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Section	A
Paper	Hydraulic Engg
Semester	6 <sup>th</sup>

QNO.1):

Solution:

As we know pressure drop  $\Delta P$  depends on  $h, d, v, \rho$  and  $\mu$ .

$$\Delta P \quad ML^{-1}T^{-2}$$

$$h \quad L$$

$$d \quad L$$

$$v \quad LT^{-1}$$

$$\rho \quad ML^{-3}$$

$$\mu \quad ML^{-1}T^{-1}$$

No. of variables  $n=6$

No. of Independent dimension  $m=3$   
(M, L and T)

No. of non-dimensional groups;  $n-m=6-3=3$

Choose  $m=3$  scaling variables;

geometric ( $d$ ); Kinematic/time-dependent ( $v$ );  
dynamic/mass-dependent ( $\rho$ )

For non-dimensionalising  $\Delta P, h$  and  $\mu$ .

$$\pi_1 = \Delta P d^a v^b \rho^c$$

$$M^0 L^0 T^0 = (ML^{-1}T^{-2})(L)^a (LT^{-1})^b (ML^{-3})^c$$

$$= M^{1+c} L^{-1+a+b-3c} T^{-2-b}$$

$$M: 0 = 1+c \quad \Rightarrow c = -1$$

$$T: 0 = -2-b \quad b = -2$$

$$L: 0 = -1+a+b-3c \quad a = 1+3c-b = 0$$

$$\pi_1 = \Delta P v^{-2} \rho^{-1} = \frac{\Delta P}{\rho v^2}$$

ID # 7794

SECTION A

(2)

$$\pi_2 = \frac{h}{d} \quad (\text{Here "h" is length})$$

$$\pi_3 = \mu d^a v^b \rho^c$$

$$M^0 L^0 T^0 = (ML^{-1}T^{-1})(L)^a (LT^{-1})^b (ML^{-3})^c$$

$$= M^{1+c} L^{-1+a+b-3c} T^{-1-b}$$

$$M: 0 = 1+c \Rightarrow c = -1$$

$$T: 0 = -1-b+0 \Rightarrow b = -1$$

$$L: 0 = -1+a+b-3c \Rightarrow a = 1+3c-b = -1$$

$$\pi_3 = \mu d^{-1} v^{-1} \rho^{-1} = \frac{\mu}{\rho v d}$$

By Reynolds number we replace  $\pi_3$  by

$$\pi_3' = (\pi_3)^{-1} = \frac{\rho v d}{\mu}$$

By dimensional analysis yields

$$\pi_1 = f(\pi_2, \pi_3')$$

i.e.

$$\frac{\Delta P}{\rho v^2} = f\left(\frac{h}{d}, \frac{\rho v d}{\mu}\right)$$

(a) Dynamic similarity requires that all non-dimensional groups be the same in model and prototypes

$$\pi_1 = \left(\frac{\Delta P}{\rho v^2}\right)_p = \left(\frac{\Delta P}{\rho v^2}\right)_m$$

$$\pi_2 = \left(\frac{h}{d}\right)_n = \left(\frac{h}{d}\right)_m$$

Hence they have similar shape  
i.e. (geometric similarity)

$$\pi_3' = \left( \frac{\rho v d}{\mu} \right)_P = \left( \frac{\rho v d}{\mu} \right)_m$$

As velocity ratio

$$\frac{v_P}{v_m} = \frac{(\mu/\rho)_P}{(\mu/\rho)_m} \frac{d_m}{d_P} = \frac{0.002/800}{1.0 \times 10^{-6}} \times \frac{1}{5} = 0.5$$

Hence

$$v_m = \frac{v_P}{0.5} = \frac{3.0}{0.5} = 6.0 \text{ m s}^{-1}$$

(b) Ratio of quantities of flow is

$$\frac{Q_P}{Q_m} = \frac{(v \times A)_P}{(v \times A)_m} = \frac{v_P}{v_m} \left( \frac{d_P}{d_m} \right)^2 = 0.5 \times 5^2$$

$$\frac{Q_P}{Q_m} = 12.5$$

(c) For pressure drop

$$\pi_1 = \left( \frac{\Delta P}{\rho v^2} \right)_P = \left( \frac{\Delta P}{\rho v^2} \right)_m$$

$$\Rightarrow \frac{(\Delta P)_P}{(\Delta P)_m} = \frac{\rho_P}{\rho_m} \left( \frac{v_P}{v_m} \right)^2$$

$$= \frac{800}{1000} \times 0.5^2$$

$$= 0.2$$

Hence

$$\Delta P_P = 0.2 \times \Delta P_m = 0.2 \times 60 = 12.0 \text{ kPa}$$

Q NO. 2);

Solutions:

Max depth of water in the reservoir =  $H = 77$

Specific Gravity of dam material

$$G = 3.4$$

Allowable Compressive strength of dam masonry =  $779 \text{ T/m}^2$

Height of wave  $H_w = 2.2 \text{ m}$   
 $u = 0.7, C_u = 0$

$$\textcircled{1} H_{\text{limiting}} = \frac{G_{\text{all}}}{\gamma_w(G - C_u + 1)} = \frac{779 \times 1000}{1000(3.4 - 0 + 1)} = 324.583 \text{ m} > H_w = 77$$

So it is low gravity dam.

② Top width "a"

$$\text{Free board} = 1.5 h_{\text{wave}} = 1.5 \times 2.2$$

$$\boxed{F.B = 3.3 \text{ m}}$$

$$\text{height of Dam} = H_D = H_w + F.B = 77 + 3.3$$

$$\boxed{H_D = 80.3 \text{ m}}$$

$$a = 14\% \text{ of } H_D$$

$$a = 0.14 \times 80.3$$

$$\boxed{a = 11.24 \text{ m}}$$

3) Base width "b" (without offset)

(i) For No Sliding Criteria.

$$b' = \frac{HW}{\mu G} = \frac{77}{0.7 \times 3.4}$$

$$b' = 32.35$$

$$\boxed{b' = 32.4}$$

(ii) For No tension criteria

$$b' = \frac{HW}{\sqrt{G}} = \frac{77}{\sqrt{3.4}}$$

$$b' = 41.8 \text{ m} = 42 \text{ m}$$

use  $\boxed{b' = 42}$

4) Depth of vertical portion on U/s side.

$$h' = 2a\sqrt{G - C_u}$$

$$h' = 2 \times 11.24 \sqrt{3.4 - 0}$$

$$h' = 41.45$$

$$\boxed{h' = 42 \text{ m}}$$

5) Up stream offset =  $\frac{a}{16}$

$$= \frac{11.24}{16}$$

$$= 0.7025 \text{ m}$$

⑥ Depth <sup>ID #7794</sup> below section A-A water level to the end of inclined portion in u/s =  $3.14 a \sqrt{G}$

$$= 3.14 \times 11.24 \sqrt{3.4}$$

$$= 65.07 \text{ m}$$

⑦ Total width of the base of dam,

$$b = b' + \frac{a}{16} = 42 + 0.7025$$

$$\boxed{b = 42.7025 \text{ m}}$$

$$\textcircled{8} \quad \tan \theta = \frac{b'}{H} = \frac{42}{77}$$

$$\theta = \tan^{-1}(0.54)$$

$$\boxed{\theta = 28.37^\circ}$$

⑨ Depth of vertical portion on D/s (700m WL on u/s side)

$$\tan \theta = \frac{a}{d'} = \frac{11.24}{d'}$$

$$\tan \theta = \frac{11.24}{d'}$$

$$\frac{42}{77} d' = 11.24$$

$$d' = \frac{11.24 \times 77}{42}$$

$$\boxed{d' = 20.6 \text{ m}}$$

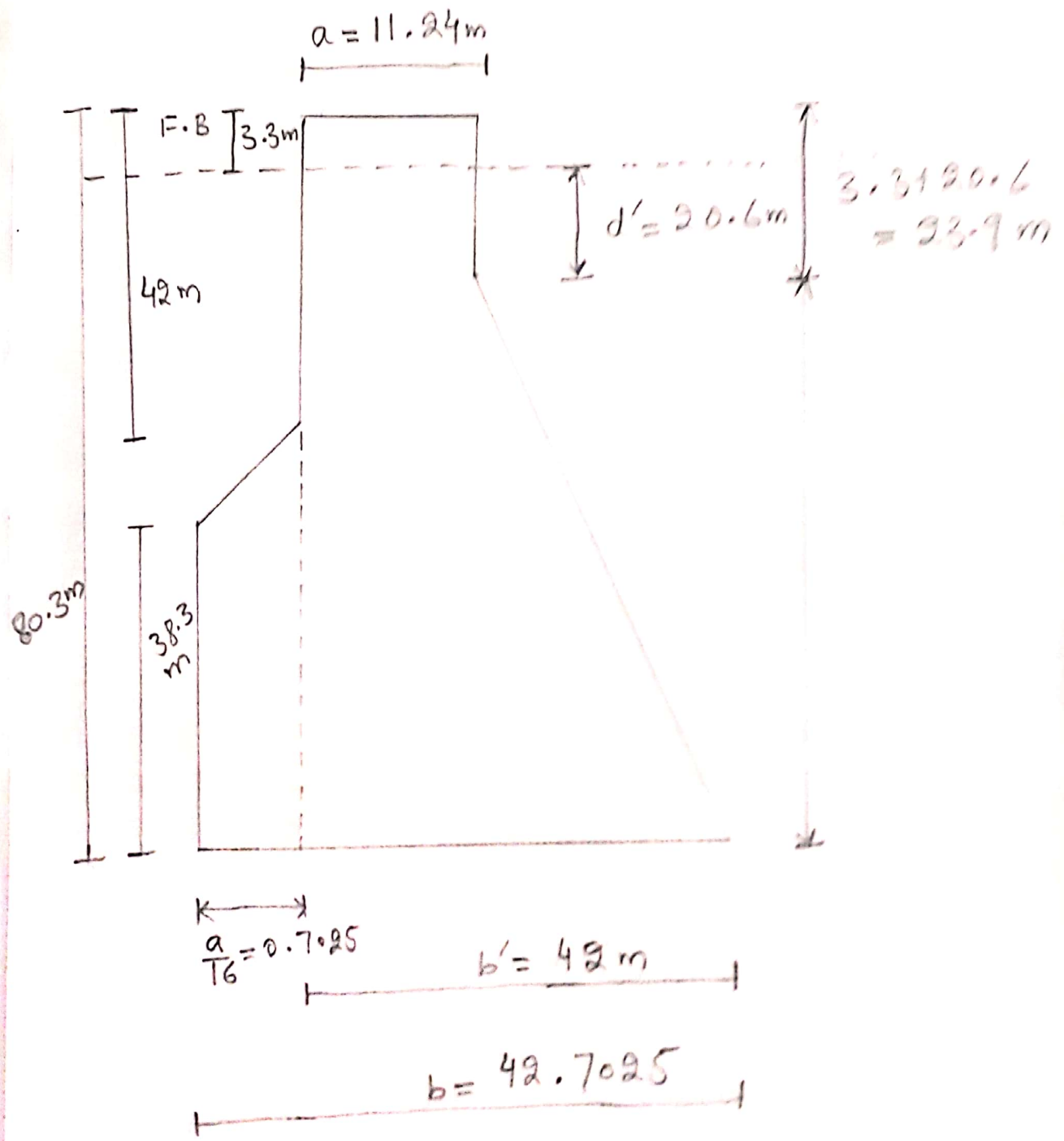
depth of vertical position (1)

$$d = d' + FB$$

$$d = 20.6 + 3.3$$

$$d = 23.9 \text{ m}$$

$$d = 24 \text{ m}$$





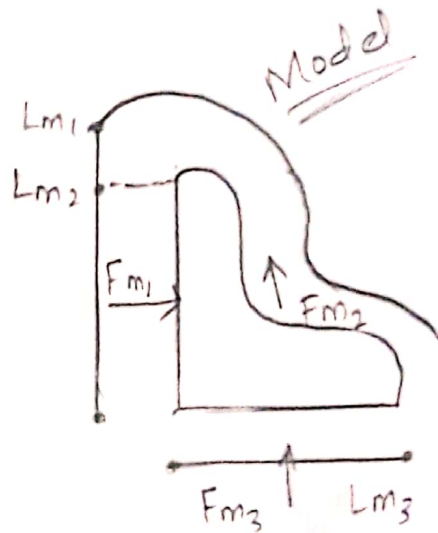
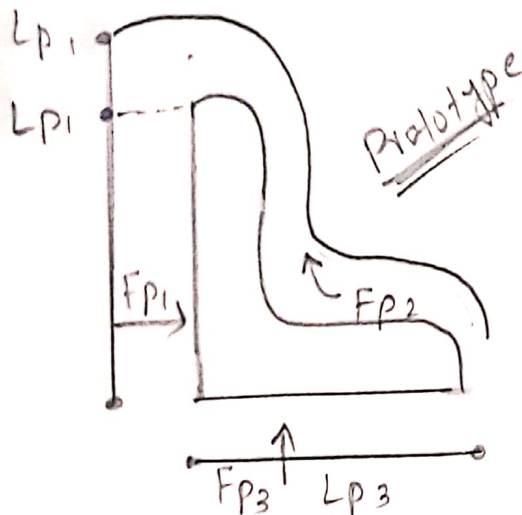
QNO. 3):

Ans:

Model Analysis:

Model: is a small scale replica of the actual structure.

Prototype: The actual structure or machine.



Model Analysis: is actually an experimental method of finding solutions of complex flow problems.

The following are the advantages of model analysis.

→ Using dimensional analysis, a relationship between the variable influencing a flow problem is obtained which help in conducting test.

→ The performance of the hydraulic structure can be predicted in advance from its model.

→ The merits of alternative design can

be predicted in advance from its model analysis to adopt most economical and safe design.

Similitude: is defined as similarity between the model and prototype in every respect, which mean model and prototype have similar properties completely similar.

Types:

- Geometric similarity
- Kinematic similarity
- Dynamic similarity.

Geometric Similarity: The linear dimension in model and prototype are equal.

$$\frac{L_p}{L_m} = \frac{B_p}{B_m} = \frac{D_p}{D_m} = L_s$$

↓                      ↓                      ↓                      ↓  
 Length              Breadth              Diameter              Scale ratio

Kinematic Similarity: similarity of motion.

$$\frac{V_{p1}}{V_{m1}} = \frac{V_{p2}}{V_{m2}} = V_s ; \frac{a_{p1}}{a_{m1}} = \frac{a_{p2}}{a_{m2}} = a_s$$

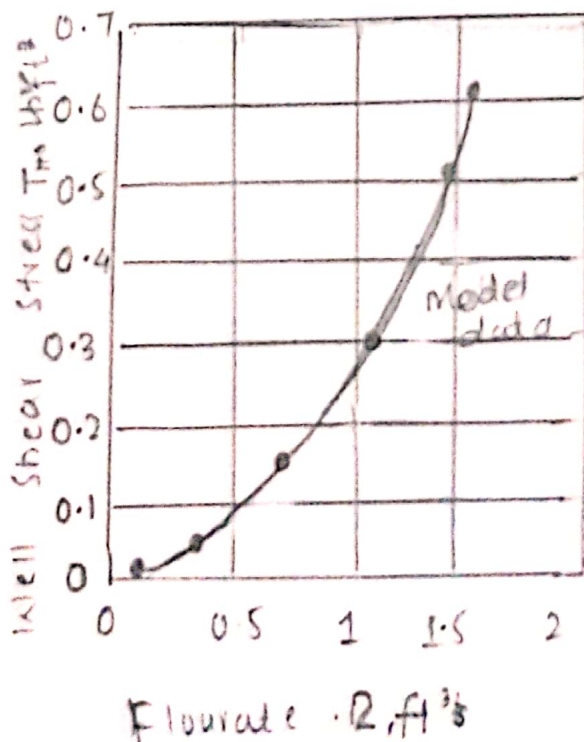
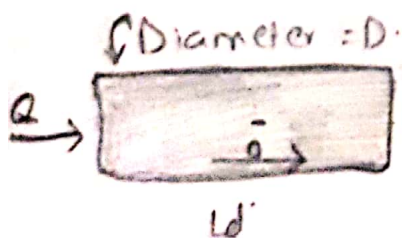
↓                      ↓                      ↓                      ↓  
 velocity              velocity ratio              acceleration ratio              acceleration ratio

Dynamic similarity: similarity of force.

$$\frac{(F_i)_p}{(F_i)_m} = \frac{(F_v)_p}{(F_v)_m} = \frac{(F_g)_p}{(F_g)_m} = F_r$$

Inertia Force
Viscous Force
Gravitational Force
Force Ratio

Example: Assume that wall shear stress  $\tau_w$  created when a fluid flows through a pipe depends on the pipe diameter,  $D$ , flow rate  $Q$ , fluid density  $\rho$  and kinematic velocity  $v$ . Some model test run in laboratory using water in a 0.2-ft diameter pipe yield the  $\tau_w$  vs  $Q$  data shown in figure. Shear stress is 0.3 ft diameter pipe through which water flow at the rate of 1.5 ft<sup>3</sup>/s.



# Dimensional Analysis:

ID# 7794

$$\tau_w = f(D, Q, P, \nu) \quad \text{repeating variables}$$

$$\tau_w = F L^{-2} \quad D = L \quad Q = L^3 T^{-1}$$

$$P = F L^{-1} T^2 \quad \nu = L^2 T^{-1}$$

From pi theorem,  $5 - 3 = 2$  pi terms required and dimension analysis yields

$$\frac{\tau_w D^4}{P Q^3} = f\left(\frac{Q}{D \nu}\right)$$

Model Analysis:

The similarity requirement is

$$\frac{Q_m}{D_m \nu_m} = \frac{Q}{D \nu}$$

So that with  $\nu_m = \nu$  and

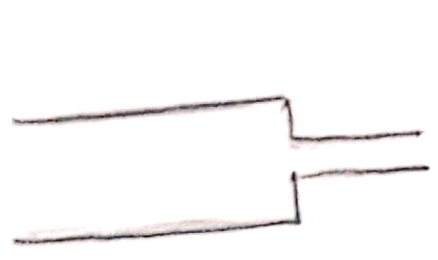
$$Q = 1.5 \text{ ft}^{3/s}$$

$$Q_m \left(\frac{D_m}{D}\right) \left(\frac{\nu_m}{\nu}\right) = \left(\frac{0.9 \text{ ft}}{0.3 \text{ ft}}\right) (1) \left(\frac{1.0 \text{ ft}^2/s}{1.0 \text{ ft}^2/s}\right)$$

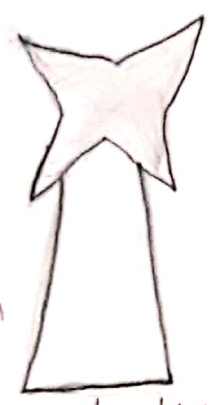
From graph  $Q_m = 1.007 \text{ ft}^3/s$ ,  $\tau_m = 0.29 \text{ lb/ft}^2$

$$\text{Thus } \frac{\tau_m D_m^4}{P_m Q_m^3} = \frac{\tau_w D^4}{P Q^3}$$

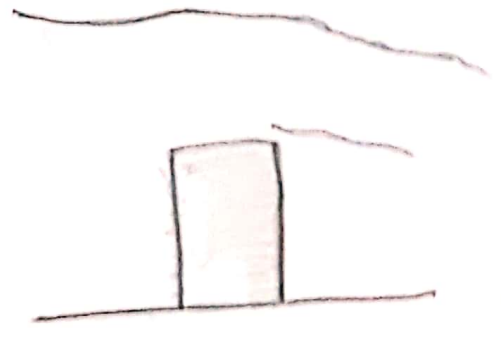
$$\begin{aligned} \text{and } \tau_w &= \left(\frac{P}{P_m}\right) \left(\frac{Q}{Q_m}\right)^3 \left(\frac{D_m}{D}\right)^4 \tau_m \\ &= (1) \left(\frac{1.5}{1.0}\right)^3 \left(\frac{0.9}{0.3}\right)^4 (0.29) \\ &= 0.129 \text{ lb/ft}^2 \end{aligned}$$



Sudden contraction in pipe



Wind turbine



Dam / spillway

- Dimensional Analysis to predict the physical parameters that will significantly influence phenomenon under study.
- Similitude and model analysis to investigate the complexity of phenomenon in details.
- Application of knowledge on actual/prototype model.

### Methodology of Dimensional Analysis:

The basic principle is Dimensional homogeneity, which means the dimensions of each terms is an equation on both sides are equal.

So such an equation, in which dimensions of each term on both

ID # 7794

SECTION A (13)

sides are equal, called dimensionally homogeneous equation.

$$V = (2gH)^{1/2}$$

$$V = LT^{-1}$$

$$(2gH)^{1/2} = LT^{-1}$$

$$L \cdot H \cdot S = R \cdot H \cdot S$$

so its dimensionally homogeneous

QNO.4):

Ans:

FALL VELOCITY:

When a grain falls down in still water it obtains a constant velocity when the upward fluid drag force on the grain is equal to the downward submerged weight of the grain.

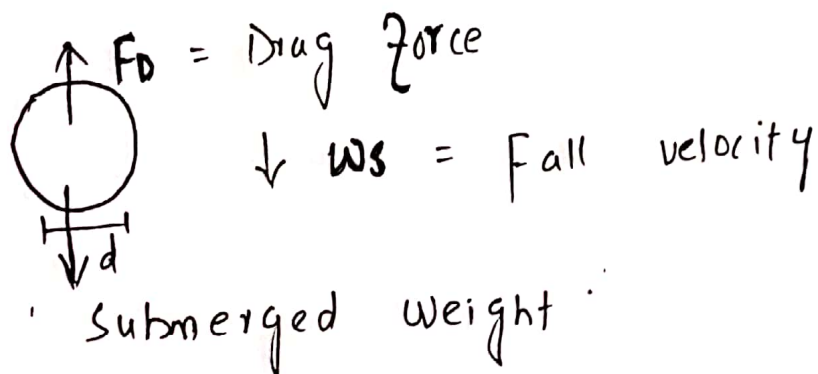
This constant velocity is defined as the fall velocity of the grain.

This is also called settling velocity.

DEPENDENCE:

Fall velocity depends on;

- (1) Particle diameter
- (2) Particle density.
- (3) Particle concentration.
- (4) Particle shape
- (5) viscosity of water (temperature)
- (6) Turbulence.



The force balance b/w the drag force and the submerged weight gives

$$F_D = \text{Submerged weight.}$$

(1) PARTICLE DIAMETER:

The diameter of the particle is directly proportional to the fall velocity because greater the size of particle so it will tend to move faster as compared to the particle of small size thus their gravitational force on particle of greater size so it will fall quickly due to its weight.

(2) PARTICLE DENSITY:

Density of the particle is directly proportional to the rate of fall velocity since particle with high density tends to settle down early compared with the particle of low density.



ID # 7794

(6) TURBULANCE OF WATER:

water effect the fall velocity of water in reservoir because the non-~~linearity~~ linearity and zig zag path effect the flow of water and cause the variation in the flow.