Linear Algabra

Summer Semester

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15243

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Q3:-

Find A^{-1} when $A = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix}$

Solution

$$|A| = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & 3 \end{bmatrix}$$

$$= 3\begin{bmatrix} -2 & 1 \\ 6 & 2 \end{bmatrix} + 2\begin{bmatrix} 5 & 2 \\ -1 & -3 \end{bmatrix} + 1\begin{bmatrix} 5 & 6 \\ 1 & 0 \end{bmatrix}$$

$$= 3x \begin{vmatrix} 2x2 & 1 \\ 6 & 2 \end{vmatrix}$$

$$= 3(-4-6) + (-15-2-) + 1(0-6)$$

$$|A| = -94$$

Now

$A_{11} = (-1)^{1+1}\begin{vmatrix} 6 & 2 \\ 0 & -3 \end{vmatrix} = -18$ $A_{12} = (-1)^{1+2}\begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} = 17$

$A_{13} = (-1)^{1+3}\begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix} = -6$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 1 \\ 0 & -3 \end{vmatrix} = -6$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 1 \\ 1 & -3 \end{vmatrix} = -10$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 1 \\ 0 & 2 \end{vmatrix} = -10, \quad A_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix} = -1$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 3 & -2 \\ 5 & 6 \end{vmatrix} = 28$$

$$\text{adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{ adj } A$$

$$A^{-1} = \frac{1}{-94} \begin{bmatrix} 18 & 6 & 16 \\ -17 & 10 & 1 \\ 6 & 2 & -28 \end{bmatrix}$$

Ans.

Q1,

Part (a):-

let $A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$ and $B \begin{bmatrix} 1 & 4 \\ 3 & -1 \\ -2 & 2 \end{bmatrix}$

Identify the (3,2) entery of AB.

sol:-

Identify $(3,2) = ?$

$Row_3\ (A) \qquad column_2\ (B)$

$= \begin{bmatrix} 0 & 1 & -3 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$

$= (0)(4) + (1)(-1) + [-2](2)$

$= 0 + (-1) + (-4)$

$= -1 - 4$

$\boxed{= -5}$ Ans.

Q21. If A and B are $n \times n$ matrices where $|A| = 2$ and $|B| = -3$

Calculate $|A^{-1} B^T|$

Solution:

$= |A^{-1} \; B^t|$

$= |A^{-1} \; B^t|$

$= \frac{1}{|A|} \; |B|$

$|B^t| = |B|$

So $A^{-1} \; b^t = \frac{1}{|A|} \; |B|$

$= \frac{1}{2} \cdot 3$

$\boxed{= \frac{3}{2}}$ Ans.

Q1.

part (B)

Find the quadratic polynomial that interpolate the point

(1, 3) (2, 4) (3, 7)

Sol.

As $a_2 x_1^2 + a_1 x_1 + a_0 = y_1$

$a_2 x_2^2 + a_1 x_2 + a_0 = y_2$

$a_2 x_3^2 + a_1 x_3 + a_0 = y_3$

Now $(x_1, y_1) = (1,3)$, $(x_2, y_2) = (2,4)$

$(x_3, y_3) = (3,7)$ Put in eqone.

$a_2 + a_1 + a_0 = 3$

$4a_2 + 2a_1 + a_0 = 4$

$9a_2 + 3a_1 + a_0 = 7$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 4 & 2 & 1 & 4 \\ 9 & 3 & 1 & 7 \end{array}\right]$$

$$R \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -8 \\ 0 & -6 & -8 & -20 \end{array}\right] \begin{array}{l} R_2 - 4R_1 \\ R_3 - 9R_1 \end{array}$$

$$R \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -8 \\ 0 & 0 & 1 & 4 \end{array}\right] R_3 - 3R_2$$

so

$$a_2 + a_1 + a_0 = 3 \quad \text{—} \ (1)$$

$$-2a_1 - 3a_0 = -8 \quad \text{—} \ (2)$$

$$a_0 = 4 \quad \text{— put in } (2)$$

$$\Rightarrow -2a_1 - 12 = 8$$

$$\Rightarrow a_1 = \frac{4}{-2} = 12$$

$$a_2 - 2 + 4 = 3 \Rightarrow \boxed{a_2 = \underline{1}}$$

Q 2
Part (B) :-

$x + y + 2z = 1$

$x - 2y + z = -5$

$3x + y + z = 3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 1 & -2 & 1 & -5 \\ 3 & 1 & 1 & 3 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & -3 & -1 & -6 \\ 0 & -2 & -5 & 0 \end{array}\right] \begin{array}{l} R_2 - R_1 \\ R_3 - 3R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 7 \\ 0 & 1 & 7/3 & 2 \\ 0 & -2 & -5 & 0 \end{array}\right] R_2/-3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 7/3 & 2 \\ 0 & -2 & -13/2 & 4 \end{array}\right] R_3 + 2R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 7/3 & 2 \\ 0 & 0 & 1 & -8/13 \end{array}\right] R_3 \times \frac{2}{-13}$$

$x + y + 2z = 1$ —— (i)

$y + 7/3\, z = 2$ —— (ii)

$z = -8/13$ —— (iii)

Now put eq (iii) in (i)

$y + 7/3 \times \frac{-8}{13} = 2$

$y - \frac{8}{39} = 2$

$y = 2 + \frac{8}{39}$

$y = \frac{78+8}{39} = \frac{86}{39}$

Now put values of y in (i)

$x + \frac{86}{39} + 2\left(\frac{-8}{13}\right) = 1$

$x + \frac{86}{39} - \frac{16}{13} = 1$

$x + \frac{39}{39} = 1$

$x = 1 - \frac{38}{39}$

$\boxed{x = \frac{1}{39}}$