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Semester

2<sup>nd</sup>

Subject

LCA

Dept

B.E (Electrical)

Submitted To:

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Assignment No # 3

## Chapter # 4

(9) Evaluate the following/det  
determine

$$(a) \begin{vmatrix} 9 & 1 \\ -4 & 3 \end{vmatrix} \quad (b) \begin{vmatrix} 0 & 9 & 11 \\ 6 & 4 & 1 \\ 3 & -1 & 5 \end{vmatrix}$$

Solution: The first determined is  
Simply evaluated as

$$\begin{vmatrix} 9 & 1 \\ -4 & 3 \end{vmatrix} = (9 \times 3) - (1 \times (-4)) \\ = 6 - (-4) \\ = 6 + 4 \\ = 10$$

$$\boxed{\det |A| = 10}$$

(b) Evaluate the determine from  
the first column.

$$\begin{vmatrix} 0 & 9 & 11 \\ 6 & 4 & 1 \\ 3 & -1 & 5 \end{vmatrix} = 0 \begin{vmatrix} 4 & 1 \\ -1 & 5 \end{vmatrix} - 6 \begin{vmatrix} 2 & 11 \\ -1 & 5 \end{vmatrix} + 3 \begin{vmatrix} 2 & 1 \\ 4 & 1 \end{vmatrix}$$

$$= 0(4 \times 5 - 1 \times (-1)) - 6(9 \times 5 - 11 \times (-1)) + 3(2 \times 1 - 11 \times 4)$$

$$= 0(22) - 6(21) + 3(-42)$$

$$= 0 - 126 - 126$$

Apply KCL to node  $V_2$  given

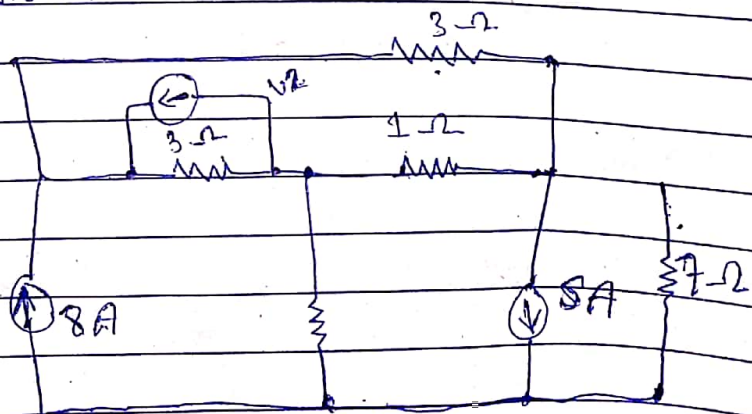
$$5 = \frac{V_3 - V_2}{10} + \frac{V_3}{200} \quad \text{--- (4)}$$

Solving the four equations

(1) (2) (3) & (4) Thus

$$V_2 = 171.639 \text{ V}$$

(13) Using the bottom node as reference determine the voltage across the  $5\text{-}\Omega$  resistor in the circuit of fig 4-39 and calculate the power dissipated by the  $7\text{-}\Omega$  resistor.

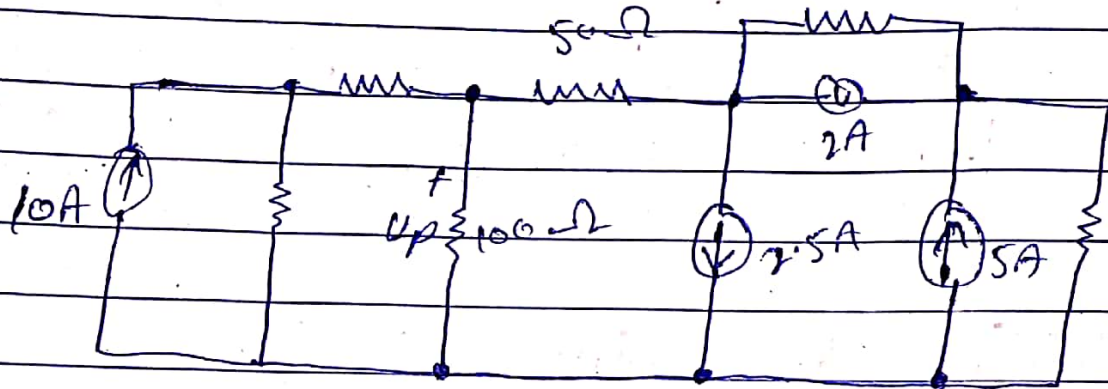


Solution

Start with the labeling the ~~total~~ nodes of the circuit.

$$\boxed{|\det B| = -252}$$

(12) Use Nodal analysis to find  $V_p$  in the circuit



Solution:

we redraw the given circuit and specify the node voltage as shown below

Apply KCL to node  $V_1$  given

$$10 = \frac{V_1}{20} + \frac{V_1 - V_p}{40} \quad \text{--- (1)}$$

Apply KCL to node  $V_p$  gives

$$0 = \frac{V_p - V_1}{40} + \frac{V_p}{100} + \frac{V_p - V_2}{50}$$

Apply KCL to node  $V_2$  gives,

$$2 - 2.5 = \frac{V_2 - V_p}{50} + \frac{V_2 - V_3}{10}$$

Solution: Start with labeling the nodes of the circuit

Let the voltage across the 8A current source to be  $V_1$  while  $V_2$  denote the voltage across the  $5\Omega$  resistor. Finally, let the voltage across the 5A current source is labeled  $V_3$

For the node  $V_1$

$$-8 - 4 \frac{V_1 - V_2}{3} + \frac{V_1 - V_3}{3} = 0$$

$$\left(\frac{1}{3} + \frac{1}{3}\right)V_1 - \left(\frac{1}{3}\right)V_2 - \left(\frac{1}{3}\right)V_3 = 12$$

$$2V_1 - V_2 - V_3 = 36 \quad \text{--- (1)}$$

For the node  $V_2$

$$4 + \frac{V_2 - V_1}{3} + \frac{V_2}{5} + \frac{V_2 - V_3}{1} = 0$$

$$-\left(\frac{1}{3}\right)V_1 + \left(\frac{1}{3} + \frac{1}{3}\right)V_2 - V_3 = -4$$

For node  $U_3$

$$5 + \frac{U_3 - U_2}{1} + \frac{U_3 - U_1}{3} + \frac{U_3}{7} = 0$$

$$-\left(\frac{1}{3}\right)U_1 - U_2 + \left[1 + \frac{1}{3} + \frac{1}{7}\right]U_3 = -5$$

$$-7U_1 - 21U_2 + 31U_3 = 105 \quad \text{--- (3)}$$

Solve the three equations:

(1) (2) and (3)

$$U_1 = 96.733V$$

$$U_2 = 2.833V$$

$$U_3 = 8.633V$$

$$\text{Thus } 45\Omega = U_2$$

$$|45\Omega = 2.833V|$$

Since the voltage across the  $7\Omega$  resistor is  $U_3$ , therefore

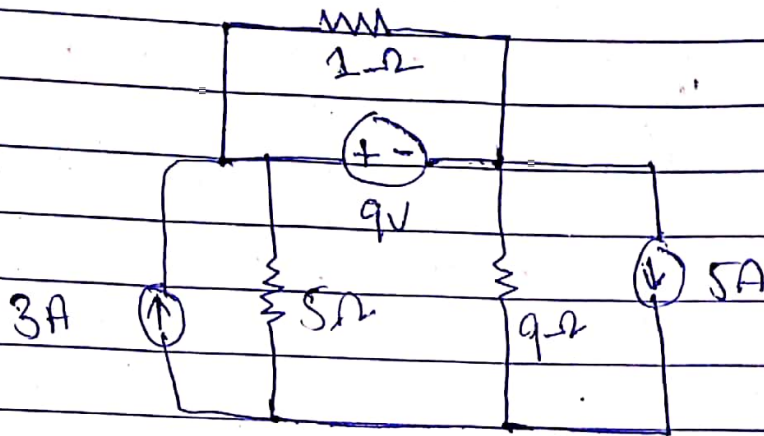
$$P_{7\Omega} = \frac{U_3^2}{7}$$

$$= \frac{8.633^2}{7}$$

$$= \frac{74.528689}{7}$$

$$P_{7\Omega} = 10.646W$$

(19) Determine a numerical voltage for the voltage labeled.



Solution :- Consider  $v_1$  and  $v_2$  as a Super node

Apply KCL on Supernode.

$$45v_1 - 45v_2 + 9v_1 + 45v_2 - 45v_1 + 9v_2 = 0$$

$$9v_1 + 9v_2 = 135 \quad \text{--- (1)}$$

So  $v_1 - v_2 = 9 \quad \text{--- (2)}$

Combine equation (1) and (2)

$$9v_1 + 9v_2 = 135$$

$$9v_1 - 9v_2 = 81$$

$$18v_1 = 216$$

$$v_1 = \frac{216}{18}$$

$$v_1 = 12V$$

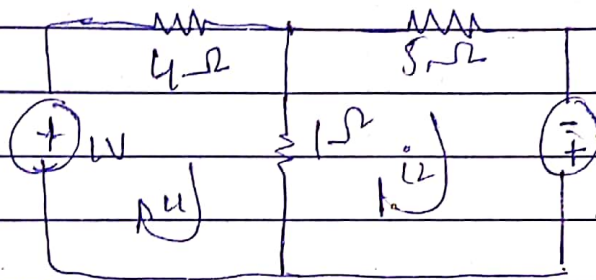
putting in eq (8)

$$V_2 = V_1 - 9$$

$$V_2 = 12 - 9$$

$$\boxed{V_2 = 3}$$

(29) Determine the current flowing out of the positive terminal of each voltage source in the circuit.



Solution by Mesh Analysis

Apply KVL on  $i_1$ .

$$4i_1 + (i_1 - i_2) = 1$$

$$5i_1 - i_2 = 1 \quad \text{--- (1)}$$

Apply KVL on  $i_2$

$$i_2(i_2 - i_1) = 1$$

$$5i_2 - i_1 = 1 \quad \text{--- (2)}$$



Apply KVL on  $i_2$

$$1(i_2 - i_1) + 5i_2 = 9$$

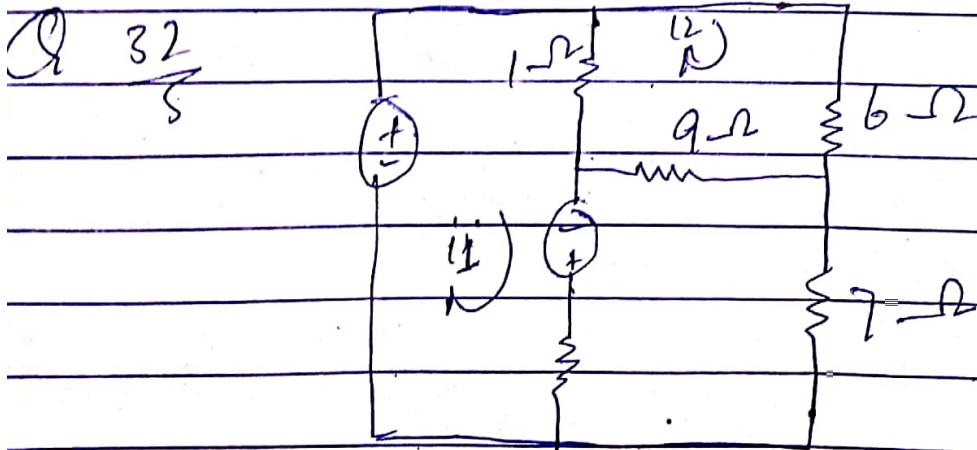
$$-i_1 + 6i_2 = 9 \quad \text{--- (1)}$$

Multiplying 5 with equation (2)  
So adding with eq (1)

$$\begin{aligned} 5i_1 - i_2 &= 1 \\ -5i_1 + 30i_2 &= 10 \\ \hline 29i_2 &= 11 \end{aligned}$$

$$i_2 = \frac{11}{29}$$

Result =  $i_1 = 0.275A$   
 $i_2 = 0.379A$



Solution:

write the first mesh equation.

$$-2 + 1(i_1 - i_2) - 3 + 5(i_1 - i_3) = 0$$

$$6i_1 - 5i_2 = 5 \quad \text{--- (1)}$$

write the 2nd mesh equation:

$$1(i_2 - i_1) + 6i_2 + 9(i_2 - i_3) = 0$$

$$-5i_1 + 16i_2 - 9i_3 = 0$$

write the third mesh equation:

$$5(i_3 - i_1) + 3 + 9(i_3 - i_2) + 7i_3 = 0$$

$$-5i_1 - 9i_2 + 21i_3 = -3 \quad \text{--- (3)}$$

Solve the three equations

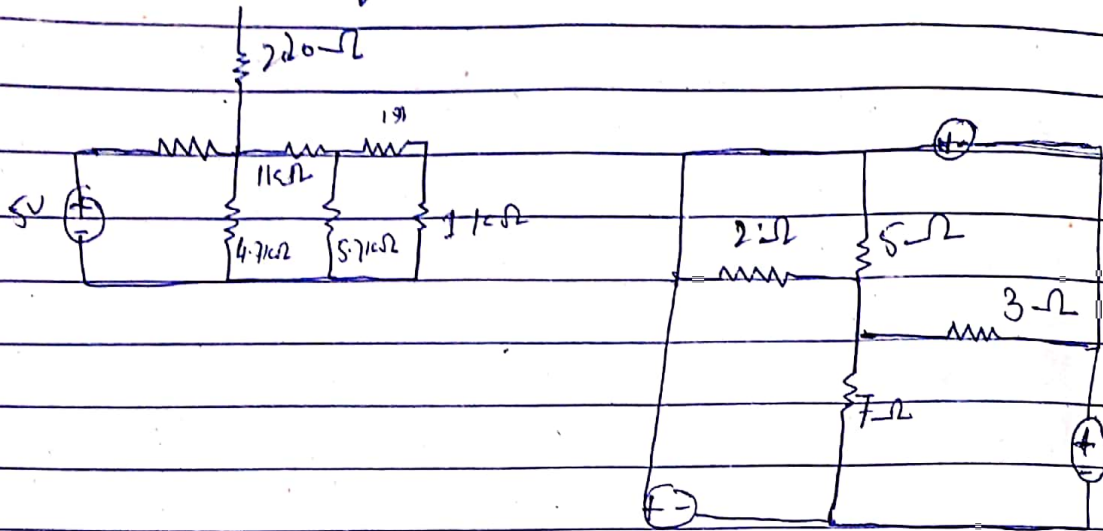
(1) (2) and (3) then

$$i_1 = 987.205 \text{ mA}$$

$$i_2 = 150.147 \text{ mA}$$

$$i_3 = 157.017 \text{ mA}$$

(Q No 35) Chose the manzero value of the three voltage source of figure so that no current any resistor in the circuit.



Sol: Lets name the source  $v_1, v_2$  and  $v_3$  from the bottom.

And the current  $i_1, i_2, i_3$

For the current in resistance to be equal to Zero we need.

$$i_1 - i_2 = 0$$

$$i_3 - i_4 = 0$$

$$i_1 - i_3 = 0$$

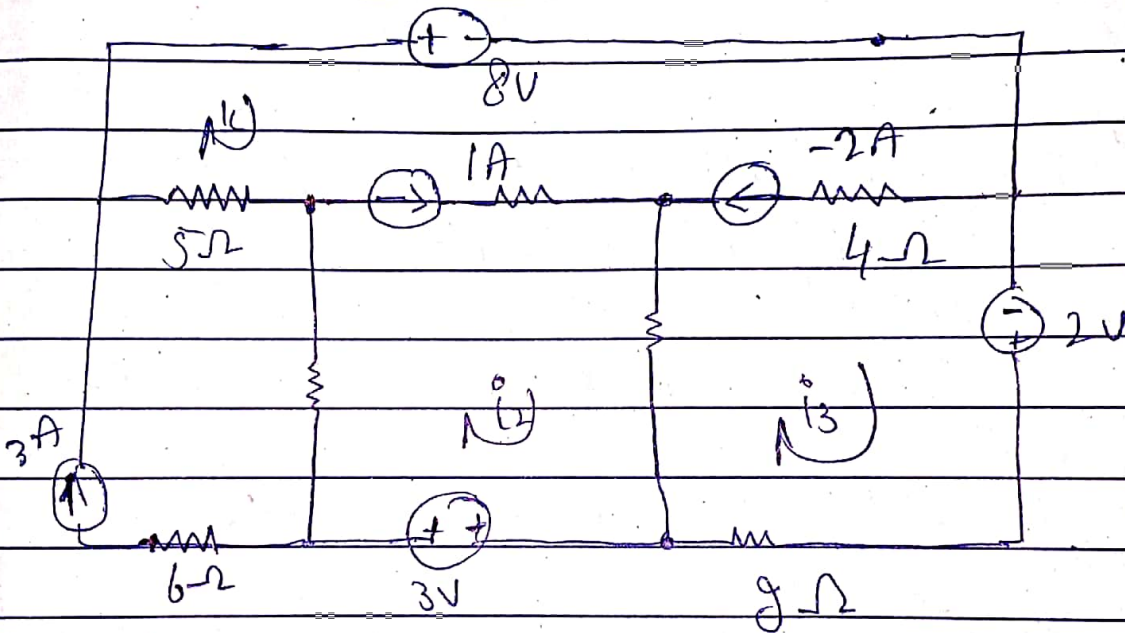
$$i_2 - i_4 = 0$$

Now we apply KVL on each of the four loop.

$$\begin{aligned} 2 \quad (i_1 - i_3) + 5 \quad (i_1 \quad i_2) &= 0 \\ 3 \quad (i_2 - i_4) + 5 \quad (i_2 \quad i_1) &= -02 \\ 7 \quad (i_3 - i_4) + 2 \quad (i_3 \quad i_3) &= v_1 \\ 3 \quad (i_4 - i_2) + 7 \quad (i_4 \quad i_3) &= v_3 \end{aligned}$$

(3)  $v_1 - v_2 - v_3 = 0$

(16)



Solution: Let's call the controlled mesh current  $i_4$  we can immediately see that

$$i_4 = 3A$$

Mesh analysis of  $i_1 - i_2 - i_3$   
Super node given us.

$$2 + 5(i_1 - 3) + 3(i_2 - 3)$$

$$-3 + 2i_1 - 2 = 0$$

By Apply KVL on the lower  
 b/w Super-Node  
 and the upper right node  
 we get:

$$i_2 - i_1 = 0$$

$$2 + i_1 - i_3 = 0$$

After solving these three  
 equation we get

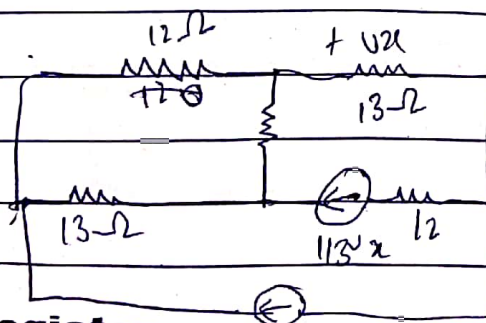
$$i_1 = 1.4 \text{ A}$$

$$i_2 = 2.4 \text{ A}$$

$$i_3 = 3.4 \text{ A}$$

Q No 47:- Figure

Through carefully application  
 of the Super mesh technique  
 obtain values for all three  
 mesh current as labeled.



We are going in the problem  
that  $I_1 = 5A$

The equation for the second mesh  
can be write as,

$$I_3 = I_1 = \frac{0x}{3}$$

$$3I_3 - 5(3) = 0x \quad 13I_3$$

$$3I_3 - 15 = 13I_3$$

$$-15 = 10I_3$$

$$I_3 = -1.5A$$

The equation of the 3<sup>rd</sup> mesh

$$-13I_1 + 36I_2 - 11I_3 = 0$$

$$-13(5) + 36I_2 - 11(-1.5) = 0$$

$$-65 + 36I_2 - 11(-1.5) = 0$$

$$36I_2 = 48.5$$

$$I_2 = 1.3A$$

Result

$$I_1 = 5A$$

$$I_2 = 1.3A$$

$$I_3 = 1.5A$$

Two meshes 1 and 2, we obtain the following matrix equation.

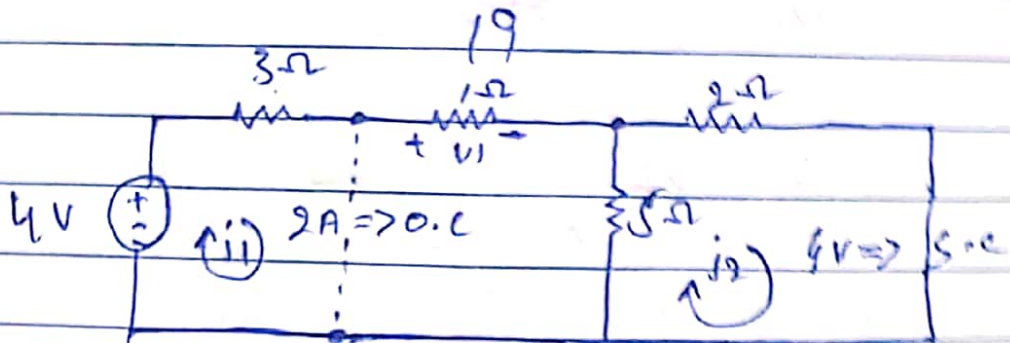
$$\begin{bmatrix} 3+1+5 & -5 \\ -5 & 5+2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

We find

$$i_1 = \frac{14}{19} \text{ A}$$

For  $1\text{-}\Omega$  resistor, ohm's law gives

$$v_1 = 1 \cdot i_1 = \frac{14}{19} \text{ V} \approx 736.84 \text{ mV}$$



To obtain  $v_2$  we set the two  $4\text{V}$  source to zero as shown below.

Apply nodal analysis to the two nodes  $v_a$  and  $v_b$  we obtain the following matrix equation

$$\begin{bmatrix} \frac{1}{3} + 1 & -1 \\ -1 & 1 + \frac{1}{3} + \frac{1}{2} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

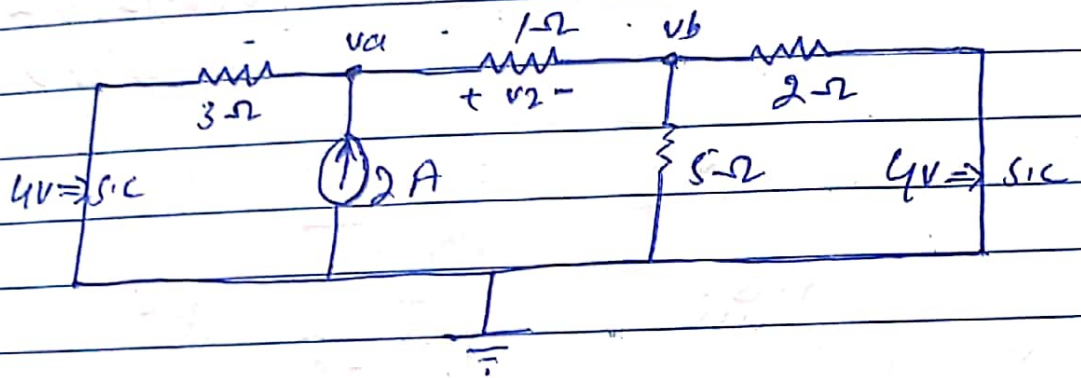
matrix equation we find:

$$v_a = \frac{51}{19} \text{ V and } v_b = \frac{21}{19} \text{ V}$$

By inspection, it is clear that

$$v_2 = v_a - v_b = \frac{51}{19} - \frac{30}{19} = \frac{21}{19}$$

$$v_2 = 1.105 \text{ V}$$



To obtain  $v_3$  we set the 2A and the left 4V source to zero as shown below

Apply mesh analysis to the two meshes 1 and 2 we obtain the following matrix equation.

$$\begin{bmatrix} 3+1+5 & -5 \\ -5 & 5+2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$

The matrix equation we find:

$$i_1 = \frac{-10}{19} \text{ A}$$



for the  $1\text{-}\Omega$  resistor, ohm's law gives

$$V_3 = I \cdot R = \frac{10}{19} \text{ V}$$

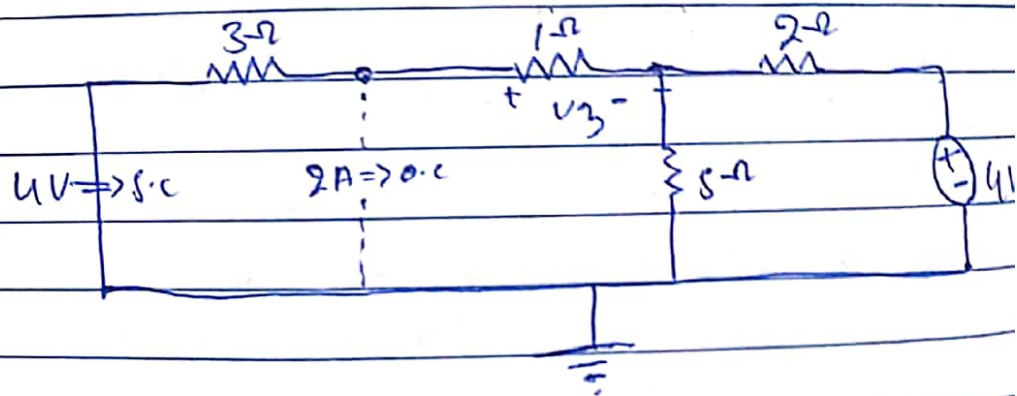
$$V_3 = -526.32 \text{ mV}$$

Therefore

$$V_x = V_1 + V_2 + V_3$$

$$V_x = 736.84 \text{ mV} + 1.105 \text{ V} - 526.32 \text{ mV}$$

$$V_x = 1.316 \text{ V}$$



It is required to evaluate the value of the current source that results in reducing the  $V_x$  values by 10%. Let the new values of  $V_x$  is  $V$ . The current source values affects only its contribution which is  $V_2$ . Let the new values

$v_2$  is  $v'_2$ . Thus

$$v'_2 = 0.9 \cdot v_x = 0.9 \cdot 1.316 \text{ V}$$

$$v'_2 = 1.1844 \text{ V}$$

and

$$v'_2 = v'_x - (v_1 + v_3) = 1.1844 \text{ V} - (736.84 \text{ mV} - 526.32 \text{ mV}) = 973.88 \text{ mV}$$

Apply nodal analysis to the two nodes  $v_a$  and  $v_b$  in the circuit shown below gives.

$$I_{CS} = \left(1 + \frac{1}{3}\right) v_a - v_b \quad \text{--- (1)}$$

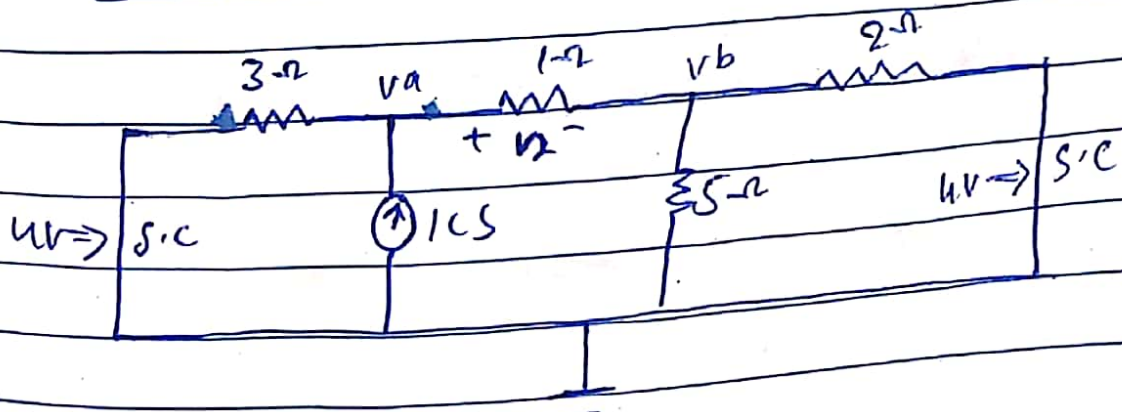
$$0 = -v_a + \left(1 + \frac{1}{5} + \frac{1}{2}\right) v_b \quad \text{--- (2)}$$

We have

$$v_a - v_b = v'_2 = 973.88 \text{ mV} \quad \text{--- (3)}$$

Solving the three equations (1), (2) and (3) we

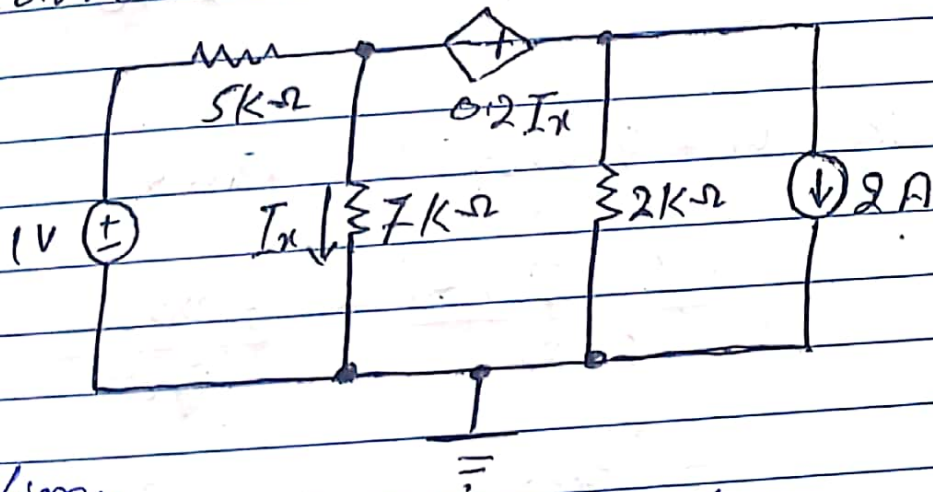
$$I_{CS} = 1.7623 \text{ A}$$



we verify our result in part  
(a) using Pspice as shown below  
we obtain.

$$V_x = 6.053 - 4.737 = \boxed{1.316V}$$

(11) Employ superposition principles  
to obtain a values for the  
current  $I_x$  as labeled.



Solution:

Since there are two independent sources, let

where  $I_{x1}$  and  $I_{x2}$  are the contribution due to the 1V voltage source and 2A current source, respectively.

To obtain  $I_{x1}$ , we set the 2A current source to zero (replacing it with an open circuit) as shown below.

Apply KCL to the super node  
 X gives

$$\frac{V_1 - 1}{5000} + \frac{V_1}{7000} + \frac{V_2}{2000} = 0$$

But  $V_2 = V_1 + 0.2 I_{X1}$ ; here

$$\frac{V_1 - 1}{5000} + \frac{V_1}{7000} + \frac{V_1 + 0.2 I_{X1}}{2000} = 0$$

But  $V_1 = 7000 I_{X1}$ ; hence,

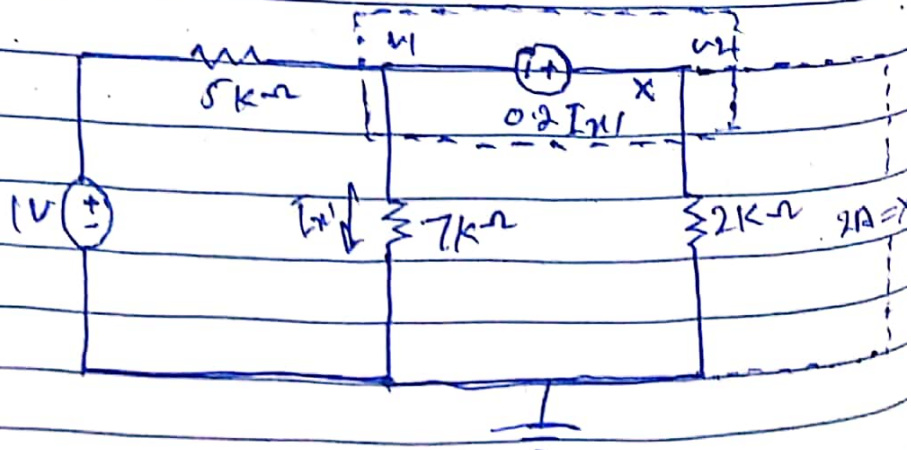
$$\frac{7000 I_{X1} - 1}{5000} + \frac{7000 I_{X1}}{7000}$$

$$+ \frac{7000 I_{X1} + 0.2 I_{X1}}{2000} = 0$$

Apply

Then

$$I_{X1} = 33.9 \mu A$$



To find  $I_{x2}$  we set the 1 V voltage source to zero (replacing it with a short circuit) as shown below.

Apply KCL to the supernode  $y$  given

$$\frac{V_1}{5000} + \frac{V_1}{7000} + \frac{V_2}{2000} = -2$$

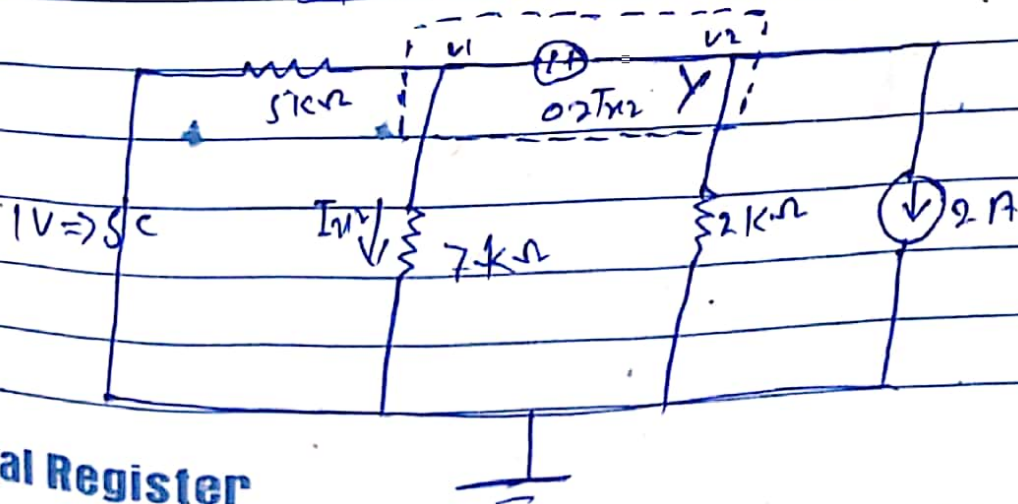
But  $V_1 = 7000 I_{x2}$ ; hence

$$\frac{7000 I_{x2}}{5000} + \frac{7000 I_{x2}}{7000} + \frac{7000 I_{x2} + 0.2 I_{x2}}{2000} = -2$$

Thus  $I_{x2} = -338.98 \text{ mA}$

Therefore;  $I_x = I_{x1} + I_{x2}$   
 $= 33.9 \mu\text{A} + (-338.98 \text{ mA})$

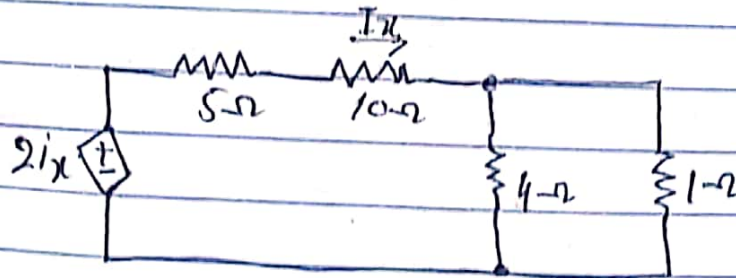
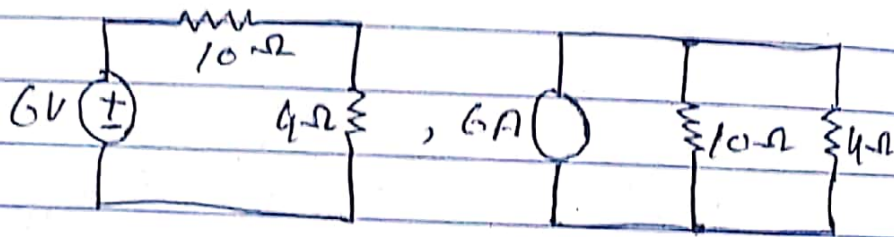
$I_x = -338.95 \text{ mA}$



Result:

$$I_x = -338.95 \text{ mA}$$

13: perform an appropriate source transformation on each of the circuit depicted in 5.58, taking care to retain the  $4\text{-}\Omega$  resistor in each final circuit.



Solution:

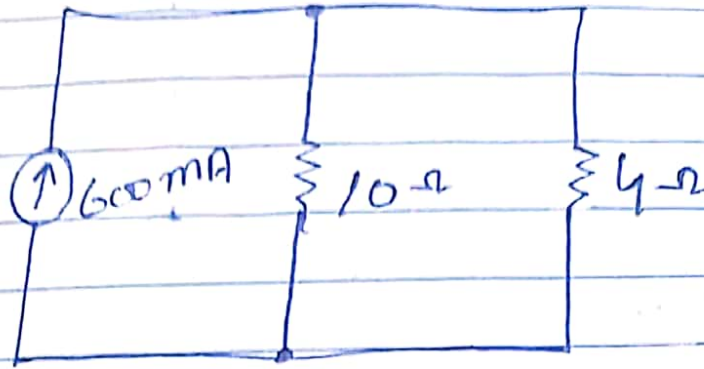
To get the values of the new source we use:

$$I = 4/R$$

$$I = 6/10$$

$$I = 0.6 \text{ A}$$

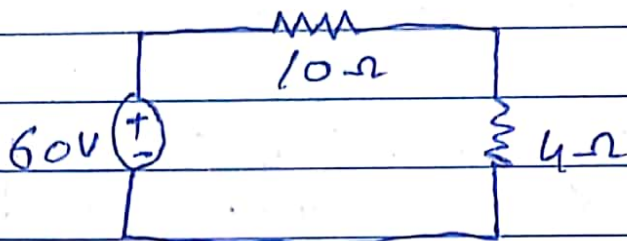
And we can draw the circuit as:



The new voltage source will have the value of:

$$U = 10 \cdot 6$$
$$U = 60 \text{ V}$$

And we draw it as

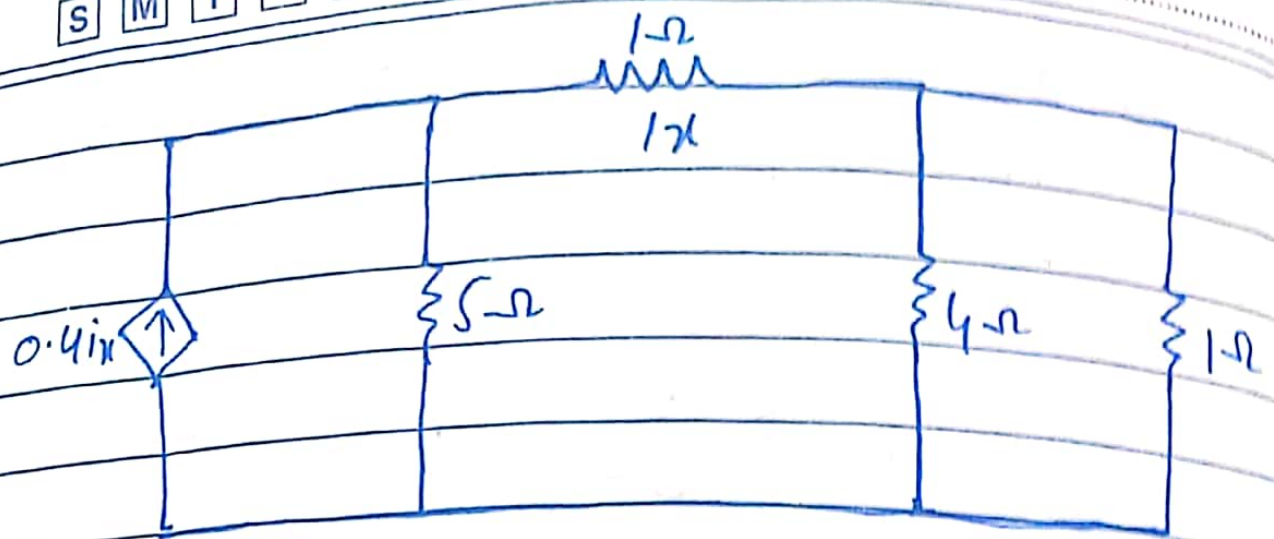


To get the value of the new current source we use

$$I = \frac{2ix}{5}$$

$$I = 0.4ix$$

And we draw it as -



Result ::

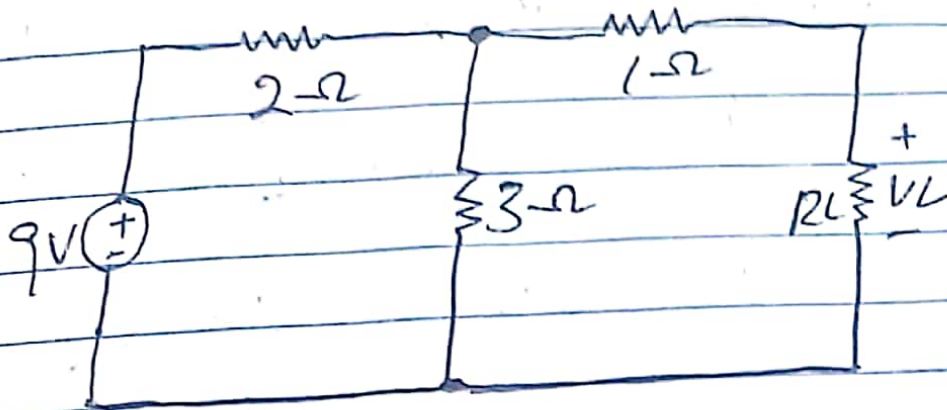
(a) we replace the  $10\ \Omega$  resistor and the voltage source with a  $600\ \text{mA}$  current source in parallel with a  $10\ \Omega$  resistor.

(b) we replace the  $10\ \Omega$  resistor and the current source with a  $60\ \text{V}$  voltage source in series with a  $10\ \Omega$  resistor.

(c) we replace the  $5\ \Omega$  resistor and the dependent voltage source with a dependent current source labeled  $0.4ix$  in parallel with a  $5\ \Omega$  resistor.



25. a) Determine the Thevenin equivalent of the network connected to  $R_L$ . b) Determine  $V_L$  for  $R_L = 1\Omega, 3.5\Omega, 6.257\Omega$  and  $9.8\Omega$



Solution:

To get  $V_{Th}$  we disconnect  $R_L$  and find the voltage between the two disconnected points. As we can see this voltage is the one on the  $3\Omega$  resistor.

$$V_{Th} = 9V \cdot \frac{3}{5}$$

$$V_{Th} = 5.4V$$

We can calculate  $R_{Th}$  as:

$$R_{Th} = 1 + 3 \parallel 2$$

$$R_{Th} = 2.2\Omega$$

then we calculate  $V_L$  as:

$$V_L = V_{Th} \cdot \frac{R_L}{R_L + R_{Th}}$$

For each value of  $R_L$  we get:

$$R_L = 1\Omega \Rightarrow V_L = 1.688V$$

$$R_L = 3.5\Omega \Rightarrow V_L = 3.316V$$

$$R_L = 6.257\Omega \Rightarrow V_L = 3.995V$$

$$R_L = 9.8\Omega \Rightarrow V_L = 4.41V$$

Result:

$$V_{Th} = 5.4V, R_{Th} = 2.2\Omega$$

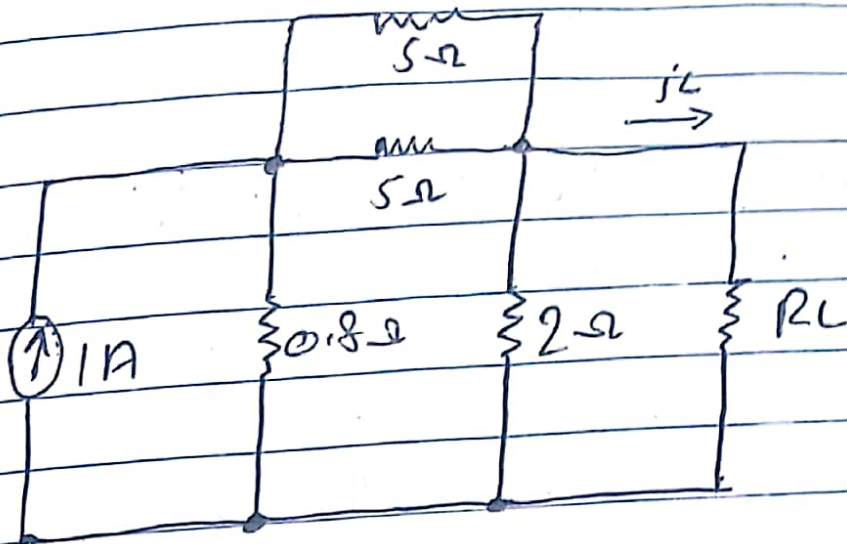
$$V_L = 1.688V, 3.316V, 3.995V, 4.41V$$

27: (a) obtain the norton equivalent of the network connected to  $R_L$ .

(b) obtain the Thevenin equivalent of the same network.

(c) use either to calculate  $i_L$  for  $R_L = 0\Omega, 1\Omega, 4.923\Omega$

cmd  $8.107 \Omega$



Solution:

(a) we calculate  $R_N$  as:

$$R_N = (0.8 + 5 \parallel 5) \parallel 2$$

$$R_N = 3.3 \parallel 2$$

$$R_N = 1.245 \Omega$$

For  $i_N$  we have:

$$i_N = \frac{0.8}{0.8 + 2.5}$$

$$i_N = 0.242 A$$

(b) now we can  $V_{TH}$  as:

$$V_{TH} = i_N \cdot R_N$$

$$V_{Th} = 0.302V$$

And

(c) using Thevenin equivalent we get:

$$i_L = \frac{V_{Th}}{R_{Th} + R_L}$$

For each value of  $R_L$  giving us:

(1)  $R_L = 0\Omega \Rightarrow i_L = 0.243A$

(2)  $R_L = 1\Omega \Rightarrow i_L = 0.135A$

(3)  $R_L = 4.923\Omega \Rightarrow i_L = 0.049A$

(4)  $R_L = 8.107\Omega \Rightarrow i_L = 0.032A$

Result :

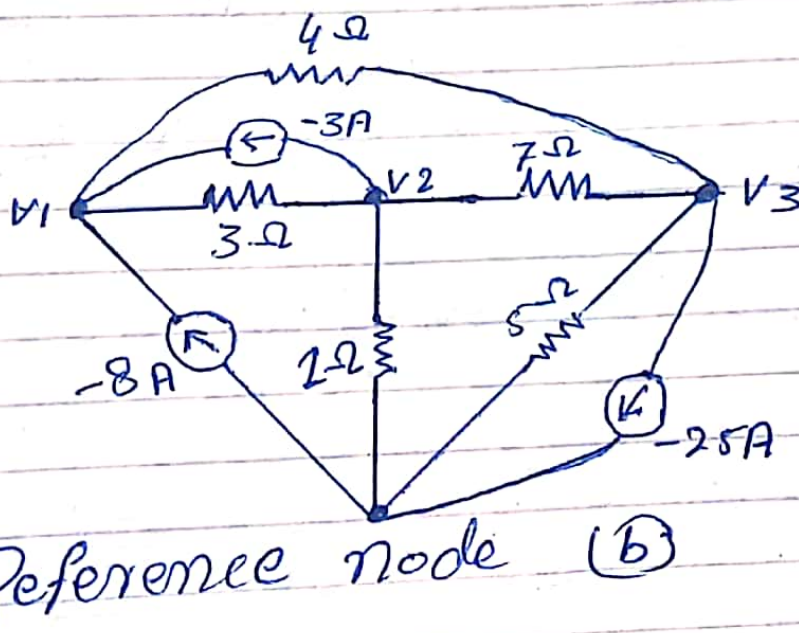
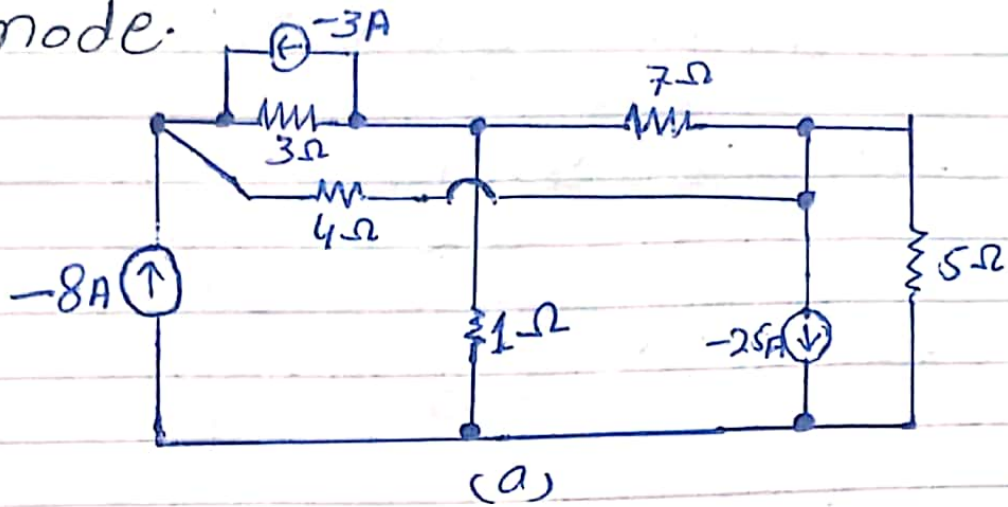
(a)  $i_N = 0.242A$ ,  $R_N = 1.245\Omega$

(b)  $V_{Th} = 0.302V$ ,  $R_{Th} = 1.245\Omega$

(c)  $i_L = 0.243A, 0.135A, 0.049A, 0.032A$ .

## Example 4.2

Determine the nodal voltages for the circuit of 4.4a, as reference to the bottom node.



**Solution:**

We have three nodes in the circuit  $v_1$ ,  $v_2$  and  $v_3$ .

Applying KCL on node 1

$$-8 - 3 = \frac{V_1 - V_2}{3} + \frac{V_2 - V_3}{4}$$

Taking L.C.M

$$-11 = \frac{4V_1 - 4V_2 + 3V_1 - 3V_3}{12}$$

$$-11 = \frac{7V_1}{12} - \frac{4V_2}{12} - \frac{3V_3}{12}$$

$$-11 = 0.5833V_1 - 0.3333V_2 - 0.25V_3$$

Applying KCL at node 2

$$3 = \frac{V_2 - V_1}{3} + \frac{V_1}{1} + \frac{V_2 - V_3}{7}$$

$$3 = \frac{7V_2 - 7V_1 + 21V_2 + 3V_3 - 3V_3}{21}$$

$$3 = \frac{-7V_1}{21} + \frac{31V_2}{21} - \frac{3V_3}{21}$$

$$3 = -0.3333V_1 + 1.4762V_2 - 0.1429V_3$$

Applying KCL at node 3

$$25 = \frac{V_3}{5} + \frac{V_2 - V_3}{7} + \frac{V_3 - V_1}{4}$$

Taking LCM

$$25 = \frac{28V_1 + 20V_2 - 20V_3 + 35V_3 - 35V_1}{140}$$

$$25 = \frac{-35V_1}{140} - \frac{20V_2}{140} + \frac{85V_3}{140}$$

$$25 = -0.25V_1 - 0.1429V_2 + 0.5929V_3$$

↳ Simplifying further by Grammer Rule

$$\begin{bmatrix} 0.5833 & -0.3333 & -0.2500 \\ -0.3333 & 1.4762 & -0.1429 \\ -0.2500 & -0.1429 & 0.5929 \end{bmatrix} \begin{matrix} V_1 \\ V_2 \\ V_3 \end{matrix} = \begin{matrix} -11 \\ 3 \\ 25 \end{matrix}$$

Let det of A =  $\Delta x$

$$\Delta x = \begin{vmatrix} 0.5833 & -0.3333 & -0.2500 \\ -0.3333 & 1.4762 & -0.1429 \\ -0.2500 & -0.1429 & 0.5929 \end{vmatrix}$$

$$\Delta x = 0.5833 \begin{vmatrix} 1.4762 & -0.1429 \\ -0.1429 & 0.5929 \end{vmatrix}$$

$$-(-0.3333) \begin{vmatrix} -0.3333 & -0.1429 \\ 0.2500 & 0.5929 \end{vmatrix} - 0.2500 \begin{vmatrix} -0.3333 & 1.4762 \\ 0.2500 & -0.1429 \end{vmatrix}$$

$$\Delta x = 0.5833(0.875 - 0.020) + 0.3333(-0.1976 - 0.0357) - 0.2500(0.6476 - 0.3696)$$

$$\Delta x = 0.49868 - 0.0777 + 0.079$$

$$\Delta x = 0.316$$

Now

$$V_1 = \frac{1}{\Delta x} \begin{bmatrix} -11 & -0.33 & -0.250 \\ 3 & 1.476 & -0.142 \\ 25 & -0.142 & 0.592 \end{bmatrix}$$

$$= -11 \begin{bmatrix} 1.476 & -0.142 \\ 0.142 & 0.592 \end{bmatrix} - 3 \begin{bmatrix} 0.33 & -0.250 \\ -0.142 & 0.592 \end{bmatrix}$$

$$+ 25 \begin{bmatrix} 0.33 & -0.250 \\ 1.476 & -0.142 \end{bmatrix}$$

$$= -11(0.875 - 0.020) - 3(-0.197 - 0.035) + 25(0.047 + 0.36)$$

$$= -10.145 + 0.69 + 10.41$$
$$= 1.714$$

$$V_1 = \frac{1.714}{0.3167} = 5.412 \text{ V}$$

$$V_2 = \frac{1}{\Delta x} \begin{bmatrix} 0.583 & -11 & -0.250 \\ -0.333 & 3 & -0.142 \\ -0.250 & 25 & 0.592 \end{bmatrix}$$

$$= 0.583 \begin{bmatrix} 3 & -0.142 \\ 25 & 0.592 \end{bmatrix} + 11 \begin{bmatrix} -0.333 & -0.142 \\ -0.250 & 0.592 \end{bmatrix}$$

$$- 0.250 \begin{bmatrix} 0.333 & 3 \\ 0.250 & 25 \end{bmatrix}$$



$$= 0.583(1.778 + 3 \cdot 57.2) + 11(0.197 - 0.035) - 0.25(-8.332 + 0.75)$$

$$= 3.86 - 2.56 + 2.83$$

$$= 2.45$$

So

$$V_2 = \frac{2.45}{0.3167} = 7.73 \text{ V}$$

Now

$$V_3 = \frac{1}{\Delta X} \begin{bmatrix} 0.5833 & -0.333 & -11 \\ -0.33 & 1.476 & 3 \\ -0.250 & -0.142 & 25 \end{bmatrix}$$

$$0.583 \begin{bmatrix} 0.333 & -11 \\ -0.250 & 1.476 & 3 \end{bmatrix} - 0.333 \begin{bmatrix} 0.33 & 3 \\ -0.25 & 25 \end{bmatrix}$$

$$-11 \begin{bmatrix} 0.333 & 1.476 \\ 0.250 & -0.142 \end{bmatrix}$$

$$= 0.583(15.238) + 0.333(-9.0825) - 11(0.107)$$

$$= 14.67$$

$$\text{So } V_3 = \frac{14.67}{0.316} = 46.32 \text{ V}$$

So

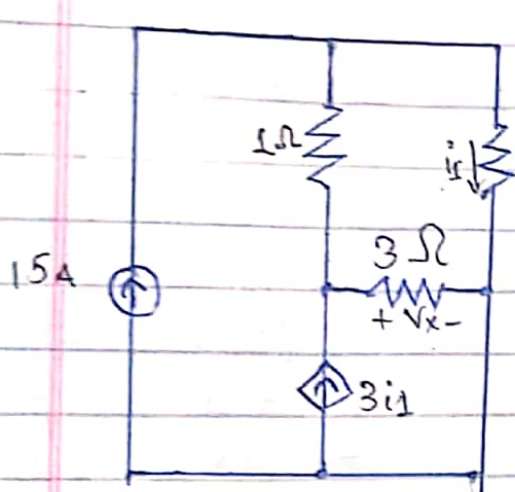
$$V_1 = 5.412 \text{ V}$$

$$V_2 = 7.73 \text{ V}$$

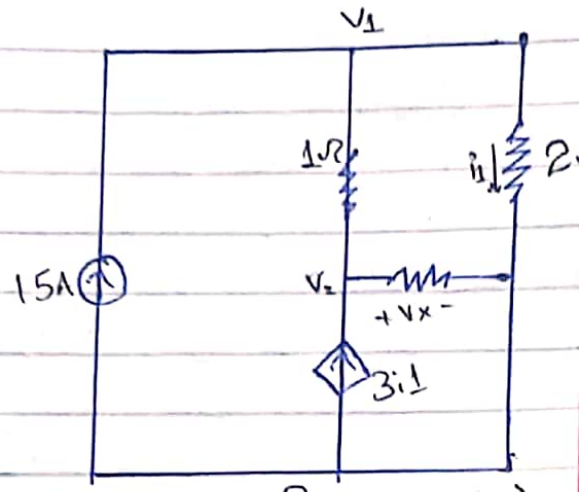
$$V_3 = 46.32 \text{ V}$$

## Example 4.3

Determine the power supplied by the dependent source of figure.



(a)



Ref (b)

## Example 4.3

Solution:

↳ Applying KCL on  $V_2$

$$15 = V_1 - V_2 = \frac{V_1}{2}$$

$$15 = \frac{2V_1 - 2V_2 + V_1}{2}$$

$$30 = 3V_1 - 2V_2 \quad \text{--- (i)}$$

↳ Applying KCL on  $V_2$

$$3i_1 = V_2 - V_1 + \frac{V_2}{2}$$

$$3i = 3v_2 - 3v_1 + v_2 \quad \dots (2)$$

$i_1$  is flowing across

$$\frac{v_1}{3\Omega} \text{ or } \frac{v_1}{2}$$

So  $i_1 = \frac{v_1}{2}$

Put eq (2)

$$\frac{3v_1}{2} = \frac{3v_2 - 3v_1 + v_2}{3}$$

$$9v_1 = 6v_2 - 6v_1 + v_2$$

$$8v_2 + 15v_1 = 0$$

$$-15v_1 + 8v_2 = 0 \quad \dots (3)$$

Multiplying (3) with eq (i) by subtraction from eq (3)

$$\begin{array}{r} 15v_1 - 8v_2 = 0 \\ -15v_1 + 10v_2 = -30 \\ \hline 3v_2 = -30 \\ v_2 = -10 \end{array}$$

Multiplying (5) with eq (i)

$$5 \times 3v_1 - 2v_2 = 30$$

$$= 15v_1 - 10v_2 = 150$$

Combining eq (1) and eq (3)

$$\begin{array}{r} 15v_1 - 10v_2 = 150 \\ \underline{-15v_1 + 8v_2 = 0} \\ -2v_2 = -150 \end{array}$$

$$v_2 = 75$$

Putting in eq 3

$$\begin{array}{r} -15v_1 + 8(-75) = 0 \\ v_1 = -40 \end{array}$$

Now:

$$i_1 = \frac{v_1}{2} = \frac{-40}{2} = -20A$$

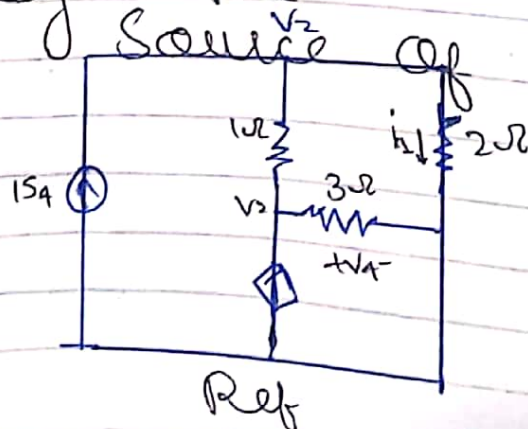
for Power,  $P = iv$

$$P = (3i_1)(v) = +3(-20)(-75)$$

$$P = 4.5 \text{ KW.}$$

## Example 4.4

Determine the power supplied by the dependent source of figure



Applying Kcl on  $v_1$

$$15 = v_1 - v_2 + \frac{v_1}{2}$$

$$\begin{aligned} 30 &= 2v_1 - 2v_2 + v_1 \\ 30 &= 3v_1 - 2v_2 \quad \dots (i) \end{aligned}$$

Apply Kcl on  $v_2$

$$3v_x = v_2 - v_1 + \frac{v_2}{3}$$

$$v_x = \frac{v_2}{3}$$

$$\frac{3v_2}{3} = \frac{3v_2 - 3v_1 + v_2}{3}$$

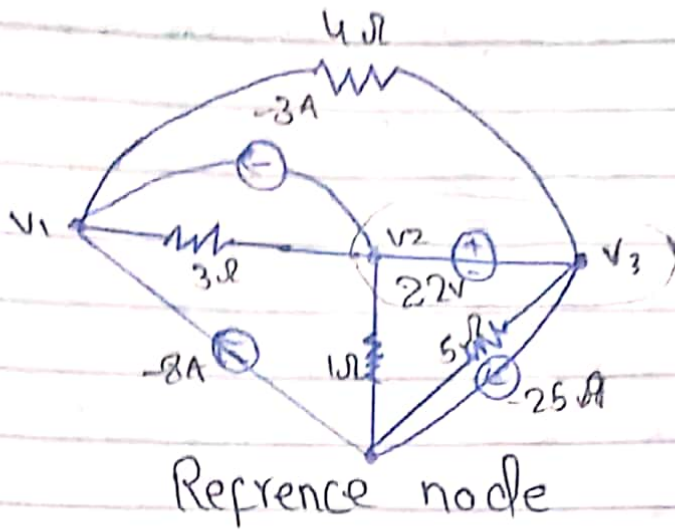
$$-3v_1 + v_2 = 0 \quad \dots (2)$$

Combine eqn (i) and (2)

$$\begin{aligned} 3v_1 - 2v_2 &= 30 \\ -3v_1 + v_2 &= 0 \\ \hline v_2 &= 30 \end{aligned}$$

Example 4.5 :

Determine the value of unknown  $v_1$  in the circuit of figure.



Solution :

Applying KCL on  $v_1$

$$-8 - 3 = \frac{v_1 - v_2}{3} + \frac{v_1 - v_3}{4}$$

$$-11 = \frac{4v_1 - 4v_2 + 3v_1 - 3v_3}{12}$$

$$\begin{aligned} (-11)(12) &= 7v_1 - 4v_2 - 3v_3 \\ -(132) &= 7v_1 - 4v_2 - 3v_3 \quad \dots (i) \end{aligned}$$

Applying KCL on node i.e

$v_2$  &  $v_3$

$$3 + 25 = \frac{v_2 - v_1}{3} + \frac{v_3 - v_1}{4} + \frac{v_3}{5} + v_2$$

$$28 = \frac{20v_2 - 20v_1 + 15v_3 + 12v_3 + 20v_2}{60}$$

$$(28)(60) = -35V_1 + 80V_2 + 27V_3$$

$$1680 = -35V_1 + 80V_2 + 27V_3 \quad \text{--- (2)}$$

Multiply 5 eq (1)

$$35V_1 - 20V_2 - 15V_3 = -660 \quad \text{--- (3)}$$

Combining eq (2) and (3)

$$\begin{array}{r} -35V_1 + 80V_2 + 27V_3 = 1680 \\ 35V_1 - 20V_2 - 15V_3 = -660 \\ \hline 60V_2 + 12V_3 = 1020 \end{array}$$

We know from Super node

$$V_2 + 22 = V_3 \quad \text{--- (4)}$$

Putting  $V_3$  in eq (4)

$$60V_2 + V_2 + 22 = 1020$$

$$61V_2 = 998$$

Dividing both side by 61

$$\frac{61V_2}{61} = \frac{998}{61}$$

$$V_2 = 998/61$$

$$v_2 = 16.4$$

putting in eq (5)

$$16.4 + 22 = v_3$$

$$v_3 = 38.4$$

putting  $v_2$  and  $v_3$  in eq (1)

$$-132 = 7v_1 - 4(16.4) - 3(38.4)$$

$$-132 = 7v_1 - 65.6 - 115.2$$

$$-132 = 7v_1 - 180.8$$

$$180.8 - 132 = 7v_1$$

~~Divided both side~~ ~~by 7~~

$$180.8$$

$$48.8 = 7v_1$$

Divided both side by 7

$$\frac{48.8}{7} = \frac{7v_1}{7}$$

$$\frac{48.8}{7} = v_1$$

$$v_1 = 6.971$$



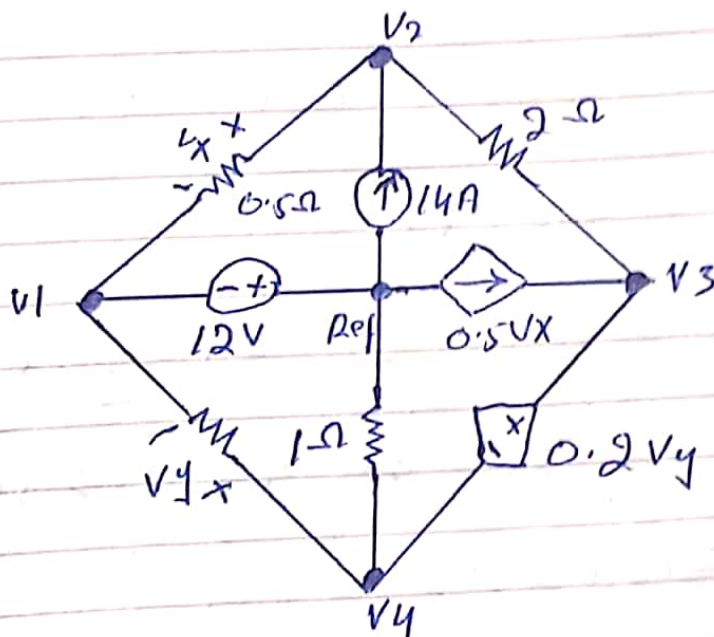
Result:

$$v_1 = 6.971 \text{ V}, \quad v_2 = 16.4 \text{ V}$$

$$v_3 = 38.4$$

### Example 4.6

Determine the node-to-reference voltage in the circuit.



Solution =

$$\text{As } v_2 = -12 \text{ V}$$

Apply KCL on  $v_2$

$$\frac{v_2 - v_1}{0.5} + \frac{v_2 - v_3}{2} = 14$$

$$\frac{2V_2 - 2V_1 + 0.5V_2 - 0.5V_3}{1} = 14$$

$$-1V_1 + 2.5V_2 - 0.5V_3 = 14 \quad \text{--- (1)}$$

Apply KCL on supper node  
i.e  $V_3$  and  $V_4$

$$0.5V_x = \frac{V_3 - V_2}{2} + \frac{V_4}{1} + \frac{V_4 - V_2}{2.5}$$

$$0.5V_x = \frac{5V_1 - 4.5V_2 + 2.5V_3 + 2V_4}{5}$$

We know that

$$0.5V_x = 0.5(V_2 - V_1)$$

$$0.5V_2 - 0.5V_1 = \frac{5V_1 - 4.5V_2 + 2.5V_3 + 2V_4}{5}$$

$$2.5V_2 - 2.5V_1 = 5V_1 - 4.5V_2 + 2.5V_3 + 2V_4$$

$$7.5V_1 - 7V_2 + 2.5V_3 + 2V_4 = 0 \quad \text{--- (2)}$$

putt the values of equation  
 $V_1$  and  $V_2$  in eq (1)  
we will get values of  $V_3$

$$-2V_1 + 2.5V_2 - 0.5V_3 = 14$$

$$-(-12) + 2.5(-4) - 0.5V_3 = 14$$

$$12 - 10 - 0.5V_3 = 14$$

$$0.5V_3 = 12$$

$$V_3 = \frac{12}{0.5}$$

$$\boxed{V_3 = 24}$$

putt in eq (2)

$$7.5(-12) - 7(-4) + 2.5(24) + 2V_4 = 0$$

$$2V_4 - 2 = 0$$

$$\cancel{2}V_4 = \cancel{2}$$

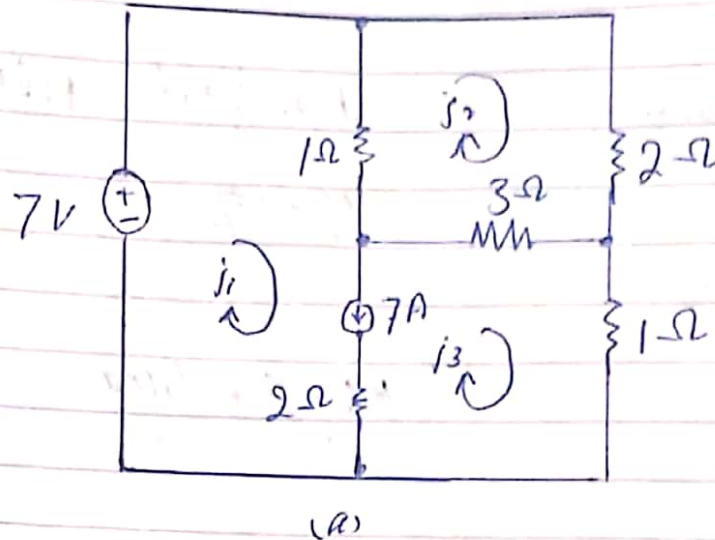
$$V_4 = 1$$

Result:

$$V_1 = -12, \quad V_2 = -4, \quad V_3 = 24, \quad V_4 = 1$$

## Example 4.11

Determine the three mesh current.



Solution:

We know that there is independent sources between node 2 and node 3 which this a super node.

Apply KVL on super mesh

$$1(i_2 - i_1) + 3(i_3 - i_2) + 2i_3 = 7$$

$$i_2 - i_2 + 3i_3 - 3i_2 + 2i_3 = 7$$

$$i_1 - 4i_2 + 5i_3 = 7 \quad \text{--- (A)}$$

Apply KVL on mesh 2

$$(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$$

$$-i_2 + 6i_2 - 3i_2 = 0 \quad \text{--- (B)}$$

Apply KCL on a node a which independent source is entering.

$$7 + i_3 = i_2$$

$$i_2 - i_3 = 7$$

Apply Cramer's rule.

$$\begin{bmatrix} 1 & -4 & 4 \\ -1 & 6 & -3 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \\ 7 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -4 & 4 \\ -1 & 6 & -3 \\ 1 & 0 & -1 \end{vmatrix}$$

$$= 1 \begin{vmatrix} 6 & -3 \\ 0 & -1 \end{vmatrix} - 4 \begin{vmatrix} -1 & -3 \\ 1 & -1 \end{vmatrix} + 4 \begin{vmatrix} -1 & 6 \\ 1 & 0 \end{vmatrix}$$

$$= 1(-6) + 4(1+3) + 4(-6)$$

$$= -6 + 16 - 24$$

$$\boxed{|A| = -14}$$

$$|Ax| = \begin{vmatrix} 7 & -4 & 4 \\ 0 & 6 & -3 \\ 7 & 0 & -1 \end{vmatrix}$$

$$= 7 \begin{vmatrix} 6 & -3 \\ 0 & -1 \end{vmatrix} + 0 + 7 \begin{vmatrix} -4 & 4 \\ 6 & -3 \end{vmatrix}$$

$$= 7(-6) + 7(12 - 24)$$

$$= -42 + 7(-12)$$

$$= -42 - 84$$

$$|Ax| = -126$$

$$|Bx| = \begin{vmatrix} 1 & 7 & 4 \\ -1 & 0 & -3 \\ 0 & 7 & -1 \end{vmatrix}$$

$$= 1 \begin{vmatrix} 0 & -3 \\ 7 & -1 \end{vmatrix} - 1 \begin{vmatrix} 7 & 4 \\ 7 & -1 \end{vmatrix} + 0$$

$$= 1(21) - 1(-7 - 28)$$

$$= 21 + 35$$

$$|Bx| = 56$$

$$C_x = \begin{vmatrix} 1 & -4 & 7 \\ -1 & 6 & 0 \\ 0 & 0 & 7 \end{vmatrix}$$

$$= 1 \begin{vmatrix} 6 & 0 \\ 0 & 7 \end{vmatrix} + 1 \begin{vmatrix} -4 & 7 \\ 0 & 7 \end{vmatrix} + 0$$

$$= 1(42) + 1(28)$$

$$= 42 + 28$$

$$\boxed{|C_x| = 70}$$

$$|C_x| = \begin{vmatrix} 1 & -4 & 7 \\ -1 & 6 & 0 \\ 1 & 0 & 7 \end{vmatrix}$$

$$= 1 \begin{vmatrix} 6 & 0 \\ 0 & 7 \end{vmatrix} + 4 \begin{vmatrix} -1 & 0 \\ 1 & 7 \end{vmatrix} + 7 \begin{vmatrix} -1 & 6 \\ 1 & 0 \end{vmatrix}$$

$$= 1(42) + 4(-7) + 7(-6)$$

$$= 42 - 28 - 42$$

$$= -28$$

Now

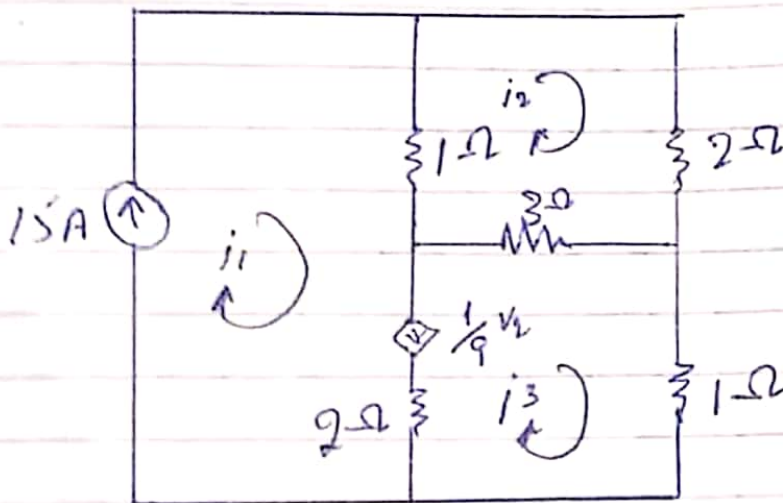
$$i_1 = \frac{|A_{x1}|}{|A|} = \frac{-126}{-14} = 9 \text{ A}$$

$$i_2 = \frac{|B_{x1}|}{|A|} = \frac{-56}{-14} = 2.5 \text{ A}$$

$$I_3 = \frac{|C_x|}{|A|} = \frac{-28}{-14} = 2 \text{ A}$$

### Example 4.12

Evaluate the three unknown currents in the circuit.



Solution:

we have one dependent current and one independent source.

Now

we have the  $v_1 = 15 \text{ A}$   
 we have to find two unknown source.

Apply KCL on a node  
 unknown is entering



$$i_2 + \frac{1}{9} v_x = i_3$$

$$\frac{v_x}{9} = i_3 - i_2$$

$v_x = 3(i_3 - i_2)$  from figure  
putting in equation

$$\frac{3(i_3 - i_2)}{9} = i_3 - i_2$$

$$\frac{1}{3} i_3 - \frac{1}{3} i_2 = i_3 - i_2$$

$$-i_2 + i_3 + \frac{1}{3} i_2 - \frac{1}{3} i_3$$

$$-i_2 + \frac{1}{3} i_2 + \frac{2}{3} i_3 = 0$$

$$\boxed{i_1 = 15}$$

$$-15 + \frac{1}{3} i_2 + \frac{2}{3} i_3 = 0$$

$$\frac{1}{3} i_2 + \frac{2}{3} i_3 = 15 \quad \text{--- (A)}$$

$$i_2 + 2i_3 = 45 \quad \text{--- (1)}$$

Apply KVL on mesh 2

$$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$$

$$i_2 - i_2 + 2i_2 + 3i_2 - 3i_3 = 0$$

$$-i_2 + 6i_2 - 3i_3 = 0$$

$$\boxed{i_2 = 15}$$

$$-15 + 6i_2 - 3i_3 = 0$$

$$6i_2 - 3i_3 = 15 \quad \text{--- (2)}$$

combining eq (1) and (2)

$$\begin{array}{r} 6i_2 + 12i_3 = 270 \\ -6i_2 - 3i_3 = -15 \\ \hline 9i_3 = 285 \end{array}$$

$$i_3 = \frac{285}{9}$$

$$i_3 = 28.33A \quad \text{and}$$

$$i_2 = 11A$$

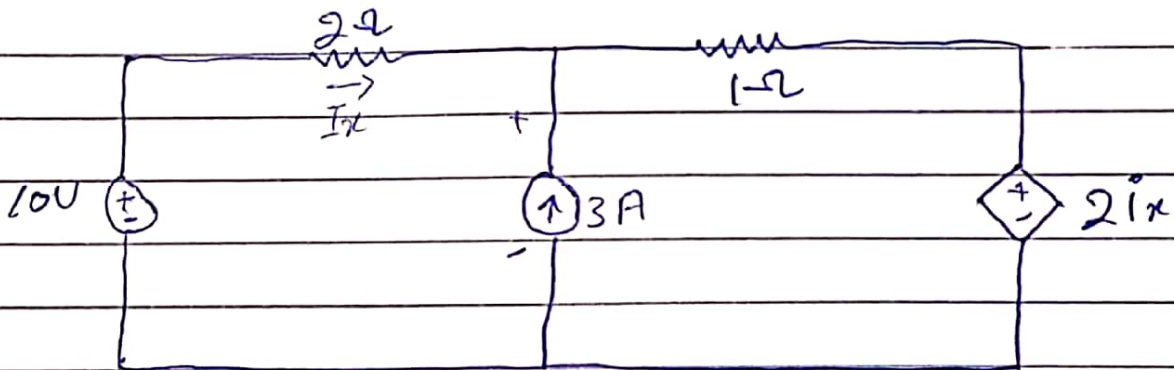
Q2

part (ii)

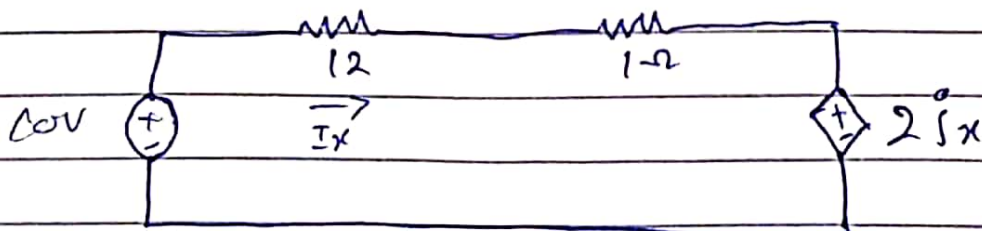
Solve following example

Example 5.3 :-

Use the superposition principle to determine the value of  $i_x$



Solution:- First we will remove current source and will make it in open circuit. Redrawing the circuit

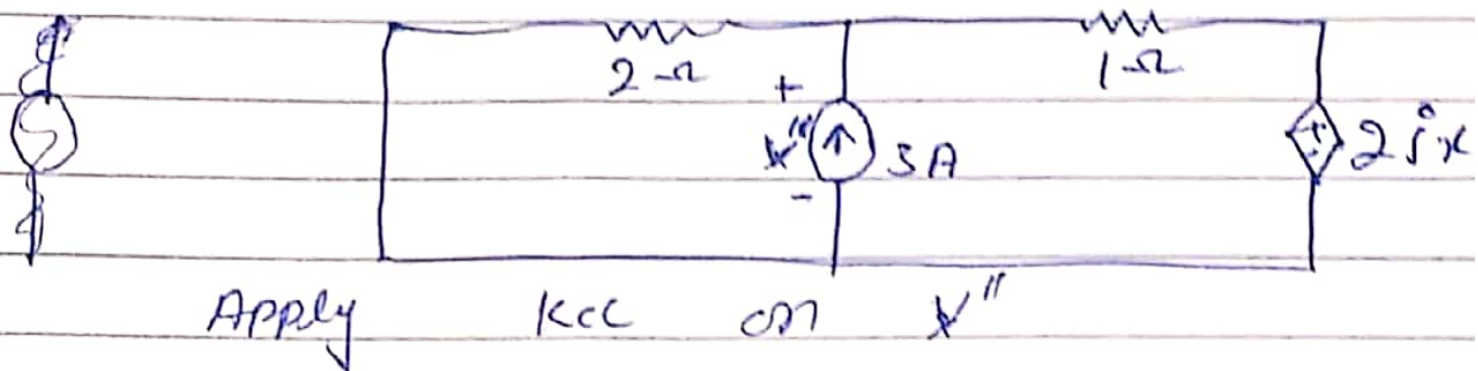


Apply KVL on mesh

$$2i_x' + 1i_x' + 2i_x' = 0$$

$$5i_x' = 10 \Rightarrow i_x' = 2A$$

Now we will remove voltage source and make it an open circuit.



$$\frac{v''}{2} + \frac{v'' - 2jx''}{1} = 3$$

$$\frac{v'' + 2v'' - 2jx''}{2} = 3$$

$$3v'' - 4jx'' = 6$$

We now from the figure

$$x'' = -2jx''$$

$$3(-2jx'') - 4jx'' = 6$$

$$-10jx'' = 6$$

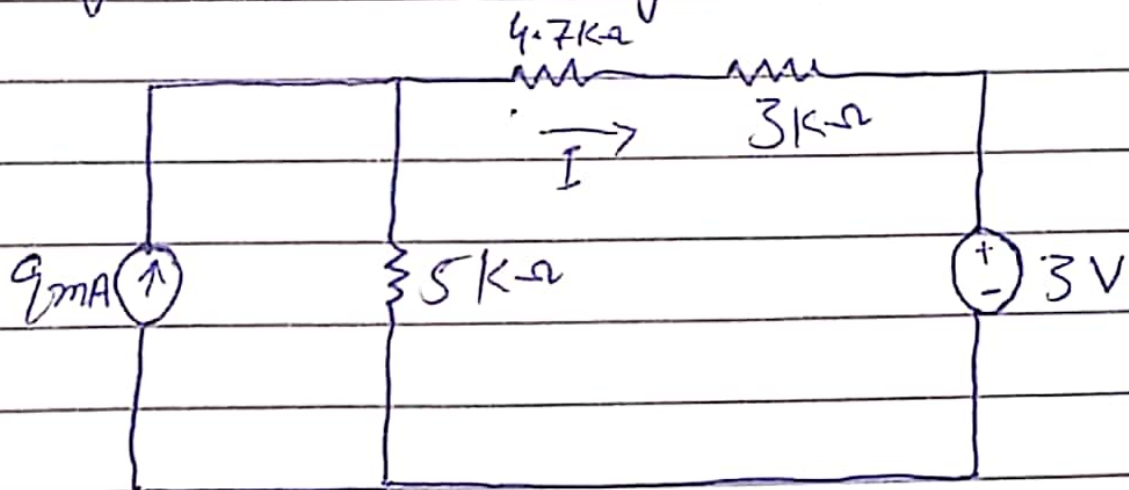
$$jx'' = -0.6 \text{ A}$$

$$jx = jx' + jx''$$

$$= 2 + (-0.6)$$

$$\boxed{I_x = 1.4 \text{ A}}$$

Example 5.4: compute the current through the  $4.7 \text{ k}\Omega$  resistor after transforming the  $9 \text{ mA}$  source in an equivalent voltage source.



Solution:

we know that

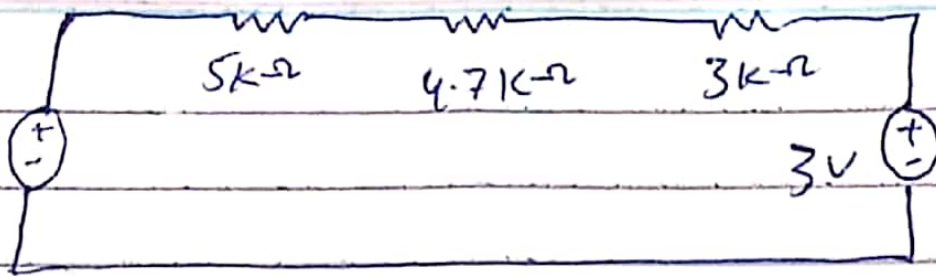
$$V = IR$$

$$V = (9 \times 10^{-3})(5000)$$

$$= (0.009)(5000)$$

$$V = 45$$

re drawing the circuit



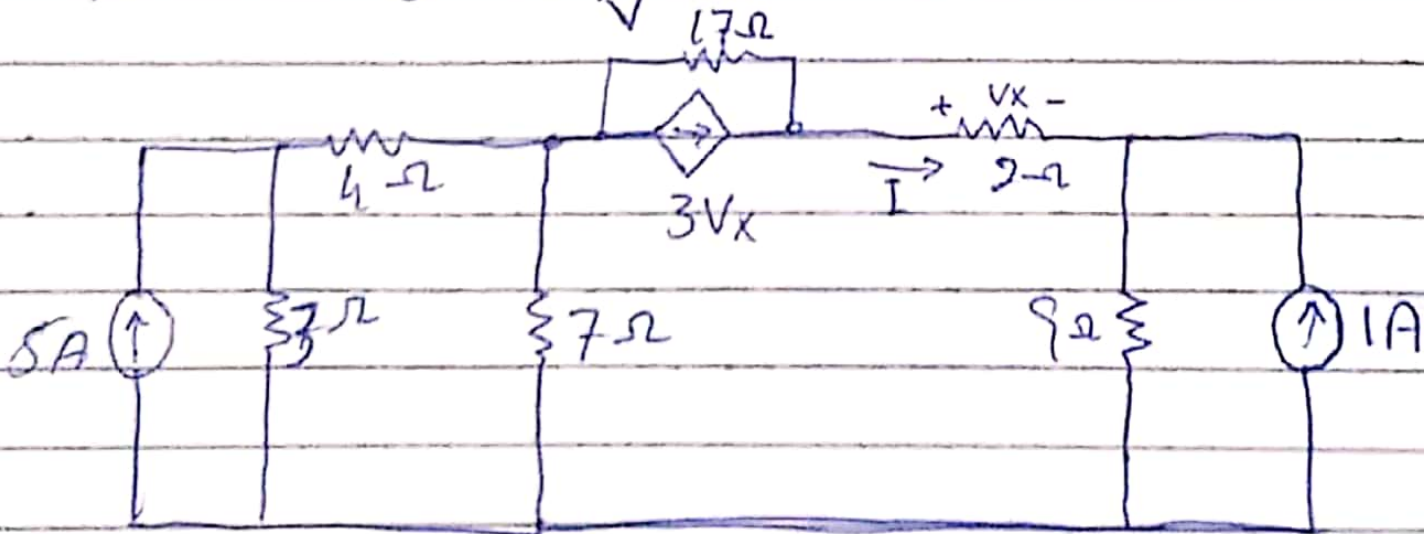
Apply KVL on mesh

$$5000i + 4700i + 3000i = 45 - 3$$

$$12700i = 42$$

$$i = 0.0033 \text{ A}$$

Example 4.5: calculate the current through the  $2\Omega$  resistor by making use of source transformation to first simplify the circuit.

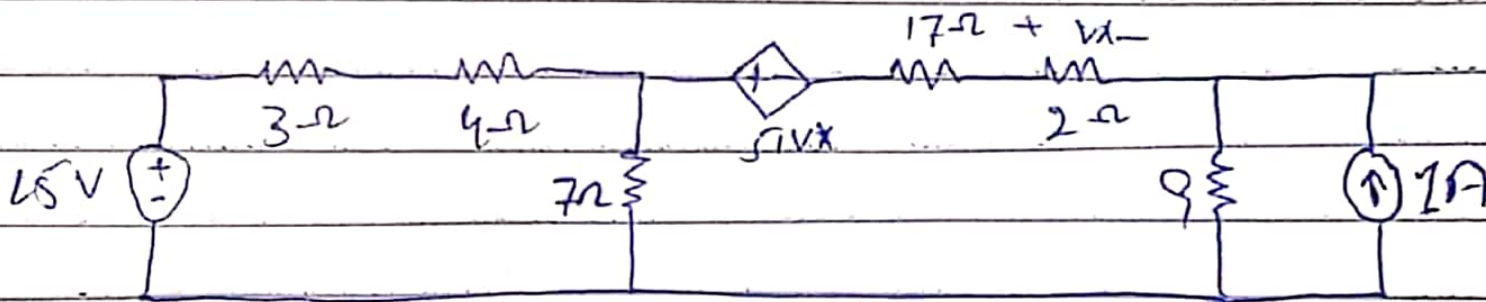


$$V = 5(3) = 15V$$

$$V_x = 3(17)$$

$$V_x = 51$$

Re drawing a circuit



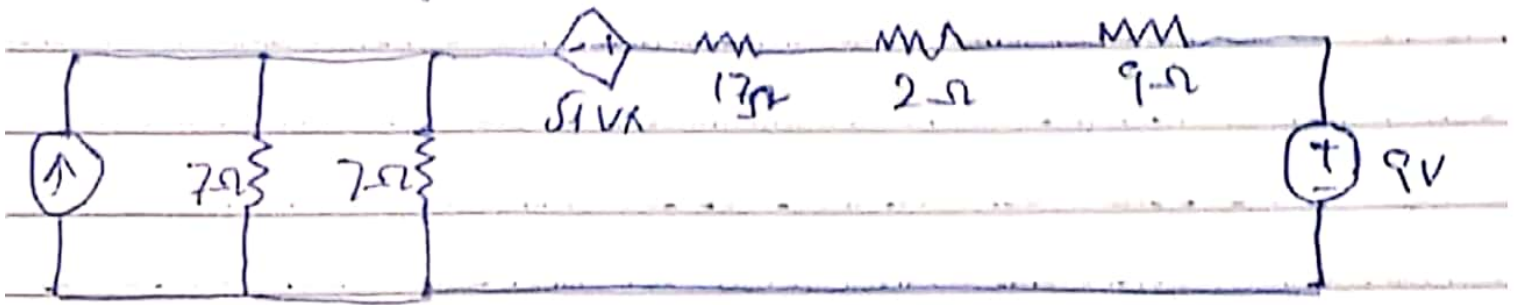
$$R = 3 + 4 \\ = 7\Omega$$

$$I = \frac{V}{R} = \frac{15}{2} = 7.5$$

$$V = IR = 9(1)$$

$$V = 9$$

le drawing a circuit

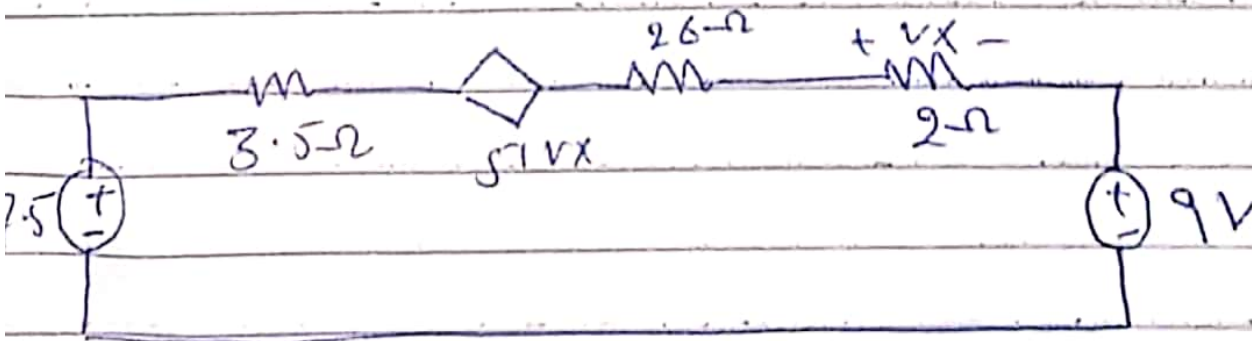


$$V = IR = \left(\frac{15}{7}\right) (7)$$

$$V = 7.5$$

$$R = 17 + 9$$

$$R = 26\Omega$$



Apply KVL on mesh i

$$3.5i - 5V + 28i = 7.5 - 9$$

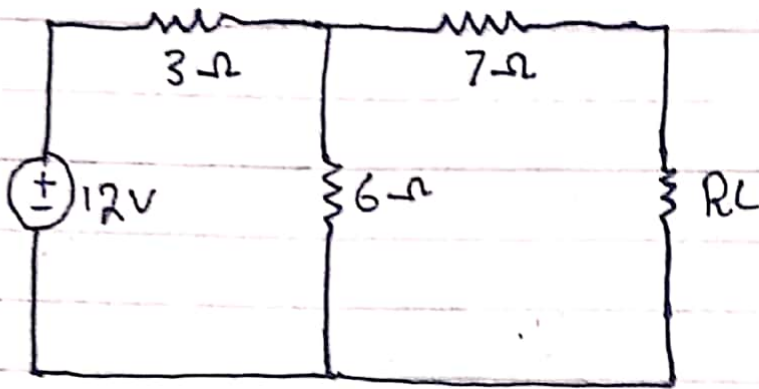
$$V_x = 21$$

$$3.5i - 51(22)i + 28i = -1.5$$

$$i = \frac{-1.5}{-1039.5} \Rightarrow \boxed{i = 0.0014A}$$

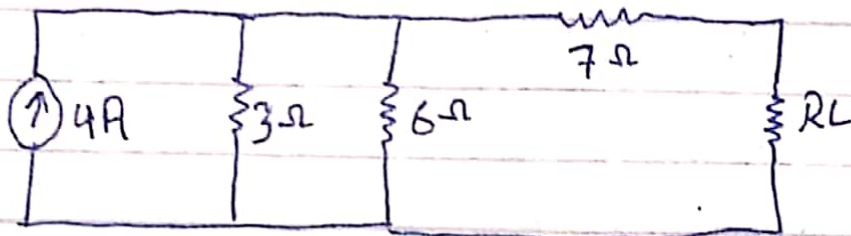


Example 5.6: consider the circuit shown in fig. Determine the Thévenin equivalent of network A, and compute the power delivered to the load resistor  $R_L$ .

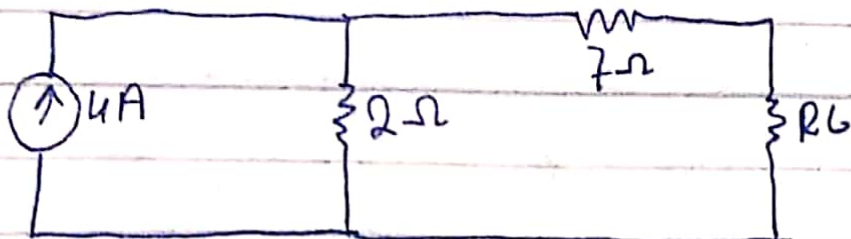


Solution:

$$I = V/R = 12/3 = 4 \text{ A}$$

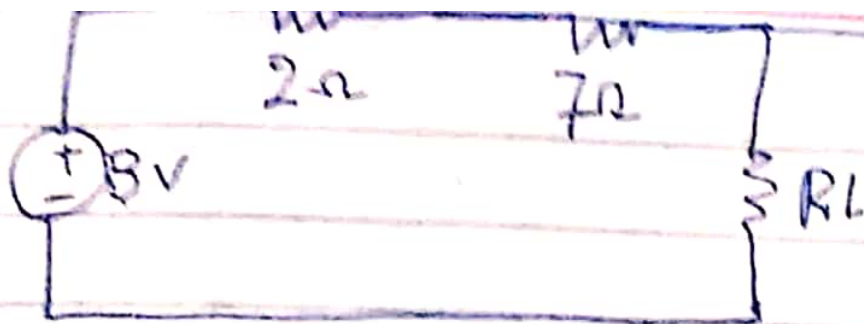


$$R_{eq} = \frac{3 \times 6}{3 + 6} = \frac{18}{9} = 2 \Omega$$

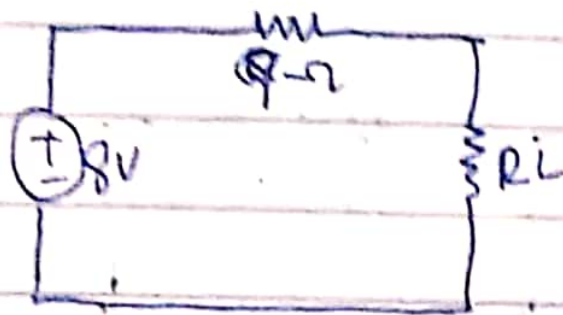


$$V = IR = 4(2)$$

$$V = 8 \text{ V}$$



$$R = 2 + 7 = 9$$



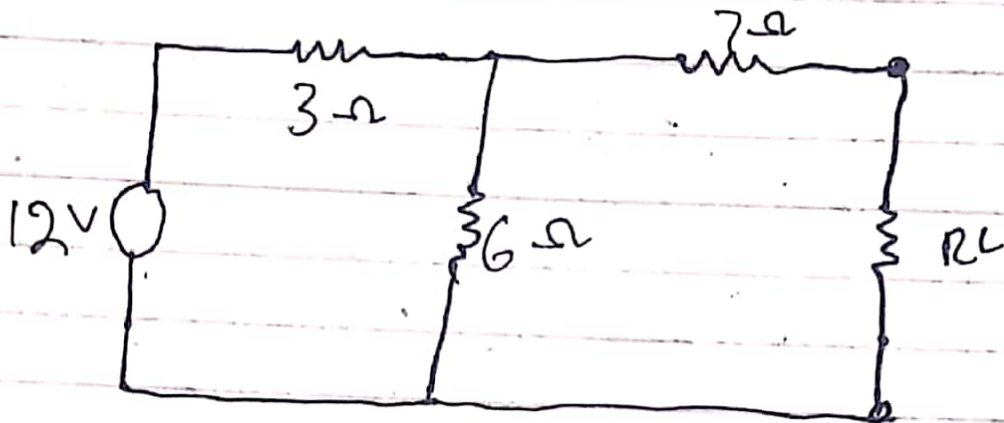
$$V_{TR} = 8V$$

$$R_{TR} = 9\Omega$$

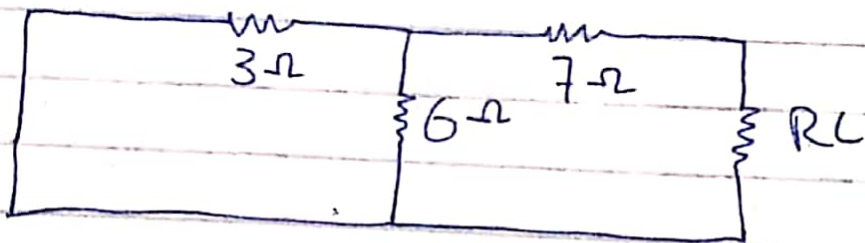
$$R.P = \left( \frac{8}{9 + R_{L}} \right)^2 R_{L}$$

For any value of RL with different solution

Example 5.7: Use Thevenin's Theorem determine the Thevenin equivalent for that part of the circuit in to the left of RL.



For finding  $R_{Th}$  we will remove voltage source and make it is a short circuit.



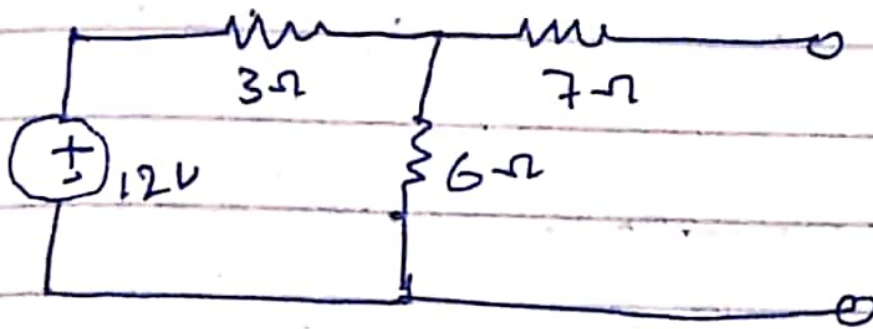
For  $R_{Th}$  we will add all the resistor except RL

$$R_{Th} = 3 \parallel 6 + 7$$

$$= \frac{18}{2} + 7$$

$$R_{Th} = 9$$

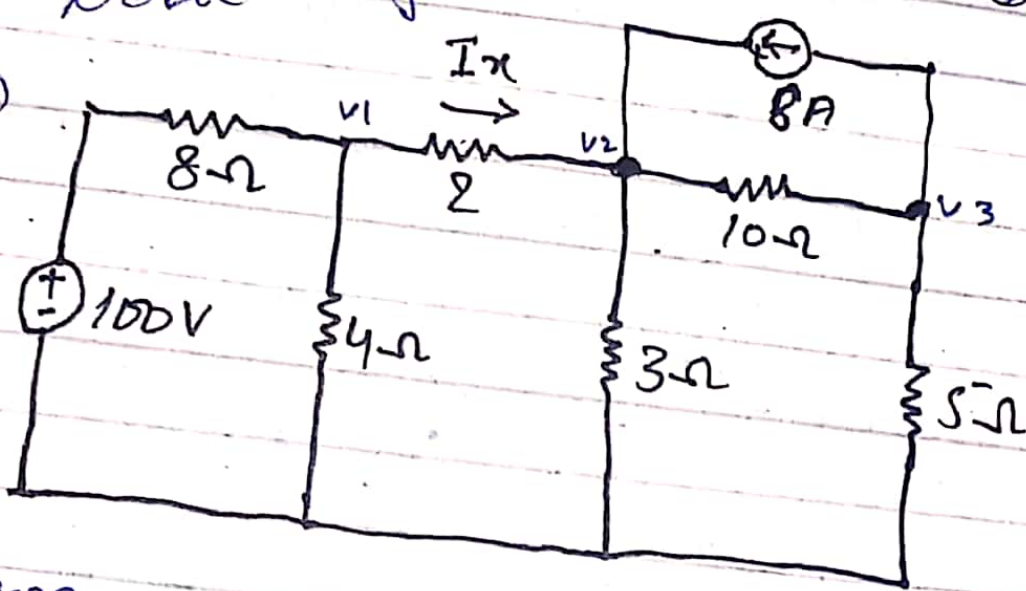
For ~~Voc~~ Voc we will remove RL and make it is an open circuit.



$$V_{oc} = 12 \left( \frac{.6}{3+6} \right)$$

$$V_{oc} = 8V$$

(part 1)



Solution ∴

Apply KCL on node 1 ∴

$$\frac{v_1 - 100}{8} + \frac{v_1}{4} + \frac{v_1 - v_2}{2} = 0$$

$$\frac{v_1 - 100 + 2v_1 + 4v_1 - 4v_2}{8} = 0$$

$$7v_1 - 4v_2 = 100 \quad \text{--- (1)}$$

Apply KCL on node 2 ∴

$$\frac{v_2 - v_1}{2} + \frac{v_2}{3} + \frac{v_2 - v_3}{10} = 8$$

$$\frac{30v_2 - 30v_1 + 20v_2 + 8v_2 - 3v_3}{60} = 8$$

60

$$-30V_1 + 53V_2 - 3V_3 = 480 \quad \text{--- (1)}$$

Apply KCL on node 3:

$$\frac{V_3 - V_2}{10} + \frac{V_3}{5} = -8$$

$$\frac{V_3 - V_2 + 2V_3}{10} = -8$$

$$-V_2 + 3V_3 = -80 \quad \text{--- (2)}$$

Taking eq (1)

$$7V_1 - 4V_2 = 100$$

$$V_1 = \frac{4V_2 + 100}{7} \quad \text{--- (a)}$$

Taking eq (2)

$$-V_2 + 3V_3 = -80$$

$$V_3 = \frac{V_2 - 80}{3} \quad \text{--- (b)}$$

Putting eq (a) and (b) in eq (1)

$$30(0.57v_2 + 14.28) + 53v_2 - 3(0.33v_2 - 26.67) - 17.1v_2 - 428.4 + 53v_2 - 0.99v_2 + 80.01 = 48$$

$$34.91v_1 = 828.39$$

$$v_2 = \frac{828.39}{34.91}$$

$$v_2 = 20.31$$

putting in eq (a)

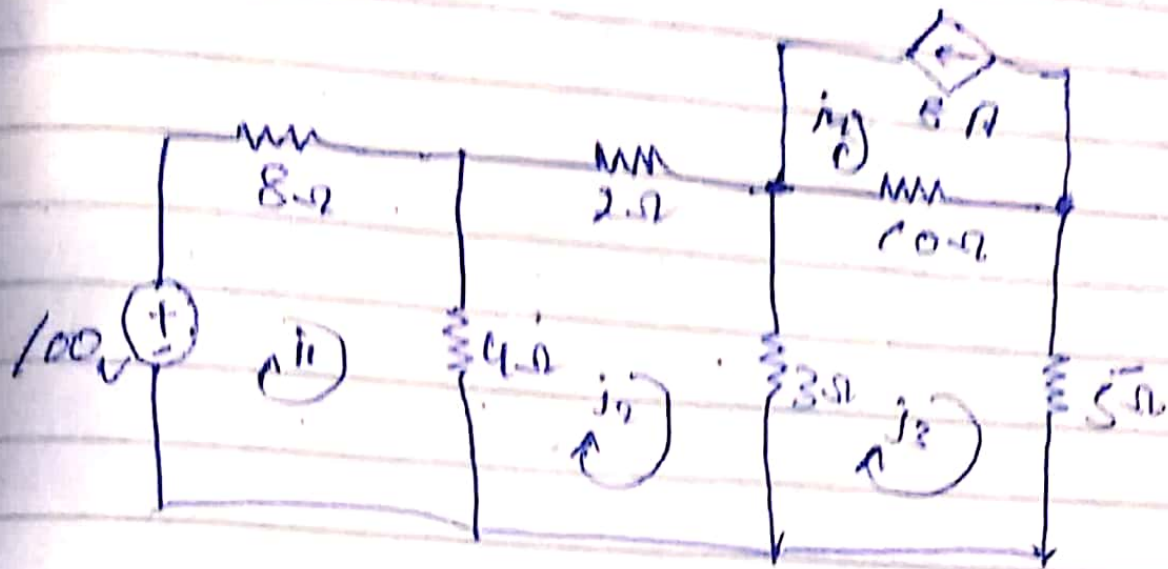
$$v_2 = \frac{4(20.31) + 100}{7}$$

$$v_2 = 25.89$$

$$i_x = \frac{v_1 - v_2}{2} = \frac{25.89 - 20.31}{2}$$

$$i_x = 2.79 \text{ A}$$

# i) Mesh analysis



Apply KVL on loop 1

$$8i_1 + 4(i_2 - i_2) = 100$$

$$8i_1 + 4i_2 - 4i_2 = 100$$

$$12i_2 - 4i_2 = 100 \quad \text{--- (1)}$$

Apply KVL on loop 2.

$$2i_2 + 4(i_2 - i_2) + 3(i_3 - i_2) = 0$$

$$2i_2 + 4i_2 - 4i_2 + 3i_3 - 3i_2 = 0$$

$$-4i_2 + 9i_2 - 3i_3 = 0 \quad \text{--- (2)}$$



Apply KVL on loop 3:

$$3(i_3 - i_2) + 10(i_3 - i_4) + 5i_3 = 0$$

$$3i_3 - 3i_2 + 10i_3 - 10i_4 + 5i_3 = 0$$

$$\text{As } i_4 = 8$$

$$-3i_2 + 18i_3 = -80 \quad \text{--- (3)}$$

Taking eq (1)

$$i_1 = \frac{4i_2 - 100}{12} \quad \text{--- (a)}$$

Taking eq (3)

$$-3i_2 + 18i_3 = -80$$

$$i_3 = \frac{-3i_2 + 80}{18} \quad \text{--- (b)}$$

Putting eq (a) and (b) on eq (2)

$$-4(0.33i_2 - 8.33) + 9i_2 - 3(0.16i_2 + 4.44) = 0$$

$$\Rightarrow -1.32i_2 + 33.32 + 9i_2 - 0.48i_2 - 13.32 = 0$$
$$7.2i_2 = -20$$

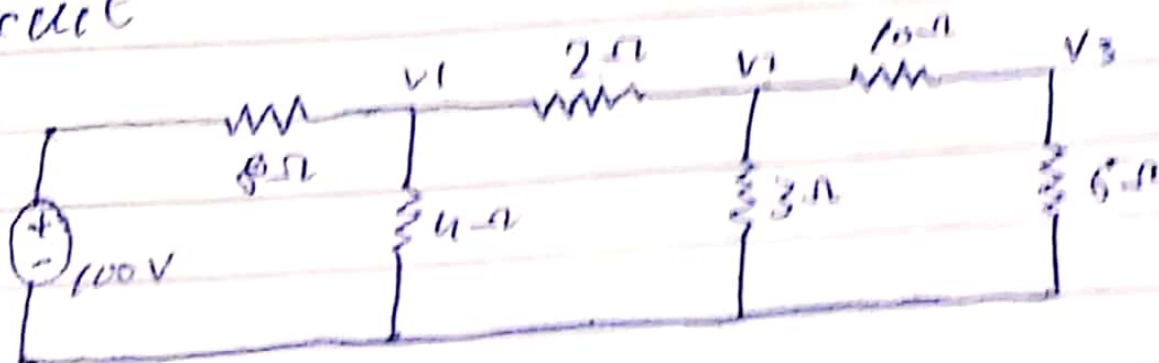
$$i_2 = \frac{20}{7.2} \Rightarrow i_2 = 2.78$$

$$i_2 = 2.79 \text{ A} \Rightarrow \boxed{i_x = 2.79 \text{ A}}$$

iii)

# Superposition Theorem:

First removing the current source and making it an open circuit



Apply KCL on node 1

$$\frac{-100 + V_1}{8} + \frac{V_1 - V_2}{2} + \frac{V_1}{4} = 0$$

$$V_1 - 100 + 4V_1 - 4V_1 + 2V_3 = 0$$

$$7V_1 - 4V_2 = +100 \quad \text{--- (1)}$$

Apply KCL on node 2:

$$\frac{V_2 - V_1}{2} + \frac{V_2}{3} + \frac{V_2 - V_3}{10} = 0$$

$$-30V_1 + 58V_2 - 3V_3 = 0$$

Apply KCL on node 3

$$\frac{V_3 - V_2}{10} + \frac{V_3}{5} = 0$$

$$\frac{V_3 - V_2 + V_3}{10} = 0$$

$$-V_2 + 2V_3 = 0 \quad \text{--- (3)}$$

Now Taking eq (1) and (2)

$$7V_1 - 4V_2 = 100$$

$$V_1 = \frac{4V_2 + 100}{7} \quad \text{--- (4)}$$

Now  $-V_2 + 3V_3 = 0$

$$V_3 = \frac{1}{3}V_2 \quad \text{--- (5)}$$

Putting in eq (2)

$$-30(0.57V_2 + 14.28) - 4V_2 + 2(0.33V_2) = 0$$

$$-17.1V_2 - 428.4 - 4V_2 + 0.60V_2 = 0$$

$$20.44V_2 = 428.4$$

$$V_2 = -20.95$$

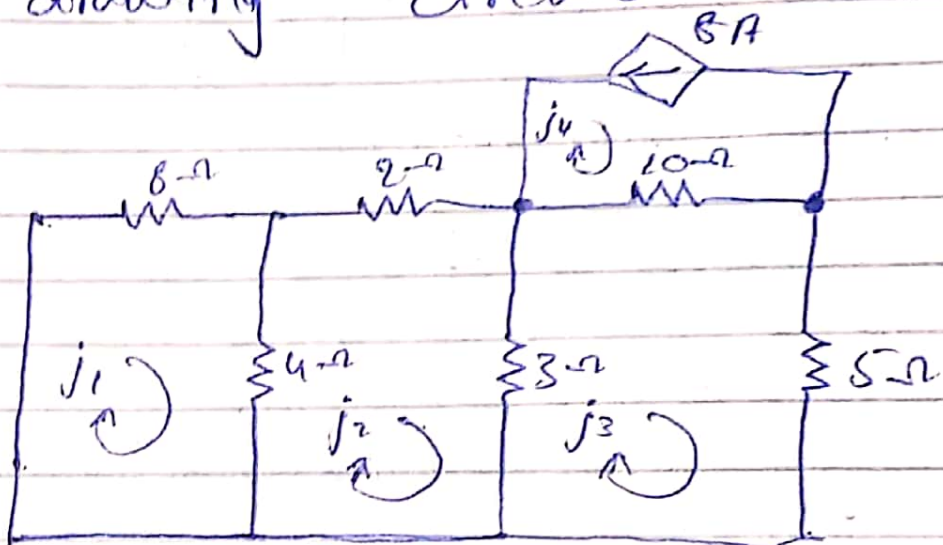
Putting in eq (4)

$$v_2 = 2.31$$

$$i_1 = \frac{2.31 + 20.95}{2}$$

$$i_1 = 11.63$$

Now Removing voltage source and making it short circuit  
and re drawing



$$i_4 = 8A$$

Apply KVL on Loop 1:

$$8i_1 + 4(i_1 - i_2) = 0$$

$$8i_1 + 4i_1 - 4i_2 = 0$$

$$12i_1 - 4i_2 = 0$$

$$3i_1 - i_2 = 0 \quad \text{--- (1)}$$

Apply KVL on loop 2:

$$2i_2 + 3(i_2 - i_3) + 4(i_2 - i_1) = 0$$

$$2i_2 + 3i_2 - 3i_3 + 4i_2 - 4i_1 = 0$$

$$-4i_1 + 9i_2 - 3i_3 = 0 \quad \text{--- (2)}$$

Apply KVL on loop 3:

$$10i_3 + 5i_3 + 3i_3 - 2i_2 + 8(10) = 0$$

$$-2i_2 + 18i_3 = -80 \quad \text{--- (3)}$$

Taking eq (1)

$$3i_1 - i_2 = 0$$

$$i_1 = 0.33i_2 \quad \text{--- (a)}$$

Taking eq (3)

$$-2i_2 + 18i_3 = -80$$

$$i_3 = \frac{3i_2 - 80}{18} \quad \text{--- (b)}$$

$$-4(0.33i_2) + 9i_2 - 3(0.16i_2 - 4.44) = 0$$

$$1.32i_2 + 9i_2 - 0.48i_2 + 13.32 = 0$$

$$j_2 = 1.354$$

Now 
$$j_x = j_1 + j_2$$

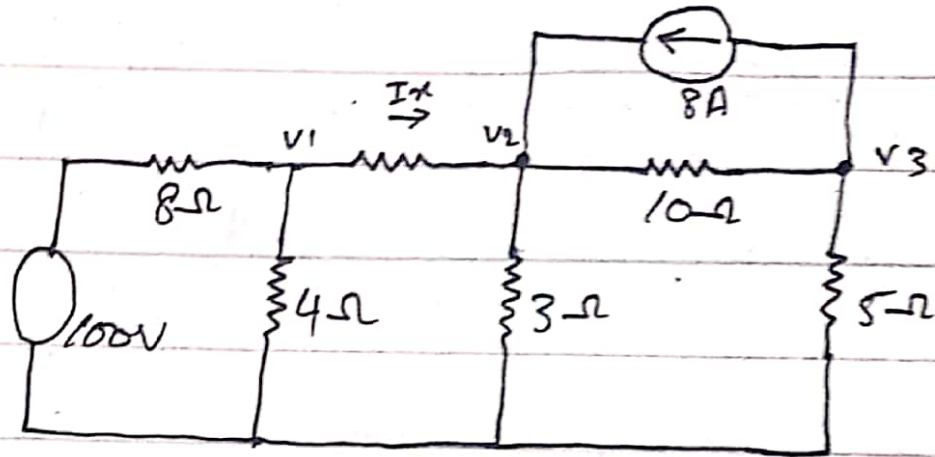
$$j_x = 1.44 + 1.35$$

$$j_x = 2.79 \text{ A}$$

Result

$$j_x = 2.79 \text{ A}$$

Q4: compare the number of steps and degree of easiness of all the three methods with each other



Solution: Node analysis  
Apply KCL on node:

$$\frac{V_1 - 100}{8} + \frac{V_1}{4} + \frac{V_1 - V_2}{1} = 0$$

$$\frac{V_1 - 100 + 2V_1 + 4V_1 - 4V_2}{8} = 0$$

$$7V_1 - 4V_2 = 100 \quad \text{--- (1)}$$

Apply KCL node 2:

$$\frac{V_2 - V_1}{1} + \frac{V_2}{3} + \frac{V_2 - V_3}{10} = 8$$

$$\frac{30V_2 - 30V_1 + 20V_2 + 3V_2 - 3V_3 = 8}{60}$$

$$-30V_1 + 53V_2 - 3V_3 = 480 \quad \text{--- (2)}$$

Apply KCL on node 3:

$$\frac{V_3 - V_2}{10} + \frac{V_3}{5} = -8$$

$$\frac{V_3 - V_2 + 2V_3}{10} = -8$$

$$-V_2 + 3V_3 = -80 \quad \text{--- (3)}$$

Taking eq (1)

~~$$7V_1 - 4V_2 = 100$$~~

$$7V_1 - 4V_2 = 100$$

$$V_1 = \frac{4V_2 + 100}{7} \quad \text{--- (a)}$$

Taking eq (2)



$$-V_2 + 3V_3 = -80$$

$$V_3 = \frac{V_2 - 80}{3} \quad \text{--- (b)}$$

putting (a) and (b) in eq (2)

$$-30(0.57V_2 + 14.28) + 53V_2 - 3(0.33V_2 - 26.67) = 480$$

$$-17.1V_2 - 428.4 + 53V_2 - 0.99V_2 + 80.01 = 480$$

$$34.91V_2 = 828.39$$

$$V_2 = \frac{828.39}{34.91}$$

$$V_2 = 20.31$$

putting in eq (a)

$$V_2 = \frac{4(20.31) + 100}{7}$$

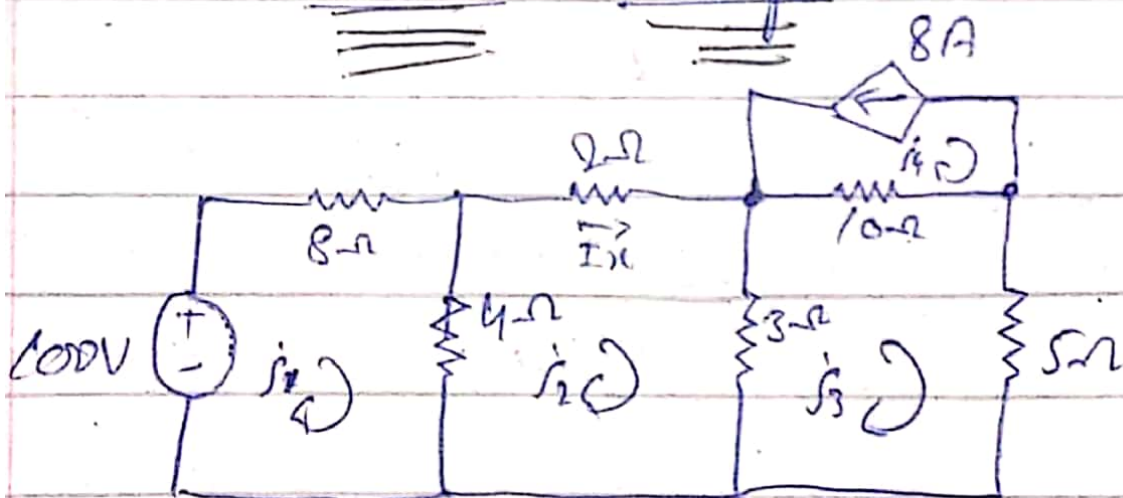
$$v_2 = 25.89$$

$$i_x = \frac{v_1 - v_2}{2}$$

$$i_x = \frac{25.89 - 20.31}{2}$$

$$i_x = 2.79 \text{ A}$$

Mesh analysis



Apply KVL on loop 1.

$$8i_1 + 4(i_2 - i_2) = 100$$

$$8i_1 + 4i_2 - 4i_2 = 100$$

$$12i_2 - 4i_2 = 100 \quad \text{--- (1)}$$

Applying KVL on loop 2:

$$2i_2 + 4(i_2 - i_3) + 3(i_3 - i_2) = 0$$

$$2i_2 + 4i_2 - 4i_3 + 3i_3 - 3i_2 = 0$$

$$-4i_2 + 9i_3 = 0 \quad \text{--- (2)}$$

Apply KVL on loop 3:

$$3(i_3 - i_2) + 10(i_3 - i_4) + 5i_3 = 0$$

$$3i_3 - 3i_2 + 10i_3 - 10i_4 + 5i_3 = 0$$

$$\text{As } i_4 = 8$$

$$-3i_2 + 18i_3 = -80 \quad \text{--- (3)}$$

Taking eq (1)

$$i_1 = \frac{4i_2 - 100}{12} \quad \text{--- (4)}$$

Taking eq (3)

$$-3j_2 + 18j_3 = -80$$

$$j_3 = \frac{-3j_2 + 80}{18} \quad \text{--- (b)}$$

Putting (a) and (b) in eq (2)

$$-4(0.33j_2 - 8.33) + 9j_2 - 3(0.16j_2 + 4.44) = 0$$

$$-1.32j_2 + 33.32 + 9j_2 - 0.48j_2 - 13.32 = 0$$

$$7.2j_2 = 20$$

$$j_2 = \frac{20}{7.2} \Rightarrow j_2 = 2.79 \text{ A}$$

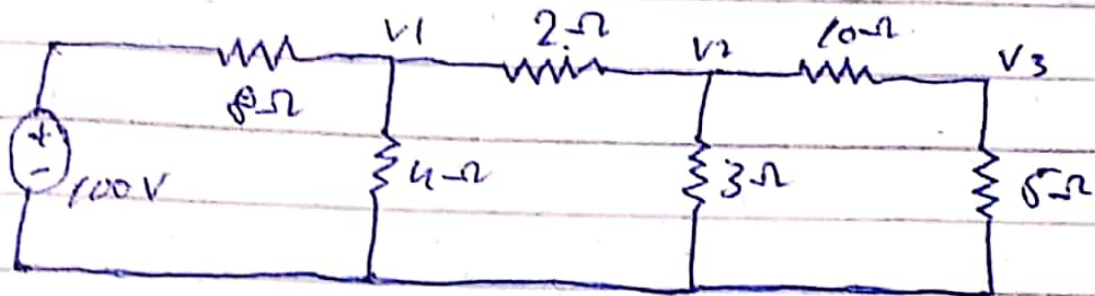
$$j_2 = j_x$$

$$\boxed{j_x = 2.79 \text{ A}}$$

(iii)

Superposition Theorem:

First removing the current source and making it an open circuit. Re drawing the circuit



Apply KCL on node 1

$$\frac{-100 + V_1}{8} + \frac{V_1 - V_2}{2} + \frac{V_1}{4} = 0$$

$$\frac{V_1 - 100 + 4V_1 - 4V_1 + 2V_3}{8} = 0$$

$$7V_1 - 4V_2 = +100 \quad \text{--- (1)}$$

Apply KCL on node 2:

$$\frac{V_2 - V_1}{2} + \frac{V_2}{3} + \frac{V_2 - V_3}{10} = 0$$

$$-30V_1 + 53V_2 - 3V_3 = 0 \quad \text{---}$$

Apply KCL on node 3

$$\frac{V_3 - V_2}{10} + \frac{V_3}{5} = 0$$

$$\frac{V_3 - V_2 + V_3}{10} = 0$$

$$-V_2 + 2V_3 = 0 \quad \text{--- (3)}$$

Now taking eq (1) and (2)

$$7V_1 - 4V_2 = 100$$

$$V_1 = \frac{4V_2 + 100}{7} \quad \text{--- (a)}$$

Now  $-V_2 + 3V_3 = 0$

$$V_3 = \frac{1}{3} V_2 \quad \text{--- (b)}$$

Putting in eq (2)

$$-30(0.57V_2 + 14.28) - 4V_2 + 2(0.33V_2) = 0$$

$$-17.1V_2 - 428.4 - 4V_2 + 0.60V_2 = 0$$

$$20.44V_2 = 428.4$$

$$V_2 = -20.95$$

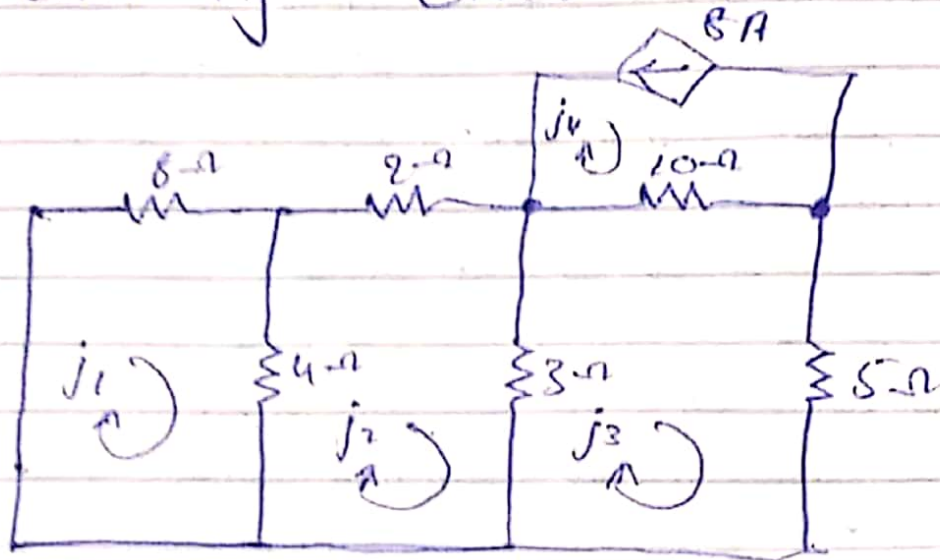
Putting in eq (a)

$$v_2 = 2.31$$

$$i_1 = \frac{2.31 + 20.95}{2}$$

$$i_1 = 11.63$$

Now and Re drawing  
 Removing voltage source and making it short circuit.



$$i_4 = 8A$$

Apply KVL on loop 1:

$$8i_1 + 4(i_1 - i_2) = 0$$

$$8i_1 + 4i_1 - 4i_2 = 0$$

$$12i_1 - 4i_2 = 0$$

$$3i_1 - i_2 = 0$$

(1)

Apply KVL on loop 2:

$$2i_2 + 3(i_2 - i_3) + 4(i_2 - i_1) = 0$$

$$2i_2 + 3i_2 - 3i_3 + 4i_2 - 4i_1 = 0$$

$$-4i_1 + 9i_2 - 3i_3 = 0 \quad \text{--- (2)}$$

Apply KVL on loop 3:

$$10i_3 + 5i_3 + 3i_3 - 2i_2 + 8(10) = 0$$

$$-2i_2 + 18i_3 = -80 \quad \text{--- (3)}$$

Taking eq (1)

$$3i_1 - i_2 = 0$$

$$i_1 = 0.33i_2 \quad \text{--- (a)}$$

Taking eq (3)

$$-2i_2 + 18i_3 = -80$$

$$i_3 = \frac{3i_2 - 80}{18} \quad \text{--- (b)}$$

$$-4(0.33i_2) + 9i_2 - 3(0.16i_2 - 4.44) = 0$$

$$1.32i_2 + 9i_2 - 0.48i_2 + 13.32 = 0$$



$$j_2 = 1.354$$

Now 
$$j_x = j_1 + j_2$$

$$j_x = 1.44 + 1.35$$

$$j_x = 2.79 \text{ A}$$

Result 
$$j_x = 2.79 \text{ A}$$