Apply limits

$= 1 + 0^2 + \not{2}2(0)$

$= 1$

For L.H.L

$$\lim_{h \to 0} g(1-h) = 2t+3$$

$= \lim_{h \to 0} 2(1-h)+3.$

$= \lim_{h \to 0} 2 - 2h + 3$

Apply limit

$= 2 - 2(0) + 3$

$= 5$

$R.H.L \neq L.H.L = g(t) = 5$

→ Now at $t = 4$

$g(4) = 2(4)+3$

$= 8+3$

$= \underline{11}$

For R.H.L

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2t +3

Q1) The function $g(t)$ is defined by

$g(t) = 0 \quad t < 0$

$t^2 \quad 0 \le t \le 3$

$2t + 3 \quad 3 < t \le 4$

$12 \quad t > 4$

a) State any point of discontinuity

b) Find, if they exist

i) $\lim_{t \to 3} g$

Sol:-

a) To check possibility of the discontinuity of the function is at $t = 0$ & 4

→ First at $t = 0$

$g(t) = t^2$

$g(0) = 0^2 = 0$

For R.H.L

$\lim g(1+h) = \lim_{h \to 0} (1+h)^2$

$= \lim_{h \to 0} 1 + h^2 + 2h$

(4)

L.H.L $\lim_{h \to 3} g(1-h) = \lim_{h \to 3} 2t + 3$

$= \lim_{h \to 3} 2(1-h) + 3$

$= \lim_{h \to 3} 2 - 2h + 3$

Apply limit

$= 2 - 2(3) + 3$

$= 2 - 6 + 3$

$= -1$

R.H.L $\neq$ L.H.L (do not exist since L.H.L is -ve)

L.H.L $\lim_{h\to 3} g(1-h) = \lim_{h\to 3} 2(1-h)+3$

$= \lim_{h\to 3}$ . . . 2

2

3 1

$\lim_{h\to 0} g(1+h) = \lim_{h\to 0} 2(1+h)+3$

$= \lim_{h\to 0} 2+2h+3$

Apply limits

$= 2+2(0)+3 \Rightarrow 5$

For L.H.L

$\lim_{h\to 0} g(1-h) = 12$

$g(4) = \text{R.H.L} \neq \text{L.H.L}$

point of discontinuity is at $t=4$

b) Find, if they exist

i) $\lim_{t\to 3} g$

For $g(t) = t^2$

R.H.L

$\lim_{h\to 3} g(1+h) = \lim_{h\to 3} (1+h)^2$

$= \lim_{h\to 3} 1+h^2+2h$

Apply limits

$= 1+3^2+2(3) \Rightarrow 16$

Q: N: 03 :->

Part II

$y = x^3(1-x)^9 e^{6x}$

Sol :->

$y = x^3(1-x)^9 e^{6x}$

Taking 'ln' on both side

$\ln y = \ln(x^3(1-x)^9 e^{6x})$

Note
$\log(mn) = \log m + \log n$

$\Rightarrow \ln y = \ln(x^3) + \ln((1-x)^9) + \ln(e^{6x})$

$\because \log x^n = n \log x$

$\Rightarrow \ln y = 3\ln x + 9\ln(1-x) + 6x$

Taking derivative on both side w.r.t. 'x'

$\frac{d}{dx}(\ln y) = \frac{d}{dx}(3\ln x + 9\ln(1-x) + 6x)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 3\frac{1}{x} + 9\frac{1}{(1-x)} + 6$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{3}{x} + \frac{9}{(1-x)} + 6$

$\Rightarrow \frac{dy}{dx} = y' = y\left(\frac{3}{x} + \frac{9}{(1-x)} + 6\right)$ $\quad \because \frac{dy}{dx} = y'$

$\Rightarrow \boxed{y' = x^3(1-x)^9 e^{6x}\left(\frac{3}{x} + \frac{9}{1-x} + 6\right)}$

Required Ans.

Now

$f''(x) = 2 - \sin x$

$f'''(x) = \frac{d}{dx}(2 - \sin x)$

$f'''(x) = 0 - \cos x$

$f'''(x) = -\cos x$

put $x = 0$

$f'''(0) = -\cos(0)$

$\boxed{f'''(0) = -1} \rightarrow (4)$

putting The value $f(0), f'(0), f''(0)$ & $f'''(0)$ in eq (1)

$\Rightarrow x^2 + \sin x = 0 + \frac{x(1)}{1!} + \frac{x^2(2)}{2!} + \frac{x^3(-1)}{3!} + \cdots\cdots$

$\Rightarrow \boxed{x^2 + \sin x = x + x^2 - \frac{x^3}{6} + \cdots\cdots}$

Required Ans.

Q: No:02:->

$$Y(x) = x^2 + \sin x$$

Sol:->

$$Y(x) = x^2 + \sin x$$

By using Maclaurin expansion series

$$f(x) = f(0) + \frac{x' f(0)}{1!} + \frac{x^2 f''(0)}{2!} + \frac{x^3 f'''(0)}{3!} + \cdots \rightarrow (A)$$

There fore

$$f(x) = x^2 + \sin x$$

Put $x = 0$

$$f(0) = 0^2 + \sin(0)$$

$$f(0) = 0 + 0 = 0 \Rightarrow \boxed{f(0) = 0} - (1)$$

Now $f(x) = x^2 + \sin x$

$$f'(x) = \frac{d}{dx}(x^2 + \sin x)$$

$$f'(x) = 2x + \cos x$$

Put $x = 0$

$$f'(0) = 2(0) + \cos(0)$$

$$f'(0) = 0 + 1 \Rightarrow \boxed{f'(0) = 1} \rightarrow (2)$$

Now $f'(x) = 2x + \cos x$

$$f''(x) = \frac{d}{dx}(2x + \cos x)$$

$$f''(x) = 2 - \sin x$$

Put $x = 0$

$$f''(0) = 2 - \sin(0) \Rightarrow \boxed{f''(0) = 2} \rightarrow (3)$$

Q: No: 3

Part (1)

$$1+xy=x^2+y^2$$

Sol:→ $1+xy=x^2+y^2$

Taking derivative on both side w.r.t 'x'

$$\frac{d}{dx}(1+xy)=\frac{d}{dx}(x^2+y^2)$$

$$\Rightarrow \frac{d}{dx}(1)+\frac{d}{dx}(xy)=\frac{d}{dx}(x^2)+\frac{d}{dx}(y^2)$$

$$\Rightarrow 0+x\,dy/dx+y\,dx/dx=2x+2y\,dy/dx$$

$$\Rightarrow xy'+y=2x+2yy' \qquad \because y'=dy/dx$$

Again taking derivative on both side with respect to 'x'

$$\frac{d}{dx}((xy')+y)=\frac{d}{dx}(2x+2yy')$$

$$\Rightarrow \frac{d}{dx}(xy')+dy/dx=\frac{d}{dx}(2x)+2\frac{d}{dx}(yy')$$

$$\Rightarrow x\,dy'/dx+y'\,dx/dx+dy/dx=2+2(y\,\frac{d}{dx}y'+y'\,dy/dx)$$

$$\Rightarrow xy''+y'+y'=2+2(yy''+y'y')$$

$$\Rightarrow xy''+2y'=2+2yy''+2y'y'$$

$$\Rightarrow xy''-2yy''=2+2y'y'-2y'$$

$$y''(x-2y)=2(y'y'-y'+1)$$

$$\boxed{y''=2(1+y'y'-y')/x-2y}$$