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Subject APPlied Calculus

Quiz No 1

Submitted to Mam Shomaila mazhar

Q:. $\int_0^1 \frac{4t^3-2t^2+3t-1}{2t^2+1}\,dt$

Sol:.

$$\int_0^1 \frac{4t^3-2t^2+3t-1}{2t^2+1}\,dt$$

$$= \int_0^1 \frac{4t^3+3t-2t^2-1}{2t^2+1}\,dt$$

$$= \int_0^1 \frac{t(4t^2+3)-(2t^2+1)}{2t^2+1}$$

$$= \int_0^1 \frac{t(4t^2+3)}{2t^2+1}\,dt \quad \int_0^1 \frac{2t^2+1}{2t^2+1}$$

$$= \int_0^1 \frac{t(4t^2+3)}{2t^2+1}\,dt - \int_0^1 1\,dt$$

$$= \int_0^1 \frac{t(4t^2+3)}{2t^2+1}\,dt - [1-0]$$

$$= \int_0^1 \frac{t(4t^2+3)}{2t^2+1}\,dt - [1-0]$$

$$= \int_0^1 t\,\frac{(4t^2+3)}{2t^2+1} - 1 \longrightarrow (1)$$

Now
= let $2t^2+1=y$
= As $t \to 1$ i.e $y=3$
= $t \to 0$ i.e $y=1$

$\Rightarrow 2t^2+1=y$
$2t^2=y-1$
$4t^2=2y-2$
$4t^2+3=2y-2+3$
$4t^2+3=2y+1$

Now Diff

$= 4t = dy/dt$

$= dt = dy/4t$

$= \int_1^3 + \frac{(2y+1)}{y} : dy/4t - 1$

$= \int_1^3 \frac{2y+1}{4y}\, dy - 1$

$= \frac{1}{4}\left[\int_1^3 \frac{2y\,dy}{y} + \int_1^3 \frac{1}{y}\, dy\right] - 1$

$= 1/4\ \left[\int_1^3 2\,dy + \int_1^3 \frac{1}{y}\, dy\right] - 1$

$= 1/4\ \left[2y\Big|_1^3 + \ln y\Big|_1^3\right] - 1$

$= 1/4\ [2(3) - 2(1) + \ln(3) - \ln(1)] - 1$

$= 1/4\ [6 - 2 + 1.0986] - 1$

$= 1/4\ [5.0986] - 1$

$= 1.27465 - 1$

$= 0.2746$

Ans

Q2: Find $\int_2^3 t \sin t^2 dt$.

Sol:-

Let $t^2 = u$

then $2t = \frac{du}{dt} \Rightarrow tdt = \frac{du}{2}$

As $t \to 2$ then $u \to 4$

$t \to 3$ then $u \to 9$

So the given question will transform as,

$$\int_2^3 t \sin t^2 dt = \int_4^9 \sin u \frac{du}{2}$$

$$= \frac{1}{2} \int_4^9 \sin u \, du$$

$$= \frac{1}{2} \left| -\cos u \right|_4^9$$

$$= \frac{1}{2} (-\cos 9 - (-\cos 4))$$

$$= \frac{1}{2} (-\cos 9 + \cos 4)$$

$$= \frac{1}{2} (\cos 4 - \cos 9)$$

$$= \frac{1}{2} (0.998 - 0.987)$$

$$= 0.0055$$

Ans