Linear Algebra

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Q1 Express The equation of Plane Passing Through The point $A(2,-2,1)$, $B=(-1,0,3)$ $C=(5,-3,4)$

Solution

The non-parallel vectors

$\overrightarrow{P_1P_2} = (-3,2,2)$

$P_1P_3 = (3,-1,3)$

The perpendicular vector is

$\overrightarrow{P_1P_2} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$\overrightarrow{P_1P_2} = \sqrt{-1-2)^2}$

$n = \overrightarrow{P_1P_2} \times P_1P_3$

$$n = \begin{vmatrix} i & j & k \\ 3 & 2 & 2 \\ 3 & -1 & 3 \end{vmatrix}$$

$m = i(6+2) - j(-9-6) + k(3-6)$

$$n = i(6+2) - j(-9-6) + k(3-6)$$

$$n = 8i + 15j - 3k$$

$$n = (8, 15 - 3)$$

Now

$$P_0(x_0, y_0, z_0) = (2, -3, 1)$$

$$n(a, b, c) = (8, 15, -3)$$

So equation of plane is

$$a(x - x_0) + b(y - y) + c(z + z_0) = 0$$

$$8(x-2) + 15(y+3) - 3(z-1) = 0$$

$$8x + 15y - 3z - 16 + 45 + 3 = 0$$

$$8x + 15y - 3z + 32 = 0$$

Ans

Q No 1

B Express a pair planes whose Intersection is the given line $x = 2 - 3t,\ y = 3 + t,\ z = 2 - 4t$

Solution

$x = 2 = -3t \Rightarrow t = \frac{x-2}{-3}$, $y = 3 = t = t = \frac{y-3}{1}$

$z = 2 - 4t \Rightarrow t = \frac{z-2}{-4}$

So $\frac{x-2}{-3} = \frac{y-3}{1} = \frac{z-2}{-4}$

for 1st plane takes 1st and 2nd

$\frac{x-2}{-3} \quad \frac{y-3}{1}$

$x - 2 = -3y + 9$

$x + 3y - 11 = 0$

for 2nd plane take 1st and 3rd

$\frac{x-2}{-3} = \frac{z-2}{-4}$

$-4x + 8 = -3z + 6$

$-4x + 3z + 2 = 0$

$4x - 3z - 2 = 0$

Q2 $L(x,y) = (x+1, y, x+y)$ illustrate That L is linear transformation?

Solution

$L(x,y) = (x+1, y, x+y)$

let $u = (x_1, y_1)$ $v = (x_2, y_2)$

$u+v = (x_1, y_1) + (x_2 + y_2)$

$u+v = (x + x_2, y_1 + y_2)$

$L(u+v) = L(x_1 + x_2, y_1 y_2)$

$L(u+v) = (x_1 + x_2 + 1, y_1 + y_2, x_1 + x_2 + y_1 + y_2)$

given That $u(x+y)$

$L(u) = L(x_1, y_1) = (x + 1, y + x_1 + y - 1)$

$L(u) = L(x_2 y_2) = (x_2 + 1, y_2\ x_2 + y_2)$

$L(u) + L(v) = (x_1 + x_2 + 2, y_1 + y_2, x_1 + x_2 + y_2 + y_2)$

Since $1 \neq 2$

Q③ Using the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{pmatrix}$ then Interpart to ~~deal~~ decode the message 77, 58 38 71 49 29 68 51 33 76 48 40 86 53 52

Solution

$$\begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix} \quad \begin{bmatrix} 71 \\ 49 \\ 29 \end{bmatrix} \quad \begin{bmatrix} 68 \\ 51 \\ 33 \end{bmatrix} \quad \begin{bmatrix} 76 \\ 48 \\ 40 \end{bmatrix} \quad \begin{bmatrix} 86 \\ 53 \\ 52 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\cancel{A^{-1} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix}} =$$

$$A^{-1} = ?$$

$$A^{-1} = \frac{1}{|A|} \operatorname{adj} A.$$

So $$A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 2 & 2 & -1 \\ -1 & 1 & 1 \end{bmatrix}$$

So, $$X_1 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix}$$

$$= \begin{bmatrix} 16 \\ 38 \\ 15 \end{bmatrix}$$

$$X_2 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 71 \\ 49 \\ 29 \end{bmatrix}$$

$$= \begin{bmatrix} 20 \\ 15 \\ 7 \end{bmatrix}$$

$$X_3 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 68 \\ 51 \\ 33 \end{bmatrix}$$

$$= \begin{bmatrix} 18 \\ 1 \\ 16 \end{bmatrix}$$

$$X_4 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 76 \\ 48 \\ 40 \end{bmatrix}$$

$$= \begin{bmatrix} 8 \\ 18 \\ 15 \end{bmatrix}$$

$$X_5 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 86 \\ 53 \\ 52 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 14 \\ 19 \end{bmatrix}$$

16 8 15 20 15 7 18 1 16 6 16 12 1 14 19

P H O T O G R A P H P L A N S

PHOTOGRPH PLANS

Ans

Q4 Find an equation of The Plane Passing through the Point $(-1, 3, 2)$ and Perpendicular to The Vector $n = (0, 1, 3)$

Solution

Equation of Plane

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

Given that

$$P = (x_0, y_0, z_0) = (-1, 3, 2)$$

$$n = (a, b, c) = (0, 1, -3)$$

So

$$0(x - (-1)) + 1(y - 3) - 3(z - 2)$$

$$0\,\underline{(x + 1)} + 1(y - 3) - 3(z - 2)$$

$$0 + y - 3 - 3z + 6$$

$$\Rightarrow y - 3z - 3 + 6$$

$$\Rightarrow y - 3z + 3 \text{ Ans}$$

Q5 Find an Eigen Values and Eigen Vectors of matrix $A=\begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$

Solution

We know that

$$Ax = \lambda x$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Then $x_1 + x_2 = \lambda x_1$ — (i)

$-2x_1 + 4x = \lambda x_1$ — (ii)

So

$$x_1 - \lambda x_1 + x_2 = 0$$

$$= (1-\lambda)x_1 + x_2 = 0$$

$=$

$$-2x_1 + 4x_2 - \lambda x_2 = 0$$

$$= 2x_1 + (4-\lambda)x_2 = 0$$

$$\begin{vmatrix} 1-\lambda & 1 \\ -2 & 4-\lambda \end{vmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$= -2x_1 + 4x_2 \neq 2x_2$ — (iii)

$= -2x_1 + 2x_2 = 0$

$x_1 - x_1 = 0$

$x_1 = x_2$

$x_1 = r$ Than $x_2 = r$

So

$$x \begin{bmatrix} x \\ x_2 \end{bmatrix} = \begin{bmatrix} r \\ r \end{bmatrix}$$

$\Rightarrow 2x_1 - x_1 = 0$

$\Rightarrow 2x_1 + 4x_2 = 3x_2$ — (ii)

$\Rightarrow -2x_1 + x_2 = 0$

$\Rightarrow 2x_1 - x_2 = 0$

$x_1 = 1/2^{x_2}$

Let $x^2 = r$

where $r \neq 0$

So $x\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1/2\,r \\ r \end{bmatrix}$

eign vector for $\lambda_2 = 2$ put in i & 2

$x_1 + x_2 = 2x_1$ — (i)

$-2x_1 + 4x_2 = 2x_2$ — (ii)

$\Rightarrow -x_1 + x_2 = 0$

$\Rightarrow x_1 - x_2 = 0$

$\Rightarrow x_1 = x^2$

$\Rightarrow -2x_1 + 4x_2 = 2x_2$ — (ii)

$= -2x_1 + 2x_2 = 0$

$x_1 - x_2 = 0$

$x_1 = x_2$

$x_1 = r$ Then $x_2 = r$

So $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} r \\ r \end{bmatrix}$