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Section B

Deptt BE(C)

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Subject Differential

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Quiz No 3

Iqra National University

## Question:

A yam merchants sells brands A, B, C of yarn each of which is a brand of Pakistani, Egyptian and American cotton in the ratio 1:2:1, 2:1:1, 2:0:2 respectively. If one kilogram of A, B, C cost 40, 50 and 60 ruppes respectively. Find the cost of a kilogram of cotton of each country.

## Solution:

1:2:1 , 2:1:1 , 2:0:2

40

| | |
|---|---|
| P | E |
| A | E |

$B_1$

50

| | |
|---|---|
| P | P |
| A | E |

$B_2$

60

| | |
|---|---|
| P | P |
| A | A |

$B_3$

Let x, y and z be the cost/kg of Pakistani, Egyptain and American cotton respectively then according to the given conditions:

$$\left.\begin{aligned} \frac{1}{4}x + \frac{2}{4}y + \frac{1}{4}z &= 40 \\ \frac{2}{4}x + \frac{1}{4}y + \frac{1}{4}z &= 50 \\ \frac{2}{4}x + \frac{2}{4}z &= 60 \end{aligned}\right\} \longrightarrow (S)$$

$$\left.\begin{aligned} 1x + 2y + 1z &= 160 \\ 2x + 1y + 1z &= 200 \\ 1x + 1z &= 120 \end{aligned}\right\} \longrightarrow (5)$$

In matrix form we can write it as.

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \quad \underline{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}$$

$$Ax = \underline{b}$$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 120 \end{bmatrix}$$

$$|A| = -2 \qquad |A| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 1(1\times1-0\times1)-2(2\times1-1\times1) + 1(2\times1-1\times1)$$

$$|A_1| = -120 \qquad |A_1| = \begin{vmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{vmatrix} = 160(1\times1-0\times1)-2(200\times1-120\times1) + 1(200\times1-120\times1)$$

$$|A_2| = -40 \qquad |A_2| = \begin{vmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{vmatrix} = 1(200\times1-120\times1) - 160(2\times1-1\times1) + 1(2\times1-1\times200)$$

$$|A_3| = 120 \qquad |A_3| = \begin{vmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 1 & 120 \end{vmatrix} = 1(1\times120-0\times200) - 2(2\times120-1\times200) + 160(2\times120-1\times1)$$

$|A| = -2$

$|A_1| = -120, \; |A_2| = -40, \; |A_3| = -120$

According to Cramer's rule:

$$x = \frac{|A_1|}{|A|} = \frac{-120}{-2} = 60$$

$$y = \frac{|A_2|}{|A|} = \frac{-40}{-2} = 20$$

$$z = \frac{|A_3|}{|A|} = \frac{-120}{-2} = 60$$

$(x, y, z) = (60, 20, 60)$.