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subject :- Applied calcalas

sec :- A.

Mid Examination

Q NO 1:-

The function $g(t)$ is defined by

$$g(t) = 0 \qquad t < 0$$
$$t^2 = \qquad 0 \le t \le 3.$$
$$2t+3 \qquad 3 < t \le 4.$$
$$12 \qquad t > 4.$$

(a) stat any point of discontuity

(b) Find, if ty exist

i. $\lim_{t \to 3} g$

SOLUTION:-

To check posibility of the discontinuity of a function at $t=0$ & 4.

First at $t=0$

$g(t) = t^2$

$g(0) = 0^2 \Rightarrow 0$.

For

R.H.L

$\lim g(1+h) = \lim (1+h)^2$

$= \lim$

$= \lim\limits_{h \to 0} 1 + h^2 + 2h$

Apply limit

$= 1 + 0^2 + 2(0)$

$= 1 + 0 + 0$

$= 1$

L.H.L

$\lim\limits_{h \to 0} (1-h) = 2t + 3$

$\Rightarrow \lim\limits_{h \to 0} 2(1-h) + 3$

$\Rightarrow \lim\limits_{h \to 0} 2 - 2h + 3$

Apply limit

$= 2 - 2(0) + 3$

$= 2 - 0 + 3$

$= 5.$

R.H.L $\neq$ L.H.L

Now at 4.

$g(t) = 2(4) + 3$

$g(4) = 2(4) + 3$

$= 8 + 3$

$= 11.$

FOR R.H.L

$\lim_{h \to 0} g(1+h) = \lim_{h \to 0} 2(1+h) + 3.$

$= \lim_{h \to 0} 2 + 2h + 3$

Apply limit

$= 2 + 2(0) + 3$

$= 2 + 3$

$= 5.$

Now for

L.H.L

$$\lim_{h \to 0} g(1-h) = D$$

$g(4) = R.H.L \neq L.H.L$

point of discontinuity is at $t=4$.

Question No 2

Find the Maclaurin series for

$y(x) = x^2 + \sin x$

SOLUTION

$y(x) = x^2 + \sin x$

according to maclaurin series

$$y(x) = y(x_0) + y'(x_0)(x - x_0) + \frac{y''(x_0)(x-x_0)^2}{2} + \cancel{\frac{(x-x_0)^2}{}} + \cdots$$

put $x_0 = 0$

$$y(x) = y(0) + (x-0)y'(0) + (x-0)y''(0) + \cdots$$

$$y_x = y(0) + x\,y'(0) + \frac{x^2 y''(0)}{2} + \cdots \quad (1)$$

Now find

$y(0) = ?$

$y(0) = x^2 + \sin x$

$y(0) = 0 + \sin 0$

$y(0) = 0 + 0$

$= 0$

$y(x) = x^2 + \sin x$

$\frac{d}{dx} y(x) = \frac{d}{dx} x^2 + \frac{d}{dx} \sin x$

$y'(x) = 2x + \cos x$

$y'(0) = 2(0) + \cos 0$

$y'(0) = 0 + 1$

$= +1.$

Since

$y'(x) = 2x + \cos x$

$\frac{d}{dx} y'(x) = 2\frac{d}{dx} x + \frac{d}{dx} \cos x$

$= 2 + \sin x$

$$y''(x) = 2 - \sin x$$

$$y''(0) = 2 - \sin 0$$
$$= 2 - 0$$
$$= 2$$

$$y''(0) = 2$$

Now

$$y''(x) = 2 - \sin x$$

$$\frac{d}{dx} y''(x) = \frac{d}{dx} 2 - \frac{d}{dx} \sin x$$

$$= 0 - \cos x$$

$$y'''(x) = -\cos x$$

$$y'''(0) = -1$$

put in eq (1)

$$y(x) = 0 + x(1) \frac{x^2(2)}{2!} + \frac{x^3}{3!}(-1) + \dots$$

$$= x + 2\frac{x^2}{2!} - \frac{x^3}{3!} + \ldots$$

$$= x + x^2 - \frac{x^3}{3!} + \ldots$$

So

$$y(x) = x + x^2 - \frac{x^3}{3!} + \ldots$$

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Q.3 Find y' given:

$$1 + xy = x^2 + y^2$$

Sol:-

$$1 + xy = x^2 + y^2 \rightarrow (1)$$

diff eq(1) b/s w.r.t x.

$$= \frac{d}{dx}(1+xy) = \frac{d}{dx}(x^2 + y^2)$$

$$= \frac{d}{dx}(1) + \frac{d}{dx}(xy) = \frac{d}{dx}x^2 + \frac{d}{dx}y^2.$$

$$= 0 + (x \cdot \frac{d}{dx}y + y\frac{d}{dx}x) = 2x + 2y \cdot \frac{dy}{dx}$$

$$= x \cdot \frac{d}{dx} + y(1) = 2x + 2y \cdot \frac{dy}{dx}$$

$$= x \cdot y' + y = 2x + 2y \cdot y'$$

$$\Rightarrow x \cdot y' + y = 2x + 2y \cdot y$$

$$\Rightarrow x \cdot y' - 2y \cdot y' = 2x - y$$

$$\Rightarrow y'(x - 2y) = (2x - y)$$

$\Rightarrow y'(x-2y) = (2x-y)$

$\Rightarrow y' = \dfrac{2x-y}{x-2y};$

Diff again w.r.t 'x'

$y'' = \dfrac{d}{dx}\left(\dfrac{2x-y}{x-2y}\right)$

$= \dfrac{(x-2y)\frac{d}{dx}(2x-y) - (2x-y)\frac{d}{dx}(x-2y)}{(x-2y)^2}$

$= \dfrac{(x-2y)(2-dy/dx) - (2x-y)(1-2\,dy/dx)}{(x-2y)^2}$

$= \dfrac{(2x-4y)(-y') - (2x-4x\frac{dy}{dx}-y+2y\,y')}{(x-2y)^2}$

$\Rightarrow y''(x-2y)^2 = 2xy + 2yy' - 2x+y$

$\Rightarrow y''(x-2y)^2 = \left(\dfrac{2x-y}{x-2y}\right)(2x-2y) - 2x+y$

$$y'' = \frac{\left(\frac{2x-y}{x-2y}\right)(2x+2y) - 2x + y}{(x-2y)^2}$$

Ans.

Q3:- $Y = x^3(1+x)^9 e^{6x}$

Take "ln" on b.sides

$$\ln y = \ln(x^3(1+x)^9) + \ln e^{(6x)}$$

$$= \ln x^3 + \ln(1+x)^9 + 6x$$

Now'

$$\frac{d}{dx}\ln(y) = \frac{d}{dx}(3\ln x + 9\ln(1+x) + 6x)$$

$$= 3\frac{d}{dx}\ln x + 9\frac{d}{dx}\ln(1+x) + 6\frac{d}{dx}.$$

$$= 3 \cdot \frac{1}{x} + 9 \cdot \frac{1}{1+x} + 6.$$

$$\frac{d}{dx} \ln(y) = \frac{3}{x} + \frac{9}{x+1} + 6.$$