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Question No# 1

$$ID = 5534$$

3rd ID $x_2 + x_3 = 0$

$2x_2 - 8x_3 = 8$

$-5x_3 = 10$

Solution

$$x_1 - 3x_2 + x_3 = 0$$
$$2x_2 - 8x_3 = 8$$
$$5x_1 - 5x_3 = 10$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & -5 & 10 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & +40 & -10 & 10 \end{array}\right] R_3 - 5R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 4 & -1 & 1 \end{array}\right] \begin{array}{l} R_2/4 \\ R_3/10 \end{array}$$

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$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & -15 & 15 \end{array}\right] R_3-4R_2$$

Consists of Because of triang.

$-15 \times 3 = -15$

$x_3 = 1$

$x_1 = -3 \times 3 = 0\ 3$

$x_2 = 3 \cdot 0 + 3x_3$

$x_2 = 6$

$x_1 - 3x_2 + x_3 = 0$

$x_1 = 3x_2 - x_3$

$\boxed{x_1 = 3x_2}$

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Question No:- 2

find the inverse of $A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 4\text{th ID} \\ 5 & -2 & 7 \end{bmatrix}$

by adjoint method.

Solution:-

My ID is 5534

$$\begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 4 \\ 5 & -2 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A| \neq 0} \text{adj}(A) = \{|A| \neq 0\}$$

$$|A| = 3\{(-1 \times 7) - (4 \times (-2))\} - 4\{(2 \times 7) - (4 \times 5)\} + 5\{(2 \times -2) - (-1 \times 5)\}$$

$$|A| = 3(-7+8) - 4(14-20) + 5(-4+5)$$

$$|A| = 3(1) - 4(-6) + 5(1)$$

$$|A| = 3 + 24 + 5$$

$$\boxed{|A| = 32}$$

Inverse of Matrix $A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 4 \\ 5 & -2 & 7 \end{bmatrix}$ exists

Now we find cofactors:-

Cofactor of 3

$\Rightarrow (-1)^{1+1}\{(-1\times 7)-(4\times(-2))\}$

$\Rightarrow (-1)^2\{-7+8\}$

$\Rightarrow 1(1)$

$\boxed{=1}$

Cofector of 4

$=(-1)^{1+2}\times\{(2\times 7)-(4\times 5)\}$

$=(-1)^3\times(14-20)$

$=-1\times(-6)$

$\boxed{=6}$

Cofector of 5

$\Rightarrow (-1)^{1+3}\times\{(2\times(-2))-[(-1)\times 5]\}$

$\Rightarrow (-1)^4\times\{-4+5\}$

$\Rightarrow 1\times(1)$

$\boxed{\Rightarrow 1}$

Cofector of 2

$\Rightarrow (-1)^{2+1} \times \{(4\times7)\cdot(5\times(-2))\}$

$\Rightarrow (-1)^3 \times \{(28+10)\}$

$\Rightarrow -1 \times (38)$

$\boxed{\Rightarrow -38}$

Cofetor of -1

$\Rightarrow (-1)^{2+2} \times \{(3\times7)-(5\times5)$

$\Rightarrow (-1)^4 \times \{(21)-(25)\}$

$\Rightarrow 1-(4)$

$\boxed{\Rightarrow -4}$

Cofetor of 4

$(-1)^{2+3} \{(2\times(-2)-(-1)\times5)\}$

$=(-1)^5 \{-4+5)$

$\Rightarrow -1\ (1)$

$\boxed{\Rightarrow -1}$

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Cofactor of 5:

$\Rightarrow (-1)^{3+1} \{(4\times4) - 5(-1)\}$

$\Rightarrow (-1)^4 (16+5)$

$\Rightarrow 1\times(21)$

$\boxed{\Rightarrow 21}$

Cofactor of -2 $\Rightarrow (-1)^{3+2} \{(3\times4)-(5\times2)\}$

$= (-1)^5 (12-10)$

$= -1\times(2)$

$\boxed{= -2}$

Cofactor of 7

$\Rightarrow (-1)^{3+3} \{(3\times(-1)-(4\times5)\}$

$\Rightarrow (-1)^6 \{-3-20\}$

$= 1 \cdot (-23)$

$\boxed{= -23}$

Question No# 3:-

$$2x + 2y + 4z = 18$$
$$x + 3y + 2z = 13$$
$$3x + 2y - 3z = 14$$

Solution:-

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{array}\right]$$

$$R_1 \leftarrow R_1 \div 2$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{array}\right]$$

$$R_2 \leftarrow R_2 - R_1$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 3 & 2 & -3 & 14 \end{array}\right]$$

$$R_3 \leftarrow R_3 - 3 \times R_1$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 0 & -1 & -9 & -13 \end{array}\right]$$

$R_2 \leftarrow R_2 \div 2$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & -1 & -9 & -13 \end{array}\right]$$

$R_1 \leftarrow R_1 - R_2$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & -1 & -9 & -13 \end{array}\right]$$

$R_3 \leftarrow R_3 + R_2$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -9 & -11 \end{array}\right]$$

$R_3 \leftarrow R_3 \div -9$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 11/9 \end{array}\right]$$

$R_1 \leftarrow R_1 - 2 \times R_3$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 41/9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 11/9 \end{array}\right]$$

i.e $x = 41/9$
$y = 2$
$z = 11/9$ Ans

Question No 4:-

find eigen values of Matrix A:-

$$[A-\lambda I]=0$$

$$\begin{vmatrix} 4-\lambda & 2 & -2 \\ 5 & (3-\lambda) & 2 \\ -2 & 4 & (1-\lambda) \end{vmatrix} = 0$$

$\because (4-\lambda)((3-\lambda)\times(1-\lambda)-2\times4)-2((5)\times(1-\lambda)-2\times(-2))+(-2)((5)\times4-(3-\lambda)\times(-2))=0$

$\because (4-\lambda)((3-4\lambda+\lambda^2)-8)-2((-5+5\lambda)-(-4))-2(-20)-(-6+2\lambda)=0$

$\therefore (4-\lambda)(-5-4\lambda+\lambda^2)-2(-1+5\lambda)-2(-14-2\lambda)=0$

$\because (-20-11\lambda+8\lambda^2-\lambda^3)-(-2+10\lambda)-(-28-4\lambda)=0$

$\because (-\lambda^3+8\lambda^2-17\lambda+10)=0$

$=-(\lambda-1)(\lambda-2)(\lambda-5)=0$

$=(\lambda-1)=0$ or $(\lambda-2)=0$ or $(\lambda-5)=0$

1) Eigen vector for $\lambda=1$

$$\cancel{A-\lambda I} \quad A-\lambda I=\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix} - I\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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$$= \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 & -2 \\ -5 & 2 & 2 \\ -2 & 4 & 0 \end{bmatrix}$$

Now reduce the matrix interchange the row $R_1 \leftrightarrow R_2$

$$= \begin{bmatrix} -5 & 2 & 2 \\ 3 & 2 & -2 \\ -2 & 4 & 0 \end{bmatrix}$$

$R_1 \leftarrow R_1 \div -5$

$$= \begin{bmatrix} 1 & -0.4 & -0.4 \\ 3 & 2 & -2 \\ -2 & 4 & 0 \end{bmatrix}$$

$R_2 \leftarrow R_2 - 3 \times R_1$

$$\begin{bmatrix} 1 & -0.4 & -0.4 \\ 0 & 3.2 & -0.8 \\ -2 & 4 & 0 \end{bmatrix}$$

$R_3 \leftarrow R_3 + 2 \times R_1$

$$\begin{bmatrix} 1 & -0.4 & -0.4 \\ 0 & 3.2 & -0.8 \\ 0 & 3.2 & -0.8 \end{bmatrix}$$

$R_2 \leftarrow R_2 \times 0.3125$

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$$\begin{bmatrix} 1 & -0.4 & -0.4 \\ 0 & 1 & -0.25 \\ 0 & 3.2 & 0.08 \end{bmatrix}$$

$R_1 \leftarrow R_1 + 0.4 \times R_2$

$$\begin{bmatrix} 1 & 0 & -0.5 \\ 0 & 1 & -0.25 \\ 0 & 3.2 & -0.8 \end{bmatrix}$$

$R_3 \leftarrow R_3 - 3.2 \times R_2$

$$\begin{bmatrix} 1 & 0 & -0.5 \\ 0 & 1 & -0.25 \\ 0 & 0 & 0 \end{bmatrix}$$

The system associated with eigen value $\lambda = 1$

$$(\lambda - I)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -0.5 \\ 0 & 1 & -0.25 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$= x_1 - 0.5x = 0, x_2 - 0.25x_3 = 0$

$= x_1 = 0.5x_3 \cdot x_2 = 0.25 x_3$

$\because$ eigen vectors corresponding to the eigen values $\lambda = 1$ is

$$V = \begin{bmatrix} 0.5x_3 \\ 0.25x_3 \\ x_3 \end{bmatrix}$$

let $x_3 = 1$

$$V = \begin{bmatrix} 0.5 \\ 0.25 \\ 1 \end{bmatrix}$$

$\therefore$ eigen vectors for λ_2

$$A - \lambda I \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & -2 \\ -5 & 1 & 2 \\ -2 & 4 & -1 \end{bmatrix}$$

Now reduce the matrix interchanging row $R_1 \leftrightarrow R_2$

$$= \begin{bmatrix} -5 & 1 & 2 \\ 2 & 2 & -2 \\ -2 & 4 & -1 \end{bmatrix}$$

$R_1 \leftarrow R_1 \div -5$

$$\begin{bmatrix} 1 & -0.2 & -0.4 \\ 2 & 2 & -2 \\ -2 & 4 & -1 \end{bmatrix}$$

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$R_2 \leftarrow R_2 = 0.2778$

$$\begin{bmatrix} 1 & -0.2 & -0.4 \\ 0 & 1 & -0.5 \\ 0 & 24 & -12 \end{bmatrix}$$

$R_1 \leftarrow R_1 + 0.2 \times R_2$

$$\begin{bmatrix} 1 & 0 & -0.5 \\ 0 & 1 & -0.5 \\ 0 & 24 & -12 \end{bmatrix}$$

$R_3 \leftarrow R_3 - 24 \times R_2$

$$\begin{bmatrix} 1 & 0 & -0.5 \\ 0 & 1 & -0.5 \\ 0 & 0 & 0 \end{bmatrix}$$

The system associated with the eigen values $\lambda = 2$

$$(A - 2I)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -0.5 \\ 0 & 1 & -0.5 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$= x_1 - 0.5x_1 = 0.x_2 - 0.5x_3 = 0$

$= x_1 = 0.5x_3 . x_2 = 0.5x_3$

eigen values corresponding the eigen value of $\lambda = 2$ is

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$$V = \begin{pmatrix} 0.5x_3 \\ 0.5x_3 \\ x_3 \end{pmatrix}$$

$$V_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

The eigen values compose the coloum of matrix P

$$P = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.25 & 0.5 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

The diagnal of matrix D is composed of the eigen values.

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

3) Now find P

$$|P| = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/4 & 1/2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

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$$= \frac{1}{2} \times \begin{bmatrix} 1/2 & 1 \\ 1 & 1 \end{bmatrix} - \frac{1}{2} \times \begin{bmatrix} 1/4 & 1 \\ 1 & 1 \end{bmatrix} + 0 \times \begin{bmatrix} 1/4 & 1/2 \\ 1 & 1 \end{bmatrix}$$

$$= \frac{1}{2} \times \left(\frac{1}{2} \times 1 - 1 \times 1 \right) - \frac{1}{2} \times \left(\frac{1}{4} \times 1 - 1 \times 1 \right) + 0 \times \left(\frac{1}{4} \times 1 - \frac{1}{2} \times 1 \right)$$

$$= \frac{1}{2} \times \left(\frac{1}{2} - 1 \right) - \frac{1}{2} \times \left(\frac{1}{4} - 1 \right) + 0 \times \left(\frac{1}{4} - \frac{1}{2} \right)$$

$$= \frac{1}{2} \left(-\frac{1}{2} \right) - \frac{1}{2} \times \left(-\frac{3}{4} \right) + 0 \times \left(-\frac{1}{4} \right)$$

$$= -\frac{1}{4} + \frac{3}{8} = 0$$

$$= \frac{1}{8}$$

$$Adj(P) = adj \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/4 & 1/2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} +\begin{bmatrix} 1/2 & 1 \\ 1 & 1 \end{bmatrix} & -\begin{bmatrix} 1/4 & 1 \\ 1 & 1 \end{bmatrix} & +\begin{bmatrix} 1/4 & 1/2 \\ 1 & 1 \end{bmatrix} \\ -\begin{bmatrix} 1/2 & 0 \\ 1 & 1 \end{bmatrix} & +\begin{bmatrix} 1/2 & 0 \\ 1 & 1 \end{bmatrix} & -\begin{bmatrix} 1/2 & 1/2 \\ 1 & 1 \end{bmatrix} \\ +\begin{bmatrix} 1/2 & 0 \\ 1/2 & 1 \end{bmatrix} & -\begin{bmatrix} 1/2 & 0 \\ 1/4 & 1 \end{bmatrix} & +\begin{bmatrix} 1/2 & 1/2 \\ 1/4 & 1/2 \end{bmatrix} \end{bmatrix}$$

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$$= \begin{bmatrix} -(1/2 \times 1 - 1 \times 1) - (1/4 \times 1 - 1 \times 1) + (1/4 \times 1 - 1/2 \times 1) \\ -(1/2 \times 1 - 0 \times 1) + (1/2 \times 1 - 0 \times 1) + (1/2 \times 1 - 1/2 \times 1) \\ -(1/2 \times 1 \cdot 0 \times 1/2)(1/2 \times 1 - 0 \times 1/4)(1/2 \times 1/2 - 1/2 \times 1/4) \end{bmatrix}$$

$$= \begin{bmatrix} +(1/2 - 1) - (1/4) + (1/4 - 1/2) \\ -(1/2 + 0) + (1/2 + 0) + (1/2 - 1/2) \\ -(1/2 + 0) - (1/2 + 0) + (1/4 - 1/8) \end{bmatrix}$$

$$= \begin{bmatrix} -1/2 & 3/4 & -1/4 \\ -1/2 & 1/2 & 0 \\ 1/2 & -1/2 & 1/8 \end{bmatrix}$$

$$\begin{bmatrix} -1/2 & -1/2 & 1/2 \\ 3/4 & 1/2 & -1/2 \\ -1/4 & 0 & 1/8 \end{bmatrix}$$

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Now $P^{-1} = \frac{1}{|P|} \times adj\ of (P)$

$$1/\tfrac{1}{8}\begin{bmatrix} -1/2 & -1/2 & 1/2 \\ 3/4 & 1/2 & -1/2 \\ -1/4 & 0 & 1/8 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -4 & 4 \\ 6 & 4 & -4 \\ -2 & 0 & 1 \end{bmatrix}$$

$$\therefore \quad P^{-1} = \begin{bmatrix} -4 & -4 & 4 \\ 6 & 4 & -4 \\ -2 & 0 & 1 \end{bmatrix}$$

$$P \times D \begin{bmatrix} 1/2 & 1/2 & 0 \\ -1/4 & 1/2 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & 1 & 0 \\ 1/4 & 1 & 5 \\ 1 & 2 & 5 \end{bmatrix}$$ Ans:-

Question No:- 5:-
Determine if the following homogenous system has a non-trivial solution. Then describe the solution set.

$$3x_1 + 5x_2 - 4x_3 = 0$$
$$-3x_1 - 2x_2 + 4x_3 = 0$$
$$6x_1 + x_2 - 8x_3 = 0$$

Solution

$$\left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ -3 & -2 & 4 & 0 \\ 6 & 1 & -8 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 5/3 & -4/3 & 0 \\ -3 & -2 & 4 & 0 \\ 6 & 1 & -8 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 5/3 & -4/3 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & -9 & 0 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 5/3 & -4/3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -9 & 0 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -4/3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

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$$x = \begin{bmatrix} 4/3 \; 8 \\ 0 \\ 8 \end{bmatrix} = 8 \begin{bmatrix} 4/3 \\ 0 \\ 1 \end{bmatrix}$$

Question No # 6

Reduce the matrix to Normal form and find its rank

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Solution:-

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Reduce matrix to reduced row-echilon form.

Swap matrix row $R_1 \leftrightarrow R_2$

$$= \begin{bmatrix} 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Cancel leading co-efficient in row R_2 by performing

$$R_2 \leftarrow R_2 - \frac{1}{3} R_1$$

$$= \begin{bmatrix} 3 & 9 & 12 & 3 \\ 0 & 0 & 0 & 2 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Cancel leading co-efficient in Row R_3 by performing.

$$R_3 \leftarrow R_3 - \frac{1}{3} \cdot R_1$$

$$\begin{bmatrix} 3 & 9 & 12 & 3 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Rank of a matrix is the number of all ~~non~~ zeros row.

$$\text{Rank of } \begin{bmatrix} 1 & 3 & 4 & 0 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix} \boxed{= 2}$$

Question No:- 5

Determine if the following homogenous