QUESTION No. 1:

The function $g(t)$ is defined by - - - - - -.

→a. State any point of discontinuity

→b. Find if they exist

SOLUTION:

a. $g(t) = 0 \quad ; t < 0$

$\quad\quad = t^2 \quad ; 0 \leq t \leq 3$

$\quad\quad = 2t + 3 ; 3 < t \leq 4$

$\quad\quad = 4 \quad ; t > 0$

Since the definition of the function changes at the point $t = 3$. Therefore, we check, the discontinuity at that point.

At $t = 3$

$g(t) = t^2$; By definition of function

$\Rightarrow g(3) = (3)^2$

$\Rightarrow g(3) = 9 \rightarrow (1)$

Now

$\lim_{t \to 3} g(t) = \lim_{t \to 3} (t)^2$

$= (3)^2$

b. $\lim_{t \to 0} (gt) = 9 — (2)$

(2)

From eq (1) and (2), we have

$$g(3) = \lim_{t \to 3} g(t)$$

$\Rightarrow g(t)$ is continues at each point.

Again we check at $t = 0$

$g(t) = t^2$ (by definition of function)

$\Rightarrow g(3) = (3)^2$

$g(3) = 9$ —— (1)

Now

$$\lim_{t \to 3} g(t) = \lim_{t \to 3}$$

$g(3) = 0$ —— (2)

From eq (1) and (2), we have

$$g(t) \neq \lim_{t \to 3} g(t)$$

So the function is discontinuous at $t = 0$.

$$\lim_{t \to 3} g(t) = \lim_{t \to 3} (t)^2$$

$$= (3)^2$$

$$\lim_{t \to 3} g(t) = 9$$

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QUESTION No.2

Find the Maclaurin's series for

$Y(x) =$

SOLUTION:

By Maclaurin's series expansion, we have

$$f(x) = f(0) + xf'(0) + \frac{x^2 f''(0)}{2!} + \frac{x^3 f'''(0)}{3!} + \text{------}$$

Now $f(x) = Y(x) = x^2 + \sin x$

$$f'(x) = 2x + \cos x$$

$$f''(x) = x - \sin x$$

$$f'''(x) = -\cos x$$

Thus $f(0) = (0)^2 + \sin(0) = 0$

$$f'(0) = 2(0) + \cos(0) = 1$$

$$f''(0) = 0 - \sin(0) = 0$$

$$f'''(0) = -\cos(0) - 1$$

Hence by Maclaurin's Expansion

$$f(x) = Y(x) = 0 + x(1) + \frac{x^2(0)}{2!} + \frac{x^3(-1)}{3!}$$

$$= 0 + x + 0 - \frac{x^3}{3!} + \text{----}$$

$$f(x) = Y(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!}$$ → Required Maclaurin's eq.

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QUESTION No. 3:

i. Find =

ii. Find by using logarithmic differentiation

SOLUTION:

Y =

(i) Differentiate wrt "x"

$$\frac{d}{dx}(1+xy) = \frac{d}{dx}(x^2+y^2)$$

$$\Rightarrow 0+\left[x\,\frac{dy}{dx}+y(1)\right] = 2x+2y\,\frac{dy}{dx}$$

$$\Rightarrow x\frac{dy}{dx}+y = 2x+2y\,\frac{dy}{dx}$$

Again differentiate w.r.t "x" we have

$$x\frac{d^2y}{dx^2}+(1)\frac{dy}{dx}+\frac{dy}{dx} = 2(1)+2\left(y\frac{d^2y}{dx^2}+\frac{dy}{dx}(1)\right)$$

$$\Rightarrow x\frac{d^2y}{dx^2}+\cancel{2\frac{dy}{dx}} = 2+2y\frac{d^2y}{dx^2}+\cancel{2\frac{dy}{dx}}$$

$$\Rightarrow x\frac{d^2y}{dx^2}-2y\frac{d^2y}{dx^2} = 2$$

$$\Rightarrow \frac{d^2y}{dx^2}(x-2y) = 2$$

$$\Rightarrow \boxed{\frac{d^2y}{dx^2} = 2/x-2y} \Rightarrow \text{Required Answer.}$$

(ii) $y = x^3\left[(1+x)^9 e^{6x}\right]$

$$\ln(y) = \ln\left[x^3(1+x)^9 e^{6x}\right]$$

$$\ln y = \ln x^3 + \ln(1+x)^9 + \ln e^{6x}$$

$$\ln y = 3\ln x + 9\ln(1+x) + \cancel{6x(\ln e)}\ 6x \ln e$$

$$\ln y = 3\ln x + 9\ln(1+x) + 6x(1)$$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}\left[3\ln + 9\ln(1+x) + 6x\right]$$

$$\frac{1}{y}(y') = 3 \times \frac{1}{x} \times 1 + 9 \times \frac{1}{1+x} \times (1) + 6$$

$$\frac{1}{y} \times y' = \frac{3}{x} + \frac{9}{1+x} + 6$$

$$\boxed{y' = y\left(\frac{3}{x} + \frac{9}{1+x} + 6\right)} \Rightarrow \text{Required Answer.}$$

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