Name # M. IKhlas Khan

ID # 7768

Section # B

Subject # Intro to structural Dynamics & Earthquake Engg

Teacher name # Engr- Yaseen Mahmood.

Q No-1

Given Data

$E = 29{,}000 \text{ Ksi}$

$I = 150 \text{ in}^4$

δ_{st} = Deflection due to 7768 lb Static load.

Beam is Pulled for # 1/2"

Required

Natural time Period of system

Develop & solve the equation of motion

Draw graphs to slow the variation of displacement with time & the variation of equivalent static forces with.

Sol :-

General EOM for SDOF system

is $Ku + C\dot{u} + m\ddot{u} = P(t)$

Since system is undamped ($c=0$) under going for vibration $p(t) = 0$

Hence general EOM becomes

$$Ku + m\ddot{v} = 0 \quad \text{—(1)}$$

$$K = \frac{3EI}{L^3} \Rightarrow \frac{3 \times 29000\ K/in^2 \times 150\ in^4}{(10 \times 12 in)^3}$$

$K = 7.55\ K/in$ $\quad K = 7.55208$

⇒ In order to eliminaite chances of mistake during calculation, it is more appropriade to use fundamental units like lb, ft sec or kg, m sec

$$K = 7.55208\ K/in \Rightarrow 90625\ lb/ft$$

$$m = W/g \Rightarrow \frac{7768}{32.2} = 241.24\ slug$$

$$W_n = \sqrt{K/m} \Rightarrow \sqrt{\frac{90625}{241.24}}$$

$$W_n = 19.382\ rad/sec$$

Scanned with CamScanner

$T_n = \frac{2\pi}{W_n} \Rightarrow \frac{2\pi}{19.382}$

$T_n = 0.324$ Sec

Put m & K in eq (1)

$90625u + 241.24\ddot{u} = 0$

where K is in lb/ft

$\Rightarrow$ General Solution to EOM for undamped free vibration is

$u(t) = u(0)\cos(W_n t + \frac{\dot{u}(0)}{W_n}\sin(W_n t)$

$u(0) = 1/2'' = 1/24$ ft & $\dot{u}(0) = 0$

$u(t) = (1/24) \times \cos(19.382t) + 0 = (1/24) \times \cos(19.382t)$

Equivalent static force at any time "t" is

$f_s(t) = K \cdot u(t) = \frac{90625 \times \cos(19.382t)}{24}$

$= 3776\cos(19.382t)$

Amplitude of dynamic displacement u_o for undamped free vibration is

$$u_o = \sqrt{[u(0)]^2 + \left(\frac{\dot{u}(0)}{\omega_n}\right)^2}$$

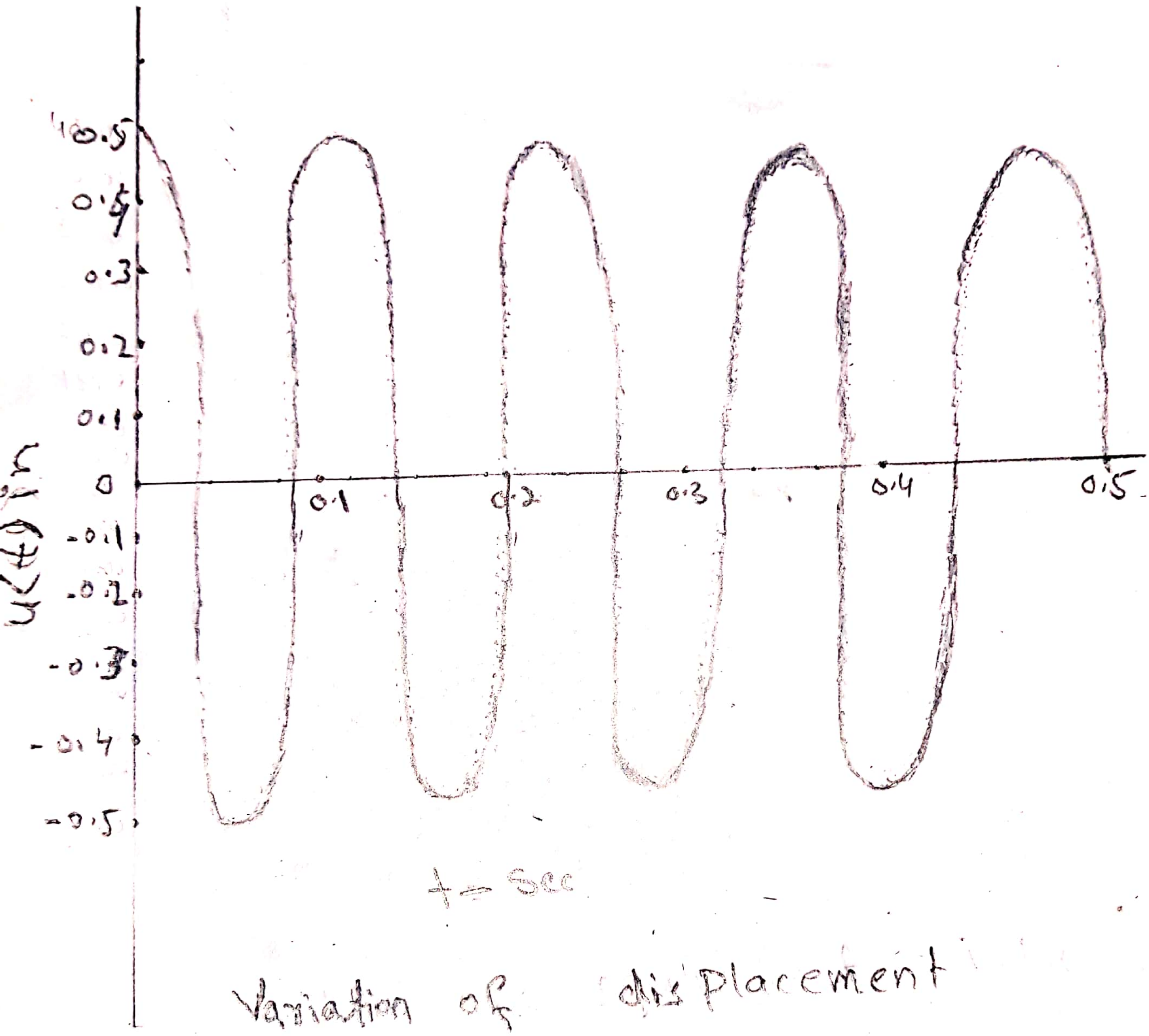
$$= \sqrt{[1/24]^2 + 0}$$

$$= 1/24 \text{ ft}$$

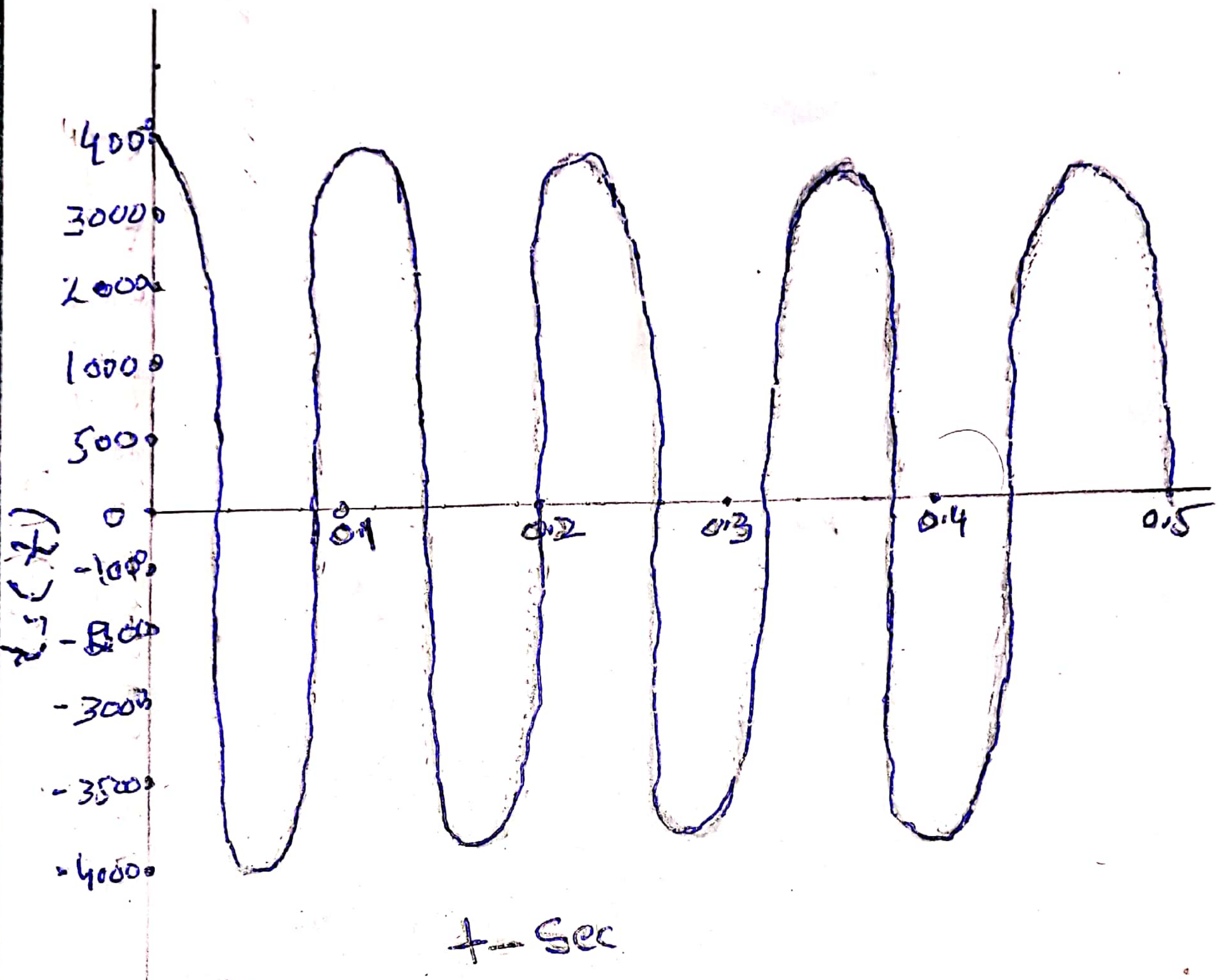
Amplitude of equivalent static force, f_{so}

$$Ku_o = 90625 \times 1/24$$

$$Ku_o = 3776$$



Variation of displacement
with time



Variation of equivalent static forces with time

Q No 02

Given data

δ (Damping ratio) of Reinforced concrete with considerable cracking = 3-5%
= 3%

Using Data of beam given in Question # 1

Required

⇒ Develope & Solve the equation showing variation in Equivalent Static force with time

⇒ Draw greaph to show the variation of displacement with & the variation of equalent static force with time.

Sol

EOM for damped free

②

Vibration is

$$Ku + C\dot{u} + m\ddot{u} = 0 \quad \text{--- (1)}$$

from Question (1)

$K = 90625$ lb/ft & $m = 241.24$ lb sec²/ft

$\omega_n = 19.382$ rad/sec

$C = \xi \times 2m\omega_n$

$C = (0.03) \times 2(241.24)(19.382)$

$C = 280.54$ sec/ft

Put values in eq (1)

$$90625 + 280.54\dot{u} + 241.24\ddot{u} = 0$$

Solution to the EOM for damped free vibration is

$$u(t) = e^{-\xi\omega_n t}\left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left\{\dot{u}(0) + u(0)\xi\omega_n\right\}\sin\omega_n t\right]$$

$\omega_D = 19.382$ rad/sec

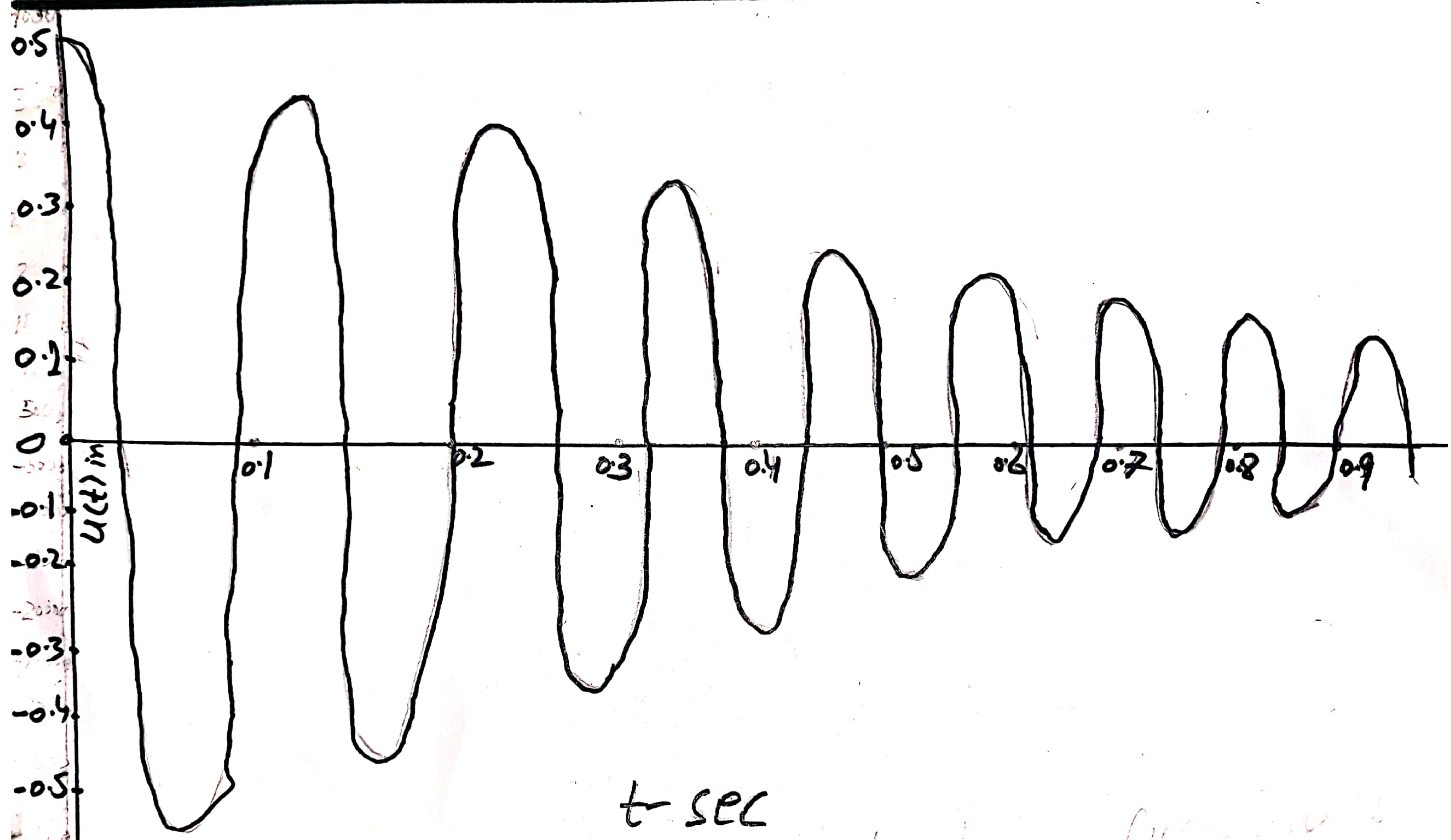
$$u(t) = e^{-0.03 \times 19.382} \left[\frac{1}{24} \times \cos(19.382t) + \frac{1}{19.382} \times \left[0 + \frac{1}{24} \times 0.03 \times 19.382 \times \sin(19.382t) \right] \right]$$

$$u(t) = e^{-0.581t} \left[0.041 \times \cos(19.382t) + 0.00125 \times \sin(19.382t) \right]$$

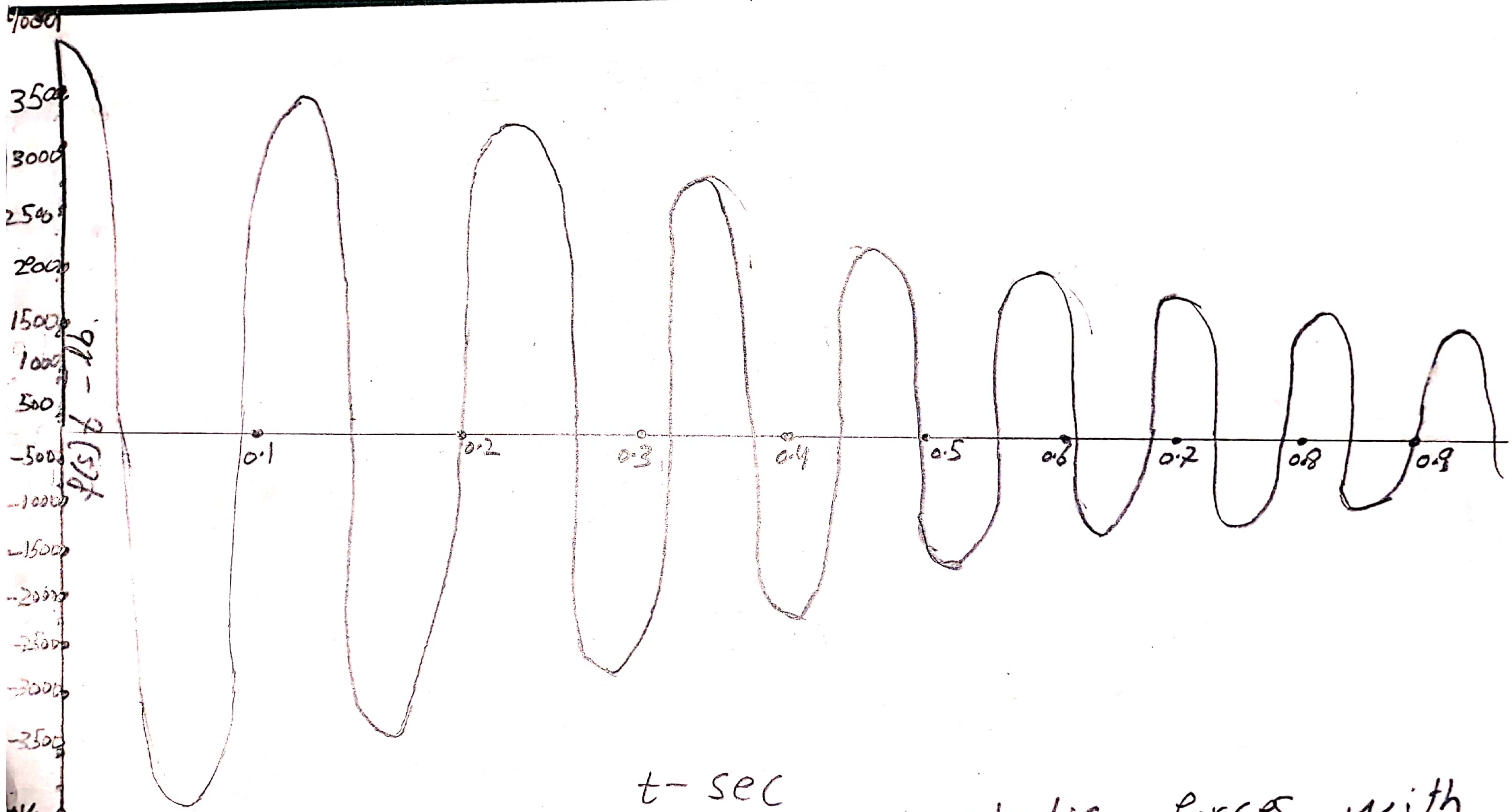
$$F_s(t) = K \cdot u(t) \Rightarrow 90625 \times u(t)$$

$$F_s(t) = e^{-0.581t} \left[(90625 \times 0.041) \cos(19.382t) + (90625 \times 0.00125 \sin(19.382t)) \right]$$

$$F_s(t) = e^{-0.582t} \left[3715.62 \cos(19.382t) + 113.28 \sin(19.382t) \right]$$



Variation of displacement with time.



variation of Equivalent static forces with time.

Q No 03

Given Data

force = 60 Kips

Displacement of tank $= \left(\frac{7768}{1000}\right)'' \Rightarrow 7.768''$

Time taken to complete 7 cycles = 3.57 sec

Amplitude of displacement = 2.286 cm

= 0.9''

Required

ⓐ Damping ratios

ⓑ Natural Period of un-damped vibration

ⓒ stiffness of structure

ⓔ Damping co-efficient

ⓑ Number of cycles to reduce the displacement amplitud.

Sol

⇒ Displacement of tank $u_1 = 7.768''$

⇒ After 7 cycles

i.e After $j = 7$ $u_{j+1} = 0.9''$

(a) Daming ratio $= \zeta = ?$

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_1+1}\right]$$

$$7 = \frac{1}{2\pi\zeta} \ln\left[\frac{7.768}{0.9}\right]$$

$\zeta = 0.0490 \Rightarrow 4.90\,\%$

(b) Natural Period of undamped Vibration $= T_n = ?$

As the 7 cycles of vibrations are completed in 3.57 sec

⇒ Time required to complete one cycle, $T_D = \frac{3.57}{7} = 0.51$ sec

$T_D = 0.51$ sec

Now

$$\omega_D = \omega_n \sqrt{(1-\xi^2)}$$

$$\frac{2\pi}{\omega_D} = \frac{2\pi}{(\omega_n \sqrt{1-\xi^2})}$$

$$\Rightarrow T_D = \frac{T_n}{(1-\xi)^2}$$

$$\Rightarrow T_n = T_D \times \sqrt{1-\xi^2}$$

$$\Rightarrow T_n = 0.51 \times \sqrt{1-(0.0490)^2} \Rightarrow 0.50 sec$$

$T_n = 0.50$ sec

(c) Stiffness of structure, K = ?

$$K = \frac{60 \times \cos 60°}{7.768} \Rightarrow 3.86 \text{ K/in}$$

$K = 3.86$ K/in $\Rightarrow 46320$ lb/ft

d) Weight of tank W = ? (4)

$$\omega_n = \sqrt{K/m} = \sqrt{\frac{K}{W/g}} \Rightarrow \sqrt{\frac{K \cdot g}{W}}$$

$$\Rightarrow \omega_n^2 = \frac{K \times g}{W}$$

$$W = \frac{K \times g}{\omega_n^2}$$

Also $\omega_n = \frac{2\pi}{T_n}$

$$W = \frac{Kg}{\left(\frac{4\pi^2}{T_n^2}\right)}$$

$$W = Kg \times \frac{T_n^2}{4\pi^2}$$

$$W = \left[46320\ lb/ft \times 32.2\ ft/sec^2\right] \times \frac{0.51\ sec^2}{4\pi^2}$$

$$W = 9826.63\ lb = 9.82\ K$$

(e) Damping co-efficient, C = ?

It is known that

$$\zeta = \frac{c}{2m\omega_n}$$

$$\Rightarrow c = \zeta \times 2m\omega_n$$

$$= \zeta \times 2m \times \left(\frac{2\pi}{T_n}\right)$$

$$\Rightarrow c = \frac{(0.0490) \times 4 \times \pi \times \left(\frac{9826.63}{32.2}\right)}{0.50}$$

$$\Rightarrow c = 375.82 \text{ lb.sec/ft}$$

(F) Number of cycles to reduce the displacement amplitude to 0.5",

$J = ?$

$$J = \frac{1}{2\pi\zeta_0} \ln\left(\frac{u_1}{u_{j+1}}\right)$$

$$\Rightarrow J = \frac{1}{2\pi \times 0.0490} \ln\left(\frac{7.768}{0.5}\right)$$

$$\Rightarrow j = 8.90 \quad \text{OR}$$

9 cycles.