Sessional Assignment
Linear Algebra

Name M. Ishfaq
Student ID 16002
Department CS (2nd Semester)

Question No. 1: Solution.

1. An orthogonal matrix.
In linear algebra, an orthogonal matrix is a square matrix whose columns and rows are orthogonal unit vectors (orthogonal vectors)... The determinant of any orthogonal matrix is either +1 or -1.

2. A basis for a vector space.
A vector space's basis is a Subset of vectors within the space that are linearly independent and span the space. A basis is linearly independent because the vectors in it cannot be defined as a linear Combination of any of the other vectors in the basis.

3. The span of a set of vectors.

In linear algebra, the linear span (also called the linear hull or just span) of a set S of vectors in a vector space is the Smallest linear subspace that Contains the set. It can be characterized either as the intersection of all linear sub spaces that Contains S, or as the set of linear combinations of elements of S.

4. The dimension of a vector space.

Dimension of a vector space If V is spanned by a finite set, then V is said to be finite-dimensional, and the dimension of V, written as dim V, is the number of vectors in a basis for V. The dimension of the zero vector space {0} is defined to be 0. If V is not spanned by a finite set, then V is said to be Infinite-dimensional.

Example: The standard basis for Rn is { e 1, ... en} where e 1.....en are the column of In. So, for example, dim R 3 = 3.

5. An eigenvector.

Eigen vectors are a special set of vectors associated with a linear system of equations (i.e., a matrix equation) that are sometimes also known as characteristic vectors, proper vectors, or latent vectors (Marcus and Minc 1988).

6. **A subspace of a vector space.**

In mathematics, and more specifically in linear algebra, a linear subspace, also known as a vector subspace is a vector space that is a subset of some larger vector space. A linear subspace is usually called simply a subspace when the context serves to distinguish it from other types of subspace.

7. The Kernel of a linear transformation.

The kernel. Related to 1-1 linear transformation is the idea of the kernel of a linear transformation.

Definition. The kernel of a linear transformation L is the set of all vectors v such that $L(v) = 0$.

8. The nullity of a linear tranformation.

The Nullity of a linear transformation of vector spaces is the dimension of its null space. The nullity and the map rank add up to the dimension of., a result sometimes known as the rank-nullity theorem.

9. The image of a linear transformation.

The image of a linear transformation or matrix is the span of the vectors of the linear transformation. (Think of it as what vectors you can get from applying the linear transformation or multiplying the matrix by a vector.) It can be written as Im(A).

10. The rank of a linear transformation.

The rank of a linear transformation L is the dimension of its image, written rank L. The nullity of a linear transformation is the dimension of the kernel, written L.

Theorem (Dimension formula). Let L : V W be a linear transformation. with V a finite-dimensional vector space 2.

11. The characteristic polynomial of a square matrix.

In linear algebra, the characteristic polynomial of a square matrix is a polynomial which is invariant under matrix similarity and has the eigenvalues as roots. It has the determinant and the trace of the matrix as coefficients.

12. An equivalence relation.

An equivalence relation is a relationship on a set, generally denoted by ?, that is reflexive symmetric and transitive for everything in the set.
Example: The relation is equal to denoted =, is an equivalence relation on the set

of real numbers since for any x, y, z R : 1.

13. A homogeneous solution to a linear system of equations.
A System of linear equations is homogeneous if all of the constant terms are zero: A homogeneous System is equivalent to a matrix equation of the form. where A is an m n matrix, x is a Column vector with n entries, and 0 is the zero vector with m entries.

14. A Particullar solution to a linear System of equations.
A Solution to the system of both equations is a Pair of numbers (x, y) that makes both equations true at once. In other words, it as a point that lies on both lines Simultaneously.

15. The general Solution to a linear System of equations.
A Solution to a System of linear equations is a Set of numbers that, when we Substitute numbers for Specified variables in the System, makes each equation in the System a true Statement. For Example, if we plug 4 in for x and 7 in for y, both of the equations in the following System are true Statements.

16. The direct sum of a pair of subspaces of a vector space.
Let all be vector sub Spaces of the vector space. We define the direct Sum of these Subspaces. \o plus U-m= V$ is defined to be a Sum of the Sub Spaces to which each element in can be uniquely written as.... lemma1: let be vector Sub spaces of

17. The orthogonal complement to a subspace of a vector space.

In the mathematical fields of linear algebra and functional analysis, the orthogonal complement of a subspace W of a vector space V equipped with a bilinear form B is the set W of all vectors in V that are orthogonal to every vector in W.... It is a sub space of V.

Question No. 2: Solution.

$$x + y + z + w = 1$$
$$x + 2y + 2z + 2w = 1$$
$$x + 2y + 3z + 3w = 1$$

- Express as $MX = V$
- Find solution set by LU decomposition.

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}_{B}$$

$$AX = B$$

Let $A = LU \Rightarrow LUX = B$.

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ l_{21} & 1 & 0 & 0 \\ l_{31} & l_{32} & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} U_{11} & U_{12} & U_{13} & U_{14} \\ 0 & U_{22} & U_{23} & U_{24} \\ 0 & 0 & U_{33} & U_{34} \end{bmatrix}$$

⇒ Since

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \end{bmatrix}$$ is not a square matrix.

So it Can not be writlen as $A = LU$. So its solution Can not be found by using LU decomposition Method.

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Question No. 3: Solution.

$$\Rightarrow \det\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix} = 4-6=-2$$

$$\Rightarrow \det\begin{pmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{pmatrix} = \text{Let } A.$$

$$|A| = \begin{matrix}1 & 2 & 3 & 1 & 2\\ 4 & 5 & 6 & 4 & 5\\ 7 & 8 & 9 & 7 & 8\end{matrix}$$

$$= (45+84+96)-(105+48+72)$$

$$= 225-225=0$$

$$= \text{So} \quad \det\begin{pmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{pmatrix} = 0$$

$$\Rightarrow \det\begin{pmatrix}1 & 2 & 3 & 4\\ 5 & 6 & 7 & 8\\ 9 & 10 & 11 & 12\\ 13 & 14 & 15 & 16\end{pmatrix}_{4\times 4} = A$$

$$|A| = 1\begin{vmatrix}6 & 7 & 8\\ 10 & 11 & 12\\ 14 & 15 & 16\end{vmatrix} - 2\begin{vmatrix}5 & 7 & 8\\ 9 & 11 & 12\\ 13 & 15 & 16\end{vmatrix}$$

$$+3\begin{vmatrix}5 & 6 & 8\\ 9 & 10 & 12\\ 13 & 14 & 16\end{vmatrix} - 4\begin{vmatrix}5 & 6 & 7\\ 9 & 10 & 11\\ 13 & 14 & 15\end{vmatrix} = 0$$

Finding by Row operations now.

Step 0:

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{vmatrix}$$

Step 01: $R_2 - 5R_1$

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -4 & -8 & -12 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{vmatrix}$$

Step 02: $R_3 - 9R_1$

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -4 & -8 & -12 \\ 0 & -8 & -16 & -24 \\ 13 & 14 & 15 & 16 \end{vmatrix}$$

Step 03: $R_4 - 13R_1$

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -4 & -8 & -12 \\ 0 & -8 & -16 & -24 \\ 0 & -12 & -24 & -36 \end{vmatrix}$$

Step 04: $R_3 + (-2R_2)$ and $R_4 + (-3R_2)$

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -4 & -8 & -12 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

Two rows are zero so by properties of determinant its determinant is zero.

So.

$$\det \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix} = 0$$

$$\Rightarrow \det \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \\ 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 25 \end{pmatrix} = 0$$

Finding by row operations.

Step 0: $\begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \\ 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 25 \end{vmatrix}$

Step 01: $R_2 + (-6R_1)$ and $R_3 + (-11R_1)$

$$\Rightarrow \begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & -5 & -10 & -15 & -20 \\ 0 & -10 & -20 & -30 & -40 \\ 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 25 \end{vmatrix}$$

step 02: $R_4 + (-16R_1)$ and $R_5 + (-21R_1)$

$$\begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & -5 & -10 & -15 & -20 \\ 0 & -10 & -20 & -30 & -40 \\ 0 & -15 & -30 & -45 & -60 \\ 0 & -20 & -40 & -60 & -80 \end{vmatrix}$$

step 03: $R_3 + (-2R_2)$, $R_4 + (-3R_2)$ and $R_5 + (-4R_2)$

$$\begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & -5 & -10 & -15 & -20 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

which Confirms that the determinant is Zero.

So

$$\det \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \\ 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 25 \end{pmatrix} = 0$$

Question No 4. Answer

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Since by a displacement

$r) \; x = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

here

$$r = \begin{pmatrix} -x-y \\ -x-2y-z \\ -y-z \end{pmatrix}$$

we can think like that f_1 x any acting on r and f.

The matrix will be matrix of the transformation.

$f_1(x,y,z) = (-x-y, -x-2y-z, -y-z)$

$f_1(1,0,0) = (-1-1 0) = -1(1,0,0) - 1(0,1,1)$
$+ 0(0,0,1)$

$f_1 = (0,1,0) = (-1,-2,-1) = -1(1,0,0)$
$-2(0,1,0) - 1(0,0,1)$

$f_1 = (0,0,1) = (0,-1,-1) = 0(1,0,0) - 1(0,1,0)$
$-(0,0,1)$

Since $\{(1,0,0), (0,1,0), (0,0,1)$

$$\boxed{m = \begin{bmatrix} -1 & -1 & 0 \\ -1 & -2 & -1 \\ 0 & -1 & -1 \end{bmatrix}}$$ Ans

(b) for eigen value of M.

$$\det(M - xI) = \begin{vmatrix} -1-x & -1 & 0 \\ -1 & -2-x & -1 \\ 0 & -1 & -1-x \end{vmatrix}$$

$$= (-1-x)\left((x+2)(x+1)-1\right) + 1\left((x+1)-0\right)$$
$$= \left(-(x+1)^2(x+2) + 2(x+1)\right)$$
$$= (x+1)\left[2 - \cancel{x}(x^2+3x+2)\right]$$
$$= -(x+1)(x^2+3x)$$

Hence $\det(M - xI) = 0 \Rightarrow x = -1, 0, -3$

$\therefore$ eigen value of M here $\boxed{-3, 0, -1}$ Ans

for $x = -3$

$$(M+3I)X = 0 \quad \begin{bmatrix} 2 & -1 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$\therefore 2x - y = 0 \Rightarrow y = 2x$

$-x + y - z = 0 \Rightarrow -x + 2x - z = 0 \Rightarrow z = x$

$-y + 2z = 0 \Rightarrow -2x + 2x = 0$

So $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ 2x \\ z \end{bmatrix} = x \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

for $x=0$

$$(M+0I)x=0 \Rightarrow \begin{bmatrix} -1 & -1 & 0 \\ -1 & -2 & -1 \\ 0 & -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=0$$

$$\Rightarrow -x-y=0 \Rightarrow y=-x$$

$$-x-2y-z=0 \Rightarrow -x+2x-z=0 \Rightarrow z=x$$

$$-y-z=0$$

$$\therefore\ x=\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} a \\ -x \\ x \end{bmatrix}=\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}x$$

for $x=-1$

$$(M+1I)x=0 \Rightarrow \begin{bmatrix} 0 & -1 & 0 \\ -1 & -1 & -1 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=0$$

$$\because\ y=0$$

$$-x-y-z=0 \Rightarrow x=-z$$

$$-y=0$$

$$\because\ x\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{bmatrix} x \\ 0 \\ -x \end{bmatrix}=x\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

for $\boxed{\begin{array}{ll} x=-3 \text{ eigen vector} & \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \\ x=0 \text{ eigen vector} & \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \\ x=-1 \text{ eigen vector} & \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \end{array}}$

Ans

(c) Since given $M\begin{pmatrix}a\\b\\c\end{pmatrix} = -\omega^2\begin{pmatrix}a\\b\\c\end{pmatrix}$

we here $\omega^2 = 3, 1$ or 0

$\therefore |\omega| = \sqrt{3}, 1, 0$

Hence the characteristic

$\boxed{0, 1, \sqrt{3}}$ Ans

(d) Since by b we three eigen vectors corresponding to $-3, -1, 0$, we have

$$MP = PD$$

where

$$\boxed{P = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 0 & -1 \\ 1 & -1 & 1 \end{bmatrix} \text{ and } D = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}}$$

Answer

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THE END