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(Differential Equation).

Q1 MCQ's :-

1) $AB = m \times n$

2) 1 non-zero row = called Rank of matrix.

3) $a = 8$.

4) $|A| = 3$

5) $\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$ is Scalar matrix

6) $\log(y) = x - x^2 + C$

7) order 1, degree 3.

8) order 2 degree is 1

9) Homogenous eq.

10) $bc(c-d) - ac(c-a) + ab(b-a)$.

Q10) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

Solution :-

$= 1\begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - 1\begin{vmatrix} a & a^2 \\ c & c^2 \end{vmatrix} + 1\begin{vmatrix} a & a^2 \\ b & b^2 \end{vmatrix}$

$= (bc^2 - cb^2) - 1(ac^2 - a^2c) + (ab^2 - a^2b)$.

$= bc(c-b) - ac(c-a) + ab(b-a)$ Answer.

Q2) Express the determinant

i) $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$

Solution:-

Expand w.r.t R_1.

$= a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$

$= a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2)$
$+ c(a^2b^3 - a^3b^2)$

$= ab^2c^3 - ab^3c^2 - a^2bc^3 - a^3bc^2 +$
$a^2b^3c - a^3b^2c$

Common abc.

$\Rightarrow abc(bc^2 - b^2c - ac^2 + a^2c +$
$ab^2 - a^2b)$

$= abc(bc(c-b) - ac(c+a) + ab(b-a))$

Answer.

ii) Find the Eigen value

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Solution:-

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

characteristic eq,u $\rightarrow |A - \lambda I| = 0 \rightarrow i$

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now taking determinent

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

Expand by R_1. (4)

$$= 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-1 \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} = 0 \longrightarrow (B)$$

Again

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-1 \end{vmatrix}$$

Expand by R_1.

$$= 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - 1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (3-\lambda)\Big[\big((3-\lambda)(2-\lambda)-(-1)(-1)\big) + 1\big((-1)(2-\lambda)-(-1)(-1)\big)$$

$$-1\big((-1)(-1)-(-1)(3-\lambda)\big)\Big\}$$

$$= (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1)+(-2+\lambda-1)-(1+3-\lambda)$$

$= (3-\lambda)(\lambda^2-5\lambda+5)+(-3+\lambda)-(4-\lambda)$ 5

$= 3\lambda^2-15\lambda+15-\lambda^3+5\lambda^2-5\lambda-3+\lambda-4+\lambda$

$$\boxed{= -\lambda^3+8\lambda^2-18\lambda+8 \rightarrow a}$$

Now

$$+1\begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1.

$$= -1\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$= -1(6-3\lambda-2\lambda+\lambda^2-1)+1(-2+\lambda-1)$

$= -\lambda^2+5\lambda-5-3+\lambda$

$$\boxed{= -\lambda^2+6\lambda-8 \rightarrow b}$$

Now

$$-1\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand

$$= -\left[-1\begin{vmatrix}-1 & -1\\ -1 & 2-\lambda\end{vmatrix} - (-1)\begin{vmatrix}3-\lambda & -1\\ -1 & 2-\lambda\end{vmatrix} + 0\right]$$

$$= -[-(-2+\lambda-1)+1(6-3\lambda-2\lambda+\lambda^2-1)$$

$$= -(3-\lambda+\lambda^2-5\lambda+5)$$

$$\boxed{= -\lambda^2+6\lambda-8 \rightarrow c}$$

put a, b and c in B

$$= (2-\lambda)[-\lambda^3+8\lambda^2-18\lambda+8]-\lambda^2+6\lambda-8-\lambda^2+$$
$$6\lambda-8$$

$$= -2\lambda^3+(6\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2-8\lambda-\lambda^2$$
$$+6\lambda-8-\lambda^2+16\lambda-8=0.$$

$$= \lambda^4-2\lambda^3-8\lambda^3+16\lambda^2+16\lambda^2-\lambda^2-\lambda^2-36\lambda$$
$$-8\lambda+6\lambda+6\lambda+16-16=0$$

$$= \lambda^4-10\lambda^3+32\lambda^2-32\lambda=0$$

By Synthatic division we get

$$\lambda(\lambda-2)(\lambda^2-8\lambda+16)=0.$$

$$(\lambda=0)$$

$$\lambda-2=0 \Rightarrow \boxed{\lambda=2.}$$

$\lambda^2 - 8\lambda + 16 = 0$

By factorization method.

$\lambda^2 - 4\lambda - 4\lambda + 16 = 0$

$\lambda(\lambda - 4) - 4(\lambda - 4) = 0$

$(\lambda - 4)(\lambda - 4) = 0$

$(\lambda - 4) = 0 \quad \lambda - 4 = 0$

$\lambda = 4 \quad , \lambda = 4$

$\boxed{\lambda_1 = 0}$, $\boxed{\lambda_2 = 2}$, $\boxed{\lambda_3 = 4}$, $\boxed{\lambda_4 = 4}$

Ans.

Q3) The rate of change .....?

$(x^2 + 3y^2)dx - 2xydy = 0$. Find general Solution at $x = 2$ and $y = 6$.

Solution:-

$(x^2 + 3y^2)dx - 2xydy = 0$

$(x^2 + 3y^2)dx = 2xydy$

Dividing b/s by $2xydx$.

we get

$$\frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^{\cancel{2}}}{2xy} + \frac{3y^{\cancel{2}}}{2xy} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \rightarrow \star$$

Let $y = vx$.

Diff

$$dy = v\,dx + x\,dv$$

Dividing by dx

$$\frac{dy}{dx} = v + \frac{x\,dv}{dx} \rightarrow (a)$$

put (a) in $\star$

$$v + \frac{x\,dv}{dx} = \frac{1}{2}\left[\frac{\cancel{x}}{\cancel{x}v} + 3\frac{v\cancel{x}}{\cancel{x}}\right.$$

$$v + \frac{x\,dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

Multiplying b/s by 2.

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + v$$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v}$$

Multiplying b/s by $\frac{dx}{dv}$

$$2x\,dv = \frac{1+v^2}{v}\,dx$$

Xing b/s by $\frac{v}{x(1+v^2)}$

we get

$$\frac{v}{1+v^2}\,dv = \frac{1}{x}\,dx$$

Take "$\int$" on b/s

$$\int \frac{2v}{1+v^2} = \ln x + \ln c$$

Take "e" on b/s.

$$e^{\ln|1+v^2|} = e^{\ln(xc)}$$

$$1+v^2 = xc$$

put $v = y/x$

$$1+\left(y/x\right)^2 = xc$$

$$\frac{x^2+y^2}{x^2} = xc$$

$$x^2+y^2 = x^3c \rightarrow \star_1$$

put $x=2$, $y=6$ in eq $\Rightarrow \star_1$

$$(4)+(36) = 8c$$

$$c = \frac{40}{8}$$

$c = 5 \rightarrow$ put in $\star_1$

So

$$x^2+y^2 = 5x^3$$

$$y^2 = 5x^3 - x^2$$

$$y^2 = x^2(5x-1)$$

Taking squreroot on b/s.

$y = +x\sqrt{5x-1}$, $y = -x\sqrt{5x-1}$