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Numerical Analysis **Assignment# O1**

1

Ans(1) #

$$A = \begin{array}{c} \begin{pmatrix} 4 & 2 & 0 \\ 2 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix} \\ \begin{matrix} A_1 & A_2 & A_3 \end{matrix} \end{array}$$

So $A = QR$

Wher

$$Q = \frac{Q_1}{\|Q_1\|} \cdot \frac{Q_2}{\|Q_2\|} \cdot \frac{Q_3}{\|Q_3\|}$$

Now

$$Q_1 = A_1 \qquad Q_1 = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}$$

$$Q_2 = A_2 - \text{Proj}\ Q_1\,(Q_1)$$

$$Q_2 = A_2 - \frac{A_2 \cdot A_1}{A_1 \cdot A_1} \cdot (A_1)$$

$$= \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} - \frac{\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}}{\begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}} \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} - \frac{12}{20} \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} - \begin{pmatrix} \frac{12}{5} \\ \frac{12}{10} \\ 0 \end{pmatrix} = \begin{pmatrix} -2/5 \\ 4/5 \\ 1 \end{pmatrix}$$

(2)

$Q_3 = A_3 - \text{Proj}\ Q_1 (Q_1) - \text{Proj}\ Q_2 - Q_2$

$$= \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 12/5 \\ 12/10 \\ 0 \end{pmatrix} - \frac{A_3 \cdot A_2}{A_2 \cdot A_2} (A_2)$$

$$= \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 12/5 \\ 12/10 \\ 0 \end{pmatrix} - \frac{\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}}{\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}} \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 12/5 \\ 12/10 \\ 0 \end{pmatrix} - \frac{\cancel{3}\,1}{\cancel{9}\,3} \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 12/5 \\ 12/10 \\ 0 \end{pmatrix} - \begin{pmatrix} 2/3 \\ 2/3 \\ 1/3 \end{pmatrix}$$

$$Q_3 = \begin{bmatrix} -46/15 \\ -13/5 \\ 0 \end{bmatrix}$$

$$Q_1 = \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix} \qquad Q_2 = \begin{bmatrix} -2/5 \\ 4/5 \\ 1 \end{bmatrix} \qquad Q_3 = \begin{bmatrix} -46/15 \\ -13/5 \\ 0 \end{bmatrix}$$

3

Now find the length of Q_1, Q_2 & Q_3

$$\|Q_1\| = \sqrt{(4)^2 + (2)^2 + (0)^2}$$
$$= 4.47$$

$$\|Q_2\| = \sqrt{\left(-2/5\right)^2 + \left(4/5\right)^2 + (1)^2}$$
$$= 1.34$$

$$\|Q_3\| = \sqrt{\left(\frac{-46}{15}\right)^2 + \left(\frac{-13}{15}\right)^2 + (0)^2}$$
$$= 3.186$$

$$Q = \frac{1}{4.47}\begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix} \cdot \frac{1}{1.34}\begin{bmatrix} -2/5 \\ 4/5 \\ 1 \end{bmatrix} \cdot \frac{1}{3.186}\begin{bmatrix} -46/15 \\ -13/15 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{4.47} \\ \frac{2}{4.47} \\ \frac{0}{4.47} \end{bmatrix} \cdot \begin{bmatrix} \frac{-2/5}{1.34} \\ \frac{4/5}{1.34} \\ \frac{1}{1.34} \end{bmatrix} \cdot \begin{bmatrix} \frac{-46/15}{3.186} \\ \frac{-13/15}{3.186} \\ \frac{0}{3.186} \end{bmatrix}$$

4

$$= \begin{bmatrix} 0.89 & -0.29 & -0.96 \\ 0.44 & 0.59 & -0.27 \\ 0 & 0.74 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.89 & 0.44 & 0 \\ -0.29 & 0.59 & 0.74 \\ -0.96 & -0.27 & 0 \end{bmatrix}^T$$

$$R = A\,Q^T$$

$$R = \begin{bmatrix} 4 & 2 & 0 \\ 2 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0.89 & 0.44 & 0 \\ -0.29 & 0.59 & 0.74 \\ -0.96 & -0.27 & 0 \end{bmatrix}$$

$$R = \begin{bmatrix} 2.94 & 2.94 & 1.48 \\ 0.22 & 1.79 & 1.48 \\ -1.25 & 0.32 & 1.48 \end{bmatrix}$$

Q2(a) Ans#

Point $\frac{\pi}{4}$ for the function $f(x) = \sin(x)$

The formula for a taylor series of degree n at a point for any function is as follow

$$F(x) = f(a) + \frac{f'(a)}{1!}(x-a)^1 + \frac{f^2(a)}{2!}(x-a)^2 + \frac{f^3(a)}{3!}(x-a)^3 + \frac{f^4(a)}{4!}(x-a)^4 + \frac{f^5(a)}{5!}(x-a)^5 \cdots \frac{f^n(a)}{n!}(x-a)^n$$

Differentiating the function $\sin(x)$ 5 time and the values of each derivatives at $\frac{\pi}{4}$.

$$f(x) = \sin(x) \qquad f\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$f'(x) = \cos(x) \qquad f'\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$f^2(x) = -\sin(x) \qquad f^2\left(\frac{\pi}{4}\right) = \frac{-1}{\sqrt{2}}$$

$$F^3(x) = -\cos(x) \qquad f^3\left(\frac{\pi}{4}\right) = \frac{-1}{\sqrt{2}}$$

$$F^4(x) = \sin(x) \qquad f^4\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$F^5(x) = \cos(x) \qquad F^5\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$T_5(x) = f\left(\frac{\pi}{4}\right) + \frac{f'(\pi/4)}{1!}(x-\pi/4)^1 + \frac{f^2(\pi/4)}{2!}(x-\pi/4)^2 + \frac{f^3(\pi/4)}{3!}(x-\pi/4)^3 + \frac{F^4(\pi/4)}{4!}(x-\pi/4)^4 + \frac{F^5(\pi/4)}{5!}(x-\pi/4)^5$$

(6)

So the taylor polynomial of degree 5 at point $\frac{\pi}{4}$ for $\sin(x)$ is

$$T_5(x) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2} \times 1!}\left(x - \frac{\pi}{4}\right) + \frac{-1}{\sqrt{2} \times 2!}\left(x - \frac{\pi}{4}\right)^2 + \frac{-1}{\sqrt{2} \times 3!}\left(x - \frac{\pi}{4}\right)^3 + \frac{1}{\sqrt{2} \times 4!}\left(x - \frac{\pi}{4}\right)^4 + \frac{1}{\sqrt{2} \times 5!}\left(x - \frac{\pi}{4}\right)^5$$

simplifies to

$$T_5(x) = \frac{1}{\sqrt{2}} + \frac{\left(x - \frac{\pi}{4}\right)}{\sqrt{2}} - \frac{\left(x - \frac{\pi}{4}\right)^2}{2\sqrt{2}} - \frac{\left(x - \frac{\pi}{4}\right)^3}{6\sqrt{2}} + \frac{\left(x - \pi/4\right)^4}{24\sqrt{2}} + \frac{\left(x - \pi/4\right)^5}{120\sqrt{2}}$$

(b)