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Subject :- Linear Algebra

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Question No 1:-
(A part):-

The non Parallel vectors:-

$\overline{P_1P_2} = (-3, 2, 3)$
$\overline{P_1P_2} = (3, -1, 3)$

$(-1, 0, 3)-(2, -2, 1)$
$(-1, 0, 3)-2, +2, -1$
$(-3, +2, 2)$

The perpendicular vector is:-
$m = \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3}$

$$n = \begin{vmatrix} i & j & k \\ -3 & 2 & 2 \\ 3 & -1 & 3 \end{vmatrix}$$

$\overrightarrow{P_1P_2} = \sqrt{(x_2-x_1)+(y_2-y_1)}$
$\overrightarrow{P_1P_2} = \sqrt{(-1-2)^2}$

$m = i(6+2) - j(-9-6) + k(3-6)$

$n = 8i + 15j - 3k$
$m = (2, 15, -3)$

Now:- $P_1(x_0, y_0, z_0) = (2, -3, 1)$

$m(a, b, c) = 8, 15, -3)$

So, equation of plane is
$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

$$8(x-2)+15(\cancel{y+3}\ y+2)-3(z-1)=0$$

$$8x+15y-3z-16+\cancel{45}\ 30+3=0$$

$$8x+15y-3z+\cancel{32}\ 17=0$$

Q1 (b)

$$x = 2-3t$$
$$y = 3+t$$
$$z = 2-4t$$

Solution:-

$$x-2=-3t \Rightarrow t = x-2$$
$$y=3=t \Rightarrow t=\frac{y-3}{1}$$

$$z-2=-4t \Rightarrow t=\frac{z-2}{-4}$$

So $\frac{x-2}{-3}=\frac{y-3}{1}=\frac{z-2}{-4}$

For 1st plane takes 1st and 2nd

$$\frac{x-2}{-3} \times \frac{y-3}{1}$$

$$x-2=-3y+9$$

for 2nd plane take 1st. Eq
3rd

$$\frac{x-2}{-3} = \frac{z-2}{-4}$$

$$-4x+8 = -3z+6$$

$$-4x+3z+2=0$$

or

$$4x-3z+2=0$$

Question:- 2

$$L(x,y) = (x+1, y, x+y)$$

Solution:-

$$L(x,y) = (x+1, y, x+y)$$

Let $u = (x_1, y_1)$ $v = (x_2, y_2)$

$$u+v = (x_1, y_1) + (x_2, y_2)$$

$$u+v = (x_1 + x_2, y_1 + y_2)$$

$$L(u+v) = L(x_1 + x_2, y_1 + y_2)$$

$$L(u+v) = (x_1 + x_2 + 1, y_1 + y_2, x_1 + x_2 + y_1 + y_2)$$

given that $u = (x, y)$

$$L(u) = L(x_1, y_1) = (x+1+y+, x1+y-1)$$

$$L(u) = L(x_2, y_2) = (x_2+1, y_2, x_2+y_2)$$

$$L(u) + L(v) = (x_1 + x_2 + 2, y_1 + y_2, x_1 + x_2 + y_2 + y_2)$$

Since $1 \neq 2$

Question No 3

Using the matrix $A = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{vmatrix}$

then interpit to decode the message.

Answer:-

$$A \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 1 & 1 & 2 \end{bmatrix}$$

To decode the above message we have to break it into five vectors in $\mathbb{R}^3$.

$$\begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix} \begin{bmatrix} 71 \\ 49 \\ 29 \end{bmatrix} \begin{bmatrix} 68 \\ 51 \\ 33 \end{bmatrix} \begin{bmatrix} 76 \\ 48 \\ 40 \end{bmatrix} \begin{bmatrix} 86 \\ 53 \\ 52 \end{bmatrix}$$

So solve the equation.

$$L(x_1) = \begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix} = Ax_1$$

for x_1 Since A is nonsingular

$$x_1 = A^{-1}\begin{bmatrix}77\\54\\38\end{bmatrix} = \begin{bmatrix}0 & 1 & -1\\2 & -2 & -1\\-1 & 1 & 1\end{bmatrix}\begin{bmatrix}77\\54\\38\end{bmatrix} = \begin{bmatrix}16\\8\\15\end{bmatrix}$$

Similarly...

$$x_2 = A^{-1}\begin{bmatrix}71\\49\\29\end{bmatrix} = \begin{bmatrix}20\\15\\7\end{bmatrix}$$

$$x_3 = \begin{bmatrix}68\\51\\33\end{bmatrix} = \begin{bmatrix}18\\1\\16\end{bmatrix}$$

$$x_4 = \begin{bmatrix}76\\48\\40\end{bmatrix} = \begin{bmatrix}8\\16\\12\end{bmatrix}$$

$$x_5 = \begin{bmatrix}86\\53\\52\end{bmatrix} = \begin{bmatrix}1\\14\\19\end{bmatrix}$$

Using our corresponding between letter & number, we recieved the following message

PHOTOGRAPH PLANS

Answer.

Question No 4:-

$(-1, 3, 2)$ $n = (0, 1, -3)$

Solution:-

Equ of Plane

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

Given that:-

$P = (x_0, y_0, z_0) = (-1, 3, 2)$

$n = (a, b, c) = (0, 1, -3)$

So, $0(x - (-1)) + 1(y - 3) - 3(z - 2)$

$\underbrace{0(x+1)}_{0} + 1(y - 3) - 3(z - 2)$

$0 + y - 3 - 3z + 6$

$\Rightarrow y - 3z - 3 + 6$

$\Rightarrow y - 3z + 3$ Ans.

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Question No :- 5

Eigen values.

Sol:-

We know that $Ax = \lambda x$

$$\begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 + x_2 \\ -2x_1 + 4x_2 \end{bmatrix} = \begin{bmatrix} \lambda x_1 \\ \lambda x_2 \end{bmatrix}$$

then

$$x_1 + x_2 = \lambda x_1 \quad \text{— (i)}$$

$$-2x_1 + 4x_2 = \lambda x_2 \quad \text{— (ii)}$$

So $x_1 - \lambda x_1 + x_2 = 0$

$= (1-\lambda) x_1 + x_2 = 0$

& $-2x_1 + 4x_2 - \lambda x_2 = 0$

$= -2x_1 + (4-\lambda) x_2 = 0$

$$\begin{bmatrix} 1-\lambda & 1 \\ -2 & 4-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

Characteristic equation.

$$\begin{bmatrix} 1-\lambda & 1 \\ -2 & 4-\lambda \end{bmatrix} = 0$$

$$(1-\lambda)(4-\lambda)+2=0$$

$$4-\lambda-4\lambda+\lambda^2+2=0$$

$$\lambda^2-5\lambda+6=0$$

$$\lambda^2-3\lambda-2\lambda+6=0$$

$$\lambda(\lambda-3)-2(\lambda-3)=0$$

$$(\lambda-3)(\lambda-2)=0$$

$$(\lambda-3)(\lambda-2)=0$$

$$\lambda-3=0 \qquad \lambda-2=0$$

$$\lambda_1=3 \qquad \lambda=2$$

are eigen values.

Now find the eigen vector, of $\lambda_1=3$ put in ① & ②

Then $x_1+x_2=3x_1$ — ①

$= -2x_1+x_2=0$

$\Rightarrow 2x_1 - x_2 = 0$

$\Rightarrow -2x_1 + 4x_2 = 3x_2$ — (ii)

$\Rightarrow -2x_1 + x_2 = 0$

$\Rightarrow 2x_1 - x_2 = 0$

$x_1 = \frac{1}{2} x_2$

let $x_2 = r$

where $r \neq 0$

So $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} r \\ r \end{bmatrix}$

eigen vector for $\lambda_2 = 2$ put in i Eq 2.

$x_1 + x_2 = 2x_1$ — (i)

$-2x_1 + 4x_2 = 2x_2$ — (ii)

$= -x_1 + x_2 = 0$

$\Rightarrow x_1 - x_2 = 0$

$= x_1 = x_2$

$= -2x_1 + 4x_2 = 2x_2$ — (1)

$= -2x_1 + 2x_2 = 0$

$x_1 - x_2 = 0$

$x_1 = x_2$

$x_1 = r$ then $x_2 = r$

So $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} r \\ r \end{bmatrix}$