

Q# 01

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$$\left[\begin{array}{cccc|c} 1 & 10-3 & 3 & 0 & 5 \\ 0 & 1 & -10-6 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 10-3 \end{array} \right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & 3 & 0 & 5 \\ 0 & 1 & -6 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right]$$

As we can see that the given matrix is already in Echelon form.

$$\therefore 6R_3 + R_2$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & 3 & 0 & 5 \\ 0 & 1 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right]$$

$$-2R_2 + R_1$$

$$2 \quad \left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 5 \\ 0 & 1 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right]$$

$$-3R_3 + R_2$$

$$2 \quad \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right]$$

$$\therefore 1x_1 + 0x_2 + 0x_3 + 0x_4 = 5$$

$$0x_1 + 1x_2 + 0x_3 + 0x_4 = 7$$

$$0x_1 + 0x_2 + 1x_3 + 0x_4 = -6$$

$$0x_1 + 0x_2 + 0x_3 + 1x_4 = 2$$

$$\Rightarrow \left[\begin{array}{l} 1x_1 = 5 \\ x_2 = 7 \\ x_3 = -6 \\ x_4 = 2 \end{array} \right]$$

Result

Ans 2 part A

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix} \quad \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix} \quad R_3 - 2R_2$$

$$\underline{B} = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

$$\underline{B} = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix} \quad R_3 + 2R_2$$

ANS NO 2 Part NO B 4 parts

Part NO 1 A

$$\begin{pmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{pmatrix}$$

Sol:-

$$\begin{pmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{pmatrix} \text{ is in echelon form}$$

yes in echelon form because
no of zero increases as
we goes down row
by row before 1st non-
zero.

Q2(b)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

is in echelon form

yes in echelon form because number of zero increase row by row before 1st non-zero.

$$c. \begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

is in reduced row echelon form

No because the 1st element in R_1 lie C_1 is not 1 so not in reduce echelon form.

$$C = \begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

Solution

$$\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

is in echelon form because it satisfies the condition that is in a column that contains the leading entry of row all the other elements are zero

$$d = \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

Solution:-

$$\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} \textcircled{1} & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \textcircled{1} & 4 \end{bmatrix}$$

is in echelon form because
 satisfies the 1st condition
 is in a column that
 contains the leading entry
 row all the other
 elements are zero

Q. part A

Reduce Row Echelon Form

- A matrix is said to be in reduced row echelon form when it satisfies following conditions
- The matrix satisfies condition for a row echelon form.
- The leading entry in each row is the only non-zero entry in its row.

For example

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Q3 Part A

Row Echelon Form:- A matrix is said to be in row echelon form when it satisfies the following conditions.

- (1) The first non-zero element in each row (leading entry) is 1
- (2) Each leading entry is in a column to the right of the leading entry in the previous row.
- (3) Rows with all zero elements if any are below (after) the row with a non-zero element

For example

$$\begin{bmatrix} 1 & 4 & 6 \\ 0 & 0 & 2 \\ 0 & 0 & 4 \end{bmatrix}$$

Q.3(B) Echelon form by using Row Operations:-

$$\begin{bmatrix} 1 & 102 & 8 \\ 2 & 8 & -1 \\ -102 & 0 & 0 \\ 1 & -4 & 10-9-1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 5 & 8 \\ 2 & 8 & -1 \\ -7 & 0 & 0 \\ 1 & -4 & 15 \end{bmatrix}$$

$$\begin{array}{l} \sim \\ \sim \end{array} \begin{array}{l} -2R_1 + R_2 \\ -2R_1 + R_3 \\ -2R_1 + R_4 \end{array} \begin{array}{l} \left[\begin{array}{ccc} 1 & 5 & 8 \\ 0 & -2 & -17 \\ -7 & 0 & 0 \\ 1 & -4 & 15 \end{array} \right] \\ \left[\begin{array}{l} -2(1) + 2 = 0 \\ -2(5) + 8 = -2 \\ -2(8) - 1 = -17 \end{array} \right] \end{array}$$

$$\begin{array}{l} \sim \\ \sim \end{array} \begin{array}{l} 7R_1 + R_3 \\ 7R_1 + R_4 \end{array} \begin{array}{l} \left[\begin{array}{ccc} 1 & 5 & 8 \\ 0 & -2 & -17 \\ 0 & 35 & 56 \\ 1 & -4 & 15 \end{array} \right] \\ \left[\begin{array}{l} 7(1) + (-7) = 0 \\ 7(5) + (0) = 35 \\ 7(8) + (0) = 56 \end{array} \right] \end{array}$$

$$\begin{array}{l} \rightarrow \\ \rightarrow \end{array} \begin{array}{l} -1R_1 + R_4 \\ -1R_1 + R_2 \end{array} \begin{array}{l} \left[\begin{array}{ccc} 1 & 5 & 8 \\ 0 & -2 & -17 \\ 0 & 35 & 56 \\ 0 & -9 & 7 \end{array} \right] \\ \left[\begin{array}{l} -1(1) + 1 = 0 \\ -1(5) + (-4) = -9 \\ -1(8) + (15) = 7 \end{array} \right] \end{array}$$

$$\begin{array}{l} \sim \\ \sim \end{array} \begin{array}{l} \times \text{ply } R_2 \text{ by } -1/2 \\ \left[\begin{array}{ccc} 1 & 5 & 8 \\ 0 & 1 & 17/2 \\ 0 & 35 & 56 \\ 0 & -9 & 7 \end{array} \right] \\ \left[\begin{array}{l} 0 \times -1/2 = 0 \\ 17 \times -1/2 = 17/2 \\ -17 \times -1/2 = 17/2 \end{array} \right] \end{array}$$

$$-35R_2 + R_3$$

$$\sim \begin{bmatrix} 1 & 5 & 8 \\ 0 & 1 & 17/2 \\ 0 & 0 & -483/2 \\ 0 & -9 & 7 \end{bmatrix}$$

$$\begin{cases} -35(0) + 0 = 0 \\ -35(1) + 35 = 0 \\ -35(17/2) + 56 = \frac{-595 + 112}{2} \\ = -483/2 \end{cases}$$

$$9R_2 + R_4$$

$$\sim \begin{bmatrix} 1 & 5 & 8 \\ 0 & 1 & 17/2 \\ 0 & 0 & -483/2 \\ 0 & 0 & 167/2 \end{bmatrix}$$

$$\begin{cases} 9(0) + 0 = 0 \\ 9(1) + (-9) = 0 \\ 9(17/2) + (7) = 167/2 \end{cases}$$

$$\text{xply } R_3 \text{ by } -2/483$$

$$\sim \begin{bmatrix} 1 & 5 & 8 \\ 0 & 1 & 17/2 \\ 0 & 0 & 1 \\ 0 & 0 & 167/2 \end{bmatrix}$$

$$\begin{cases} 0 \times -2/483 = 0 \\ 0 \times -2/483 = 0 \\ +483/2 \times 1 + 2/483 = 1 \end{cases}$$

$$-167/2 \cdot R_3 + R_4$$

$$\sim \begin{bmatrix} 1 & 5 & 8 \\ 0 & 1 & 17/2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{cases} -167/2(0) + 0 = 0 \\ -167/2(0) + 0 = 0 \\ -167/2(1) + 167/2 = 0 \end{cases}$$

Result

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Linear Algebra

BS-SE 2nd Semester

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