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Dept :: BSSE

Subject :: A Linear Algebra.

Q1

(1)
$$\begin{bmatrix} 1 & 2 & \text{2nd-ID} \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

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2nd number is 5

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

$$A_{adj} = ?$$

Replace all values its cofactor then take transpose also equal to adjoint of A.

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = (1) \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = 6 - 1 = 5$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = (-1) (4 - 3) = -1$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} = (1) (2 - 9) = -7$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 2 & 2 \\ 1 & 2 \end{vmatrix} = (-1) (4 - 2) = -2$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} = (1) (2 - 6) = -4$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = (-1) (1 - 6) = 5$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} = (1) (2 - 6) = -4$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = (-1) (1 - 4) = 3$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = (1) (3 - 4) = -1$$

$$A \text{ cofactor} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & -7 \\ -2 & -4 & 5 \\ -4 & 3 & -1 \end{bmatrix}$$

Transpose

$$A \text{ adj} = A^T = \begin{bmatrix} 5 & -2 & -4 \\ -1 & -4 & 3 \\ -7 & 5 & -1 \end{bmatrix}$$

Ans.

Q1

(ii) $B = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 8 \end{bmatrix}$

Sol $A \text{ adj} = ?$

$$A_{11} = (-1)^{1+1} \begin{vmatrix} -1 & 8 \\ -2 & 8 \end{vmatrix} = (1)(-8+16) = 8$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 8 \\ 5 & 8 \end{vmatrix} = (1)(16-40) = -24$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix} = (1) (-4+5) = 1$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 4 & 5 \\ -2 & 8 \end{vmatrix} = (-1) (32+10) = -42$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 5 \\ 5 & 8 \end{vmatrix} = (1) (24-25) = -1$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 3 & 4 \\ 5 & -2 \end{vmatrix} = (-1) (-6-20) = 26$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 4 & 5 \\ -1 & 8 \end{vmatrix} = (1) (32+5) = 37$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 5 \\ 2 & 8 \end{vmatrix} = (-1) (24-10) = -14$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = (1) (-3-8) = -11$$

Now replace all value factors.

$$B = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 24 & 1 \\ -42 & -1 & 26 \\ 37 & -14 & -11 \end{bmatrix}$$

Now transpose

$$B_{adj} = B^t = \begin{bmatrix} 8 & -4 & 2 & 3 \\ 2 & 4 & -1 & -1 \\ 4 & 1 & 2 & 6 \\ -1 & 1 & 2 & -1 \end{bmatrix}$$

Ans.

Q2 Find the cofactor of A_{21}, A_{31}

A_{33} if

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$$

Solution:

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 3 \\ -3 & 2 \end{vmatrix} = (-1)(-4+9)$$

$$A_{21} = -5$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 3 \\ 3 & 1 \end{vmatrix} = (1)(-2-9)$$

$$A_{31} = -11$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -2 \\ -2 & 3 \end{vmatrix} = (1)(3-4)$$

$$A_{33} = -1$$

Q3 $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix}$ & $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Sol

$$\boxed{|A - \lambda I| = 0} \rightarrow \text{formula}$$

$$\begin{vmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{vmatrix} - \lambda \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{vmatrix} - \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} = 0$$

$$\begin{vmatrix} 2 - \lambda & 1 & 1 \\ 1 & 3 - \lambda & 2 \\ -1 & 1 & 2 - \lambda \end{vmatrix} = 0$$

Now take Determinant.

$$2 - \lambda ((3 - \lambda)(2 - \lambda) - 2) - 1(2 - (2 - \lambda)) + 1(1 + (3 - \lambda)) = 0$$

$$2 - \lambda (6 - 3\lambda - 2\lambda + \lambda^2 - 2) - 1(2 - 2 + \lambda) + 1(1 + 3 - \lambda) = 0$$

$$2 - \lambda (\lambda^2 - 5\lambda + 4) - 1(\lambda) + 1(4 - \lambda) = 0$$

$$2x^2 - 10x + 8 - x^3 + 5x^2 - 4x - x + 4 - x = 0$$

by ordering

$$-x^3 + 7x^2 - 16x + 12 = 0$$

×ing by (-1)

$$x^3 - 7x^2 + 16x - 12 = 0 \rightarrow (A)$$

Now put $x - 2 = 0$

then $x = 2$ in above eq (A)

$$x^3 - 7x^2 + 16x - 12 = 0$$

put $x = 2$

$$(2)^3 - 7(2)^2 + 16(2) - 12 = 0$$

$$8 - 7(4) + 32 - 12 = 0$$

$$8 - 28 + 32 - 12 = 0$$

$$-20 + 20 = 0$$

$$0 = 0$$

Eqn value = $x = 2$

or $x - 2 = 0$

~~Ans.~~