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(1)

Q1 The function g(t) is defined

$g(t) = 0 \quad t < 0$

$t^2 \quad 0 \le t \le 3$

$2t + 3 \quad 3 < t \le 4$

$12 \quad t > 4$

(a) State any point of discontinuity.

To check the possibility of discontinuity of the functions at $x = 4$

$g(4) = 2(4) + 3$

$= 8 + 3$

$\boxed{= 11} \rightarrow (1)$

L.H.L

$\lim g(t) = \lim_{t \to 4} (2t + 3)$

$= 2(4) + 3)$

$= 11$

R.H.L $\lim_{t \to 4} g(t) = 12$

L.H.L $\neq$ R.H.L

So the function is discontinous at $x=4$

(b) Find, if they exist

$\lim_{t \to 3} g$

$\lim_{t \to 3} g(t) = \lim_{t \to 3} t^2 \quad 0 \leq t \leq 3$

$= 3^2 = 9$

R.H.L

$\lim_{t \to 3} g(t) = \lim_{t \to 3} (2t+3)$

$= 2(3) + 3$

$= 9$

L.H.L = R.H.L

So $\lim_{t \to 3}$ exist.

Q2(i) Find the Maclaurin's series form. (3)

$$Y(x) = x^2 + \sin x$$

Sol^n $f(x) = x^2 + \sin x$ $\qquad f(0) = 0$

taking derivative

$\cancel{f'(x)} = \cancel{2x} + \cancel{\cos x}$

$$\frac{dy}{dx} = \frac{d}{dx}(x^2 + \sin x)$$

$$y' = \frac{dx^2}{dx} + \frac{d}{dx}\sin x$$

$$y' = 2x + \cos x \Rightarrow f'(0) = 1$$

taking second derivaties

$$y'' = \frac{d}{dx}(2x + \cos x)$$

$$y'' = 2 - \sin x \Rightarrow f''(0) = 2$$

taking third derivate

$$y''' = \frac{d}{dx}(2 - \sin x)$$

$$y''' = -\cos x \Rightarrow f'''(0) = -1$$

taking fourth derivate

$$y^{iv} = \cancel{\sin x}\ \frac{d}{dx}(-\cos x)$$

$$y^{iv} = -(-\sin x)$$

$$y^{iv} = \sin x \Rightarrow f^{iv}(0) = 0$$

taking fifth desivate (4)

$y^{v} = \frac{d}{dx} \sin x$

$\boxed{y^{v} = \cos x} \Rightarrow f^{v}(0) = 1$

taking sixth desivates

$y^{vi} = \cancel{\sin x} \frac{d}{dx} \cos x$

$\boxed{y^{vi} = -\sin x} \Rightarrow f^{vi}(0) = 0$

taking seventh desivates

$y^{vii} = \frac{d}{dx} -\sin x$

$\boxed{y^{vii} = -\cos x} \Rightarrow f^{vii}(0) = -1$

Mac lausin's sesies

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \frac{x^4}{4!} f^{iv}(0) + \frac{x^5}{5!} f^{v}(0)$$

$$+ \frac{x^6}{6!} f^{vi}(0) + \frac{x^7}{7!} f^{vii}(0) \ldots$$

$$\boxed{x^2 + \sin x = x + x^2 - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} \ldots}$$

Ans!

Q3(i) Find y" given

(i) $1+xy = x^2+y^2$

$1+xy = x^2+y^2$

Taking derivaties on B.S

$$\frac{d}{dx}(1+xy) = \frac{d}{dx}(x^2+y^2)$$

$$0 + x\frac{dy}{dx} + y\frac{dx}{dx} = \frac{dx^2}{dx} + \frac{dy^2}{dx}$$

$$x\frac{dy}{dx} + y(1) = 2x + 2y\frac{dy}{dx}$$

$$x\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x - y$$

$$\frac{dy}{dx}(x-2y) = (2x-y)$$

$$\frac{dy}{dx} = \frac{2x-y}{x-2y}$$

$$\boxed{y' = \frac{2x-y}{x-2y}}$$

Now taking second derivaties

$$y'' = \frac{1}{(x-2y)^2}\left[(x-2y)\frac{d}{dx}(2x-y) - (2x-y)\frac{d}{dx}(x-2y)\right]$$

(6)

$$= \frac{1}{(x-2y)^2}\left[\frac{(x-2y)\left(2-\frac{dy}{dx}\right)-(2x-y)\left(1-2\frac{dy}{dx}\right)}{(x-2y)^2}\right]$$

Put $\quad dy/dx = \dfrac{2x-y}{x-2y}$

$$y''(x) = \frac{1}{(x-2y)^2}\left[(x-2y)\left(2-\frac{2x-y}{x-2y}\right)-2x-y\left(1-2\left(\frac{2x-y}{x-2y}\right)\right]\right/(x-2y)^2$$

$$= \frac{1}{(x-2y)^2}\left[\cancel{(x-2y)}\left[\frac{2x-4y-2x+y}{\cancel{x-2y}}\right]-2x-y\left[\frac{x-2y-4x+2y}{x-2y}\right]\right]$$

$$= \frac{1}{(x-2y)^2}\left[-3y-(2x-y)\frac{(-3y)}{x-2y}\right]$$

$$\boxed{= \frac{1}{(x-2y)^2}\left[\frac{-3y(x-2y)+3y(2x-y)}{(x-2y)}\right]}$$

Ans

Q3(ii) Find y' by using logarithmic differentiation

$$y = x^3 (1+x)^9 e^{6x}$$

Sol^n $y = x^3 (1+x)^9 e^{6x}$

taking ln on B.S

$$\ln y = \ln (x^3 (1+x)^9 e^{6x}$$

$$\ln y = \ln x^3 + \ln (1+x)^9 + \ln e^{6x}$$

$$\ln y = 3 \ln x + 9 \ln (1+x) + 6x$$

$$\frac{d \ln y}{dx} = 3 \frac{d}{dx} \ln x + 9 \frac{d}{dx} \ln (1+x) + 6 \frac{dx}{dx}$$

$$\frac{1}{y} \frac{dy}{dx} = 3 \frac{1}{x} + \frac{9}{1+x} + 6$$

$$\boxed{\frac{dy}{dx} = \left(\frac{3}{x} + \frac{9}{1+x} + 6 \right) y}$$ Ans

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