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Section: A

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Part A

Question 1

The function of $g(t)$ is defined by $g(t) = 0 \quad t < 0$

$t^2 \quad 0 \leq t \leq 3$

$2t+3 \quad 3 < t \leq 4$

$12 \quad t > 4$

(a) state any point of discontuity:

Solution

At point $t = 3$ $g(t)$ is not continues

because $g(3) = 9 \neq \lim_{t \to 3^+} g(t)$

and limit (t)

So $\lim_{t \to 3}$ doesnot exist

in the Case

Thus $t = 0$ is discontinuty point in the domain $g(t)$.

Part b

Solution

$g(0) = 0$ and

$$\lim_{t \to 0} g(t) = 0$$

and $\lim_{t \to 0} g(t) = 0$

i.e $\lim g(t) = \lim_{t \to 0} g(t)$

$\Rightarrow \lim_{t \to 0} g(t)$ exist

and $\lim_{t \to 0} g(t) = g(0)$

So $g(t)$ is continues at point $t=0$.

Q2

Find the maclurin Series for

$y(x) = x^2 + \sin x$

$f(x) = x^2 + \sin x$

$f(0) = 0$

$f'(x) = 2x + \cos x$

$f'(0) = 2(0) + \cos(0)$

$f'(0) = 1$

$f''(x) = 2 - \sin x$

$f''(0) = 2 - \sin(0)$

$f''(0) = 2$

$f'''(x) = -\cos x$

$f'''(0) = \cos(0)$

$f'''(0) = -1$

Subsitituting values the formula

$F(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f''(0)x^3}{3!} - - -$

$x^2 + \sin x = 0 + 1x + \frac{\cancel{2}x^2}{\cancel{2}!} + \left(\frac{(-1) \times 3}{3!}\right) + - - - - - -$

$x^2 + \sin x = x + x^2 - \frac{x^3}{6} + - - - - -$

Q3

Find y'' given

$1+xy = x^2+y^2$

$d/dx(1) + d/dx\, xy = d/dx\, x^2 + d/dx\, y^2$

$0 + x\, dy/dx + y = 2x + 2y\, dy/dx$

$x\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x - y$

$\frac{dy}{dx}(x-2y) = 2x-y$

$\frac{dy}{dx} = \frac{2x-y}{x-2y}$

Now taking second derivation using quatients formula.

$y'' = V.U' - U.V'$

$U = 2x - y$

$U' = 2$

$V = x - 2y$

$V' = 1$

Put in formula

$y'' = \frac{(x-2y)(2) - (2x-y)(1)}{(x-2y)^2}$

$y'' = \frac{2x - 4y - 2x + y}{(x-2y)^2}$

$y'' = \frac{-3y}{(x-2y)^2}$

Part b

Find y' by using logrithamic differtiation

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$y = x^3 (1+x)^9 e^{6x}$

Taking log on both side

$\log y = \log x^3 (1+x)^9 e^{6x}$

$\frac{1}{y} \frac{d}{dx} = \frac{1}{x^3 (1+x)^9 e^{6x}} \frac{d}{dx} (x^3 (1+x)^9 e^{6x})$

$\frac{dy}{dx} = y' = \frac{y}{x^3 (1+x)^9 e^{6x}} [x^3 (1+x)^9 \frac{d}{dx} e^{6x} + x^3 \cdot$

$e^{6x} \frac{d}{dx} (1+x)^9 + (1+x)^9 e^{6x} \frac{d}{dx} x^3]$

$= \frac{y}{x^3 (1+x)^9 e^{6x}} [x^3 (1+x)^9 (6e^{6x}) + 9x^3 e^{6x} (1+x)^8$

$+ 3x^2 e^{6x} (1+x)^9$

$= \frac{y}{x^3 (1+x)^9 e^{6x}} [6x^3 e^{6x} (1+x)^9 + 9x^3 \cdot e^{6x} (1+x)^8$

$+ 3x + (1+x)]$

$= \frac{y}{x(1+x)} (3(2x(1+x)^9 + 4x+1)$

$= 3y [(2x (1+x)^9 + 4x+1]$

$= \frac{3y [(2x (1+x)^9 + 4x+1]}{x(1+x)}$

Q3

Part b

Second method

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$$y = x^3(1+x)^9 e^{6x}$$

$$y' = x^3(1+x)^9 \frac{d}{dx} e^{6x} + x^3\left(\frac{d}{dx}(1+x)^9\right) \cdot e^{6x}$$
$$+ \left(\frac{d}{dx} x^3\right)\left[(1+x)^9\right) \cdot e^{6x}$$

$$= x^3(1+x)^9(6e^{6x}) + x^3(9(1+x)^8) \cdot e^{6x}$$
$$+ 3x^2(1+x)^9 \cdot e^{6x}$$

$$= 6x^3e^{6x}(1+x)^9 + 9x^3 \cdot e^{6x}(1+x)^8 + 3x^2 e^{6x}$$
$$(1+x)^9$$

$$= 3x^2 \; e^{6x}(1+x)^8\left[2x(1+x)^9 + 3x + (1+x)\right]$$

$$= 3x^2 e^{6x}(1+x)^8\left[2x(1+x)^9 + 3x + 1 + x\right]$$

$$= 3x^2 e^{6x}(1+x)^8\left[2x(1+x)^9 + 4x + 1\right].$$