IQRA NATIONAL UNIVERSITY

Differential Equations

Final Term Assignment Summer 2020

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⇒ Question No (1)

⇒ Estimate the gernal solution of $4y'' - 20y' + 25y = 0$.

(Solution) :- $4y'' - 20y' + 25y = 0$

The auxilary equation

$$4\lambda - 20\lambda + 25 = 0$$

$$4\lambda - 10\lambda - 10\lambda + 25 = 0$$

$$2\lambda(2\lambda - 5) - 5(2\lambda - 5) = 0$$

$$(2\lambda - 5)(2\lambda - 5) = 0$$

$$2\lambda - 5 = 0 \quad , \quad 2\lambda - 5 = 0$$

$$\lambda_1 = \frac{5}{2} \quad , \quad \lambda_2 = 5/2$$

$$\lambda_1 = \lambda_2 = 5/2$$

⇒ The roots are equal

So

$$\boxed{y = (c_1 + c_2 x)e^{\lambda x} = (c_1 + c_2 x)e^{5/2 x}}$$

Answer.

⇒ **Question NO (2)**

⇒ **Part (A)**

⇒ Calculate the initial value Problem $y'' + 2y' + y = 0$

$4 \cdot y'(0) = -6$

(Solution):- Auxilary Solution

$\lambda^2 + 2\lambda + 1 = 0$

$\lambda^2 + \lambda + \lambda + 1 = 0$

$\lambda(\lambda+1) + (\lambda+1) = 0$

$(\lambda+1)(\lambda+1) = 0$

$\lambda_1 = -1 \quad , \quad \lambda_2 = -1$

⇒ Roots are real and equal

$y = (c_1 + c_2 x) e^{\lambda x}$

$y = (c_1 + c_2 x) e^{-1x}$ or $y = c_1 e^{-x} + c_2 e^{-x}$

where $x = 0 \quad , \quad y = 4$

$4 = c_1 e^{0} + c_2 e^{0}$

$(4 = c_1)$

⇒ Since

$y = c_1 e^{x} + c_2 e^{-x}$

$\Rightarrow \quad y = -C_1 e^{-x} + C_2 e^{-x} - C_2 x e^{-x}$

where

$x = 0 \quad , \quad y = -6$

$-6 = -C_1 + C_2$

$-6 = C_1 + C_2$

$-6 = -4 + C_2$

$C_2 = -6 + 4 = -2$

$(C_2 = -2)$

So particular solution

$$\boxed{\left\{ y = 4e^{-x} - 2xe^{-x} \quad \text{or} \quad y = (y - 2x)e^{-x} \right\}}$$

Answer.

$\longleftrightarrow$

⇒ Question NO (2)

⇒ Part B

Analyze the general solution of $x^2\ddot{y} + 3x\dot{y} + y = 0$

(Solution):- $a = 3, \quad b = 1$

$$m^2 + (a-1)m + b = 0$$

$$m^2 + (3-1)m + 1 = 0$$

$$m^2 + 2m + 1 = 0$$

$$m^2 + m + m + 1 = 0$$

$$m(m+1) + 1(m+1) = 0$$

$$(m+1)(m+1) = 0$$

$$m = -1, \quad m = -1$$

Roots are real and equal

So

$$y = (c_1 + c_2 \ln x)\, x^m$$

$$\boxed{y = (c_1 + c_2 \ln x) \cdot x^{-1}}$$

Answer.

⇒ Question No (3)

⇒ Examine the method of undermined cofficient method for $y'' + y' - 6y = 6x^3 - 3x^2 + 12x$.

(Solution):-

$$h^2 + h - 6 = 0$$

$$h^2 + 3h - 2h - 6 = 0$$

$$h(h+3) - 2(h+3) = 0$$

$$(h-2)(h+3) = 0$$

$$h_1 = 2$$

$$h_2 = -3$$

$$y_h = c_1 e^{2x} + c_2 e^{-3x}$$

⇒ Check for y_p

$$y_p = k_2 x^2 + k_1 x^1 + k_0$$

$$y'_p = 2k_2 x + k_1 + 0$$

$$y''_p = 2k_2$$

⇒ $2k_2 + 2k_2 x + k_1 - 6(k_2 x^2 + k_1 x + k_4)$

$$= 6x^2 - 3x^2 + 12x$$

$$\Rightarrow 2K_2 + 2K_2 x + K_1 - 6K_2 x^2 - 6K_1 x - 6K_0 = 3x^2 + 12x + 0$$

$$(2K_2 - 6K_1)x = 12 \rightarrow (1)$$

$$-6K_2 = 3 \rightarrow (2)$$

$$K_2 = -\frac{1}{2}$$

$$2K_2 + K_1 - 6K_0 = 0$$

$$2 \times -\frac{1}{2} - 6K_1 = 12$$

$$-1 - 6K_1 = 12$$

$$-6K_1 = 12 + 1$$

$$-6K_1 = 13$$

$$\boxed{K_1 = \frac{13}{6}}$$

$$\Rightarrow \cancel{2}\left(-\frac{1}{\cancel{2}}\right) - \frac{13}{6} - 6K_0$$

$$-1 - \frac{13}{6} - 6K_0 = 0$$

$$\frac{-6-13}{6} = -\frac{19}{6}$$

$$\boxed{K_0 = -\frac{19}{6}}$$ **Answer.**

⇒ Question No (4)

⇒ Examine the method of variation of parameters fo $y'' - 4y' + 4y = x^2 e^{2x}$.

(Solution):-

$$y'' - 4y' + 4y = 0$$

$$\lambda^2 - 4\lambda + 4 = 0$$

$$(\lambda - 2)^2 = 0$$

$$(\lambda - 2)(\lambda - 2) = 0$$

$$\lambda_1 = 2 \quad , \quad \lambda_2 = 2$$

⇒ Roots are equal

$$y = (c_1 + c_2 x) e^{2x}$$

$$\{ y_c = c_1 e^{2x} + c_2 x e^{2x} \}$$

⇒ Now:-

$$y_1 = e^{2x} \quad , \quad y_2 = x e^{2x}$$

$$y_1' = 2e^{2x} \quad , \quad y_2' = 2x e^{2x} + e^{2x}$$

⇒ $W = y_1 y_2' - y_1 y_2$

$$\Rightarrow \quad W = (e^{2x})(2xe^{2x} + e^{2x}) - (2e^{2x})$$

$$W = 2xe^{4x} + e^{4x} - 2xe^{4x}$$

$$\{ W = e^{4x} \}$$

$$y_P = -y_1 \int \frac{y_2\, r(x)}{W}\, dx + y_2 \int \frac{y_1 r(x)}{W}\, dx$$

$$y_P = -e^{2x} \int \frac{(xe)^{2x} (x^2 e^{2x})}{e^{4x}}\, dx + xe^{2x} \int \frac{(e)^{2x} (x^2 e^{2x})}{4x}\, dx$$

$$y_P = -e^{2x} \int \frac{x^3 e^{4x}}{e^{4x}}\, dx + xe^{2x} \int \frac{x^2 e^{4x}}{e^{4x}}\, dx$$

$$y_P = -e^{2x} \int x^3\, dx + xe^{2x} \int x^2\, dx$$

$$y_P = -e^{2x} \quad \frac{x^4}{4} + xe^{2x} \cdot \frac{x^3}{3}$$

$$y_P = \frac{-x^4 e^{2x}}{4} + \frac{x^4 e^{2x}}{3}$$

$$\Rightarrow \quad y_P = \frac{-3x^4 e^{2x} + 4x^4 e^{2x}}{12}$$

$$\Rightarrow \quad \left\{ y_P = \frac{x^4 e^{2x}}{12} \right\}$$

So

$$\boxed{y = y_h + y_P}$$ Answer.

$\longleftrightarrow$

⇒ Question No (5)

⇒ identify an ODE $y'' + ay' + by = 0$ for the basis 1, e^{-3x}.

(Solution):-

$$y_1 = e^{0x}, \quad y_2 = e^{3x}$$

$$y = c_1 e^{0x} + c_2 e^{3x}$$

So both roots are real and distinct

$$y = (c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x})$$

So $\lambda_1 = 0$, $\lambda_2 = 3$

$$\lambda_1 = 0, \quad \lambda_2 - 3 = 0$$

$$(\lambda)(\lambda - 3) = 0$$

$$\lambda^2 - 3\lambda = 0$$

So

$$\lambda^2 - a\lambda + b = 0$$

As $a = -3$, $b = 0$

So

$$y'' + ay' + by = 0$$

$$\boxed{y'' - 3y' = 0}$$

Answer. (OR)⇒

$\Rightarrow \quad y_1 = e^{0x}, \; y_2 = e^{-3x}$

$y = c_1 e^{0x} + c_2 e^{-3x}$

So roots are real and distinct

$y = (c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x})$

So, $\lambda_1 = 0, \; \lambda_2 = -3$

$\Rightarrow \lambda_1 = 0 \quad , \quad \lambda_2 + 3 = 0$

$(\lambda)(\lambda + 3) = 0$

$\Rightarrow \quad \lambda^2 + 3\lambda = 0$

So, $\lambda^2 + a\lambda + b = 0$

As, $a = 3, \; b = 0$

So $y'' + ay' + by = 0$

$\boxed{y'' + 3y' = 0}$ Answer.

Thank You.