$\text{Here } dL = 6 \times 10^{-6} aE$$

$$\text{where } \nabla E = 100ap - 200a\phi + 300az =$$

$$0.267ap$$

$$[100^2 + 200^2 + 300^2]^{1/2}$$

$$0.5352\phi + 0.802az$$

Thus:

$$dw = -(20 \times 10^{-6})(100ap - 200a\phi - 300az) \cdot$$

$$[0.267ap - 0.802az] [6 \times 10^{-6}] \cdot$$

$$= -6 [-44.9 \text{ J}]$$

(e):

In the direction of  $G = 22x - 32y + 40z$ :

In this case  $dL = 6 \times 10^{-6} dG$

where

$$\nabla G = \frac{22x - 32y + 40z}{(2^2 + 3^2 + 4^2)^{1/2}}$$

$$= 0.3712 - 0.557ay + 0.743az$$

$\int_0^7$

$$dw = -(20 \times 10^{-6}) [100ap - 200a\phi + 300az] \cdot$$

$$[6 \cdot 371ax - 0.557ay + 0.743az] [6 \times 10^{-6}]$$

$$= -(20 \times 10^{-6})$$

$$[37.1(ap \cdot ax) - 55.7(ap \cdot ay) - 74.2(20z)]$$

$$+ 111.4(a\phi \cdot ay) + 222.9 [6 \times 10^{-6}]$$

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4.2:-

Let

$$E = 400ax - 300ay - 500az$$

in the neighborhood of point  $P(6, 2, 3)$

Find the incremental work done in moving a 4-C charge a distance of 1mm in the direction specified by

(a):-

$$ax + 2y + 2z :$$

$$dw = -q_e E \cdot dl = -4(400ax - 300ay + 500az)$$

$$\left[ \frac{(ax + ay + az)}{\sqrt{3}} (10^{-3}) \right]$$

$$= -\frac{(4 \times 10^{-3}) (400 - 300 - 500)}{\sqrt{3}}$$

$$= \boxed{-1.39 \text{ J}}$$

(b) -2ax + 3ay - az :

The computation is similar to that of part (a) but we change the direction

$$dw = -q_e E \cdot dl = -4(400ax - 300ay - 500az)$$

$$= \frac{(-2ax + 3ay - az) (10^{-3})}{\sqrt{14}}$$

$$= -\frac{(4 \times 10^{-3}) (-800 - 900 - 500)}{\sqrt{14}} = \boxed{2.35 \text{ J}}$$

4.3

If  $E = 120 \text{ zp v/m}$ . Find the incremental amount of work done in moving a  $50 \mu\text{m}$  charge a distance of  $2 \text{ mm}$  from:

$P(1, 2, 3)$  toward  $Q(2, 1, 4)$ . The vector along this direction will be  $Q - P = (1, -1, 1)$   
from  $dw = -q_e E dl = -(50 \times 10^{-6}) [120 \text{ ap} (\text{ax} - \text{ay} + \text{az})]$   
 $\sqrt{3}$

$$= (50 \times 10^{-6}) (120) [\text{ap} - 2\text{az}] \frac{1}{\sqrt{3}} (2 \times 10^{-3})$$

At  $P, \theta = \tan^{-1}(2/1) = 63.4^\circ$  Thus  $\text{ap} \cdot \text{ax} = \cos(63.4) = 0.447$  &  $\text{ap} \cdot \text{ay} = \sin(63.4) = 0.894$  Substituting these we obtained

$$dw = 3.1 \mu\text{J}$$

4.5:-

Compute the value of  $\int_A^P G \cdot dl$  for  $G = 2y \text{ ax}$  with  $A(1, -1, 2)$  &  $P(2, 1, 2)$  using the path.

(a) Straight line segment  $A(1, -1, 2)$  to  $P(2, 1, 2)$   
 $\int_A^P G \cdot dl = \int_1^2 2y dx$

The change in  $x$  occurs when moving between band  $P$  during which  $y=1$

Thus

$$\int_A^P G \cdot dL = \int_A^P 2y dx = \int_1^2 2(1) dx = 2$$

(b)

Straight line segments  $A(1, -1, 2)$  to  $C(2, -1, 2)$  to  $P(2, 1, 2)$  in this case the change change in  $x$  occurs.

$$\int_A^P G \cdot dL = \int_A^C 2y dx = \int_1^2 2(-1) dx = -2$$

14.7 :-

Repeat problem 4.6 for  $G = 3xy^2 \mathbf{a}_x + 2xy \mathbf{a}_y$  Now think are different in that the path does matter

(a) Straight line:

$y = x - 1, z = 1$  we get

$$\int G \cdot dL = \int_1^4 3xy^2 dx + \int_1^3 3x(x-1)^2 dx +$$

$$\int_1^3 2(1) dy = 90$$

(b) Parabolic by  $y = x^2 + 2, z = 1$  we get

$$\int G \cdot dL = \int_2^4 3xy^2 dx + \int_1^3 2x dy = \int_2^4 3x(x^2+2)^2 dx +$$

$$\int_1^3 2(1) dy = 80 + 2 = 82.$$