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4th

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Electromagnetic field theory

ID # 14620

Q.1 The value of E at $P(\rho = 2, \phi = 40^\circ, Z = 3)$ is given as $E = 100\,a_\rho - 200\,a_\phi + 300\,a_z$ V/m. Determine the incremental work required to move a $20\,\mu C$ charged a distance of $6\,\mu m$.

Ans in the direction of a_ρ: The incremental work is given by $dW = -qE \cdot dL$, where in this case, $dL = d\rho\, a_\rho = 6 \times 10^{-6}\, a_\rho$. Thus

$$dW = -(20 \times 10^{-6} C)(100 V/m)(6 \times 10^{-6} m) = -12 \times 10^{-9} J = \underline{-12 nJ}$$

* in the direction of a_ϕ: in this case $dL = 2\,d\phi\, a_\phi = 6 \times 10^{-6}\, a_\phi$, and so

$$dW = -(20 \times 10^{-6})(-200)(6 \times 10^{-6}) = 2.4 \times 10^{-8} J = \underline{24 nJ}$$

* in the direction of a_z: Here, $dL = dz\, a_z = 6 \times 10^{-6}\, a_z$,

$$dW = -(20 \times 10^{-6})(300)(6 \times 10^{-6}) = -3.6 \times 10^{-8} J = \underline{-36 nJ}$$

* in the direction of E: Here, $dL = 6 \times 10^{-6}\, a_E$, where

$$a_E = \frac{100 a_\rho - 200 a_\phi + 300 a_z}{[100^2 + 200^2 + 300^2]^{1/2}} = 0.267 a_\rho - 0.535 a_\phi + 0.802 a_z$$

Thus

$$dW = -(20 \times 10^{-6})[100 a_\rho - 200 a_\phi + 300 a_z] \cdot [0.267 a_\rho - 0.535 a_\phi + 0.802 a_z]$$
$$(6 \times 10^{-6}) = \underline{-44.9 nJ}$$

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⇒ in The direction of $G = 2a_x - 3a_y + 4a_z$: in This case, $dl = 6\times10^{-6}\, a_G$, where

$$a_G = \frac{2a_x - 3a_y + 4a_z}{[2^2 + 3^2 + 4^2]^{1/2}} = 0.371 a_x - 0.557 a_y + 0.743 a_z$$

So now

$$dw = -(20\times10^{-6})[100 a_\rho - 200 a_\phi + 300 a_z]\cdot[0.371 a_x - 0.557 a_y + 0.743 a_z](6\times10^{-6})$$

$$= -(20\times10^{-6})[37.1(a_\rho\cdot a_x) - 55.7(a_\rho\cdot a_y) - 74.2(a_\phi\cdot a_x) + 111.4 (a_\phi\cdot a_y) + 222.9](6\times10^{-6})$$

Where, at P, $(a_\rho\cdot a_x) = (a_\phi\cdot a_y) = \cos(40^\circ) = 0.766$, $(a_\rho\cdot a_y) = \sin(40^\circ) = 0.643$, and $(a_\phi\cdot a_x) = -\sin(40^\circ) = -0.643$. Subsituting These results in

$$dw = -(20\times10^{-6})[28.4 - 35.8 + 47.7 + 85.3 + 222.9](6\times10^{-6}) = \underline{-41.8\,nJ}$$

Q3 if $E = 120\, a_\rho$ V/m, Find The incremental amount of work done in moving a 50 μm charge a distance of 2mm From.

Ans P(1,2,3) Toward Q(2,1,4): The vector along This direction will be $Q - P = (1, -1, 1)$ from which $a_{PQ} = [a_x - a_y + a_z]/\sqrt{3}$. we now write

$$dW = -qE \cdot dL = -(50 \times 10^{-6})\left[120\, a_\rho \cdot \left(\frac{a_x - a_y + a_z}{\sqrt{3}}\right)\right](2 \times 10^{-3})$$

$$= -(50 \times 10^{-6})(120)\left[(a_\rho \cdot a_x) - (a_\rho \cdot a_y)\right]\frac{1}{\sqrt{3}}(2 \times 10^{-3})$$

At P, $\phi = \tan^{-1}(2/1) = 63.4°$. Thus $(a_\rho \cdot a_x) = \cos(63.4) = 0.447$ and $(a_\rho \cdot a_y) = \sin(63.4) = 0.894$. Subsituting These, we obtain $dw = 3.1\mu J$.

Q(2,1,4) Toward P(1,2,3): A Little Thought is in order here: Note That The field has only a radial component and does not depend on ϕ or z. Note also That P and Q are at The Same radius ($\sqrt{5}$) from The z axis, but have different ϕ and z coordinates. We could Just as well Position The Two Points at The Same z location and The Problem would not change. if This were So, Then moving along a straight

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Q4 Compute The value of G - -
- - - - - - using The path.

Ans:) Striaght line of Segments A (1,-1,2) To B (1,1,2) To P (2,1,2) In general

we have

$$\int_A^P G \cdot dL = \int_A^P 2y\,dx$$

The change of x occurs when moving b/w B and P during which y = 1 Thus

$$\int_A^P G \cdot dL = \int_B^P 2y\,dx = \int_1^2 2(1)\,dx = \boxed{2}$$

(B) Straigh line Segment A (1,-1,2) C (2,-1,2) To P (2,1,2) In case The change in x occur when moving from A To C, during which y = -1 Thus

$$\int_A^P G \cdot dL = \int_A^C 2y\,dx = \int_1^2 2(-1)\,dx = \boxed{-2}$$

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Q5 For $G = 3xy^3 a_x + 2z a_y$. Now things - - -
- - - - - in that path does matter.

Ans Straight line $y = x-1, z = 1$ we obtain

$$\int G \cdot dL = \int_2^4 3xy^2 dx + \int_1^3 2z\, dy = \int_2^4 3x(x-1)^2 dx + \int_1^3 2(1)\, dy = \boxed{90}$$

B. Parabola $6y = x^2 + 2, z = 1$ we obtain

$$\int G \cdot dL = \int_2^4 3xy^2 dx + \int_1^3 2z\, dy$$

$$\Rightarrow \int_2^4 \frac{1}{12} x (x^2+2)^2 dx + \int_1^3 2(1)\, dy = \boxed{82}$$

(B) STEP 2 :

$$I_a = \frac{V}{Z_1} = \frac{138.56\angle -30^\circ}{0.96 + j1.6627} = 72.17\angle 90^\circ A$$

$$I_b = I_a \angle -120^\circ = 72.17 \angle -210^\circ A$$

$$I_c = I_a \angle +120 = 72.17 \angle 30^\circ A.$$

x ———————————— x