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Section	B
Subject	Differential Equation
Submitted To	Miss Shomaila
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Q.No1 Find the Fourier Series representation of $f(t) = 1+t$, $-\pi < t < \pi$

Here we use the formula

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nt + \sum_{n=1}^{\infty} b_n \sin nt$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)\,dt$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} (1+t)\,dt$$

$$a_0 = \frac{1}{2\pi}\left| t + \frac{t^2}{2} \right|_{-\pi}^{\pi}$$

$$a_0 = \frac{1}{2\pi}\left(\pi - (-\pi) + \frac{\pi^2}{2} - \frac{\pi^2}{2} \right.$$

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$$a_0 = \frac{1}{2\pi}\left(2\pi + \frac{2\pi^2}{2}\right)$$

$$a_0 = \frac{1}{2\pi}\left(2\pi + \pi^2\right)$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(1+t)(\cos nt)\,dt$$

$$a_n = \frac{1}{\pi}\left((1+t)\frac{\sin nt}{n}\Big|_{-\pi}^{\pi} - \int\right.$$

$$\left.\left(\frac{\sin nt}{n}\frac{d}{dt}(1+t)\right)\right)$$

$$a_n = \frac{1}{\pi}\left((1+t)\cancel{\frac{\sin nt}{n}}^{\,0} - \frac{\cos nt}{n^2}\Big|_{-\pi}^{\pi}\right)$$

$$a_n = \frac{-1}{n^2\pi}\left(\cos n\pi - \cos n(-\pi)\right)$$

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$$a_n = \frac{-1}{n^2\pi}\left(-1-(-1)\right)$$

$$\boxed{a_n = 0}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(1+t)\sin nt\, dt$$

$$b_n = \frac{1}{\pi}\left((1+t)\int_{-\pi}^{\pi}\sin nt - \int\left(\int \sin nt \cdot \frac{d}{dt}(1+t)\, dt\right)\right)$$

$$b_n = \frac{1}{\pi}\left((1+t)\left(\frac{-\cos nt}{n}\right)\Big| - \int_{-\pi}^{\pi}\left(\frac{-\cos nt\,(1)}{n}\right)\right)$$

$$b_n = \frac{1}{\pi}\left(\frac{-(1+t)(\cos nt)}{n}\Big|_{-\pi}^{\pi} + \left(\frac{\sin n\pi}{n^2}\Big|_{-\pi}^{\pi}\right)\right)$$

$$b_n = \frac{-1}{n\pi}\left((1+\pi)(\cos n\pi) - (1+(-\pi))\cos n(\pi)\right)$$

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$$bn = \frac{-1}{n\pi}\left(\cancel{\cos n\pi} + \pi\cos n\pi - \cancel{\cos n\pi} + \pi\cos n\pi\right)$$

$$bn = \frac{-1}{n\cancel{\pi}}\left(2\cancel{\pi}\cos n\pi\right)$$

Here $\cos n\pi = \frac{(-1)^{n+1}}{n}$

$$bn = \frac{2}{n}(-1)^{n+1}$$

So eqn become

$$\boxed{F(x) = \frac{1}{2\pi}\left(2\pi + \pi^2\right) + 0 + \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n}\sin t}$$

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Q.No.2 $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix}$

Sol: Step 01

We have
$$(A - \lambda I) \cancel{x} X = 0$$

Step 02

We have The chroacteristics equation given by
$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 2-\lambda \end{vmatrix} = 0$$

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Step 03

λ^3 -	Sum of Diagnal element	λ^2 +	Sum of Diagnal minor

$$|\lambda - |A|| = 0 \quad \rightarrow (B)$$

Sum of Diagnal element $= 1+1+2$
$= 4$

Sum of Diagnal minors

$$= \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$$

$= (-6) + (2) + (1)$

$= -6 + 2 + 1$

$= -3$

Putting values in eq (B)

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$$\lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0 \rightarrow (C)$$

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix}$$

$$= 1 \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0 \begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + 1 \begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$$

$$= 1(2-8) - 0 + 1(6-0)$$

$$= -6+6$$

$$= 0$$

Putting values in (C)

$$\lambda^3 - 4\lambda^2 - 3\lambda - 0 = 0$$

$$\lambda^3 - 4\lambda^2 - 3\lambda = 0$$

$$\lambda(\lambda^2 - 4\lambda - 3) = 0$$

$\lambda = 0$

$\lambda^2 - 4\lambda - 3 = 0$

Using quadratic formula

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \begin{array}{l} a = 1 \\ b = -4 \\ c = -3 \end{array}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - (4)(1)(-3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16+12}}{2}$$

$$= \frac{4 \pm \sqrt{28}}{2}$$

$$\lambda = \frac{4+\sqrt{28}}{2} \quad , \quad \lambda = \frac{4-\sqrt{28}}{2}$$

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Q No 3

Sol.

$5x + 0 + 4z + 2m = 3$
$x - y + 2z + m = 1$
$4x + y + 2x + 0 = 1$
$x + y + z + m = 0$

Sol.

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \quad R_4 \; R_2$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1+\frac{4}{5} & 1 \\ 0 & -1 & +\frac{6}{5} & +\frac{4}{5} & \frac{3}{5} \\ 0 & 2 & -1 & 0 & -1 \end{array}\right] \quad -\frac{1}{5} \times R_3$$

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$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -1 & \frac{6}{5} & \frac{4}{5} & \frac{3}{5} \\ 0 & 0 & \frac{7}{5} & \frac{8}{5} & \frac{1}{5} \end{array}\right]$$ 5 x R3 and 5xR4

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right]$$ 5R3 and 5R1

$$\left[\begin{array}{cccc|c} 1 & 0 & \frac{4}{5} & \frac{2}{5} & \frac{3}{3} \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right]$$ $\frac{1}{5} \times R_1$

$$\left[\begin{array}{cccc|c} 1 & 0 & \frac{4}{5} & \frac{2}{5} & \frac{3}{5} \\ 0 & -1 & \frac{6}{5} & \frac{1}{5} & \frac{2}{5} \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right]$$ $R_2 \times 5$

$$\left[\begin{array}{cccc|c} 1 & 0 & \frac{4}{5} & \frac{2}{5} & \frac{3}{5} \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad R_3 - R_2$$

$$\left[\begin{array}{cccc|c} 1 & 0 & \frac{4}{5} & \frac{2}{5} & \frac{3}{5} \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & \frac{8}{7} & \frac{1}{7} \\ 0 & 0 & 0 & 1 & \frac{1}{3} \end{array}\right] \quad \begin{array}{l} R_3 \leftrightarrow R_4 \\ \frac{1}{7} \times R_3 \\ \frac{1}{3} \times R_4 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & \frac{4}{5} & \frac{2}{5} & \frac{3}{5} \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -\frac{4}{21} \\ 0 & 0 & 0 & 1 & \frac{1}{3} \end{array}\right] \quad R_2 \times -5$$

$$\left[\begin{array}{cccc|c} 1 & 0 & \frac{4}{5} & \frac{2}{5} & \frac{3}{5} \\ 0 & 1 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -\frac{4}{21} \\ 0 & 0 & 0 & 1 & \frac{1}{3} \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & \frac{4}{5} & \frac{2}{5} & \frac{3}{5} \\ 0 & 1 & 0 & -5 & \frac{26}{21} \\ 0 & 0 & 1 & 0 & -\frac{11}{21} \\ 0 & 0 & 0 & 1 & \frac{1}{3} \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & \frac{1}{2} & \frac{3}{4} \\ 0 & 1 & 0 & 0 & \frac{31}{21} \\ 0 & 0 & 1 & 0 & -\frac{11}{21} \\ 0 & 0 & 0 & 1 & \frac{1}{3} \end{array}\right]$$

$\frac{5}{4} \times R_1$

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$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 126/84 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 1/2 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$(x, y, z, m) = \left(\frac{3}{4}, \frac{31}{21}, \frac{-11}{21}, \frac{1}{2}\right)$$

$$x = \frac{3}{4}$$

$$y = \frac{31}{21}$$

$$z = \frac{-11}{21}$$

$$m = \frac{1}{3}$$

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Q No4 $U(x,t) = \sin(x+2t)$

Sol $\frac{\partial^2 U}{\partial t^2} = C^2 \frac{\partial^2 U}{\partial x^2}$

$U(x,t) = \sin(x+2t)$ is

solution of $\frac{\partial^2 U}{\partial t^2} = C^2 \frac{\partial^2 U}{\partial x^2}$

it will statisfy the above equation

$\frac{\partial U}{\partial t} = \cos(x+2t) \cdot \frac{\partial}{\partial t}(x+2t)$

$\frac{\partial U}{\partial t} = 2\cos(x+2t)$

again

$\frac{\partial^2 U}{\partial t^2} = -2\sin(x+2t) \frac{\partial}{\partial t}(x+2t)$

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$$\Rightarrow \frac{\partial^2}{\partial t^2} = -4\sin(x+2t) \rightarrow A$$

Now $\frac{\partial u}{\partial x} = \cos(x+2t)$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)$$

$$\Rightarrow \frac{\partial^2 u}{\partial x^2} = -\sin(x+2t) \rightarrow B$$

Comparing A and B

$$c = 2$$

$$\Rightarrow -4\sin(x+2t) = -c^2\sin(x+2t)$$

$$\Rightarrow -4\sin(x+2t) + c^2\sin(x+2t = 0$$

this is possible if $c = \pm 2$

$$-4\sin(x+2t) + (\pm 2)^2\sin(x+2t) = 0$$

$$0 = 0$$

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This $y(x,t) - \sin(x+at)$