Final Term Examination

Course Name: Multivariate Calculus

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BS (SE-8) Section: A

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Final Term Examination

Subject Name: Multivariate Calculus
Date: 24 Sep 2020
Instructor: Shakeel

Note: Attempt all Questions

1. Evaluate

$$\int_0^5 \int_0^x x \ (x + 3x) dy dx$$

2. Evaluate

$$\int_1^4 \int_0^3 (xy + x^3y^3)dydx$$

3. Find partial derivatives w.r.t r and s

$f\,(r,s) = r.\ln(r^3 + s^2)$

4. Finding partial derivatives w.r.t "x"
$F(x,y,z) = xy^2z^4 + 3yz^2$

5. Find the value of x and y

$8x – y = -1, \quad 7x – y = -2$

Q1

Answer:

① $$\int_0^5 \int_0^x x\cdot(x+3x)\,dy\,dx.$$

Sol: $$\int_0^5 \left[\int_0^x x\cdot(x+3x)\,dy\right]dx.$$

$$= \int_0^5 \left[\int_0^x (x^2+3x^2)\,dy\right] \cancel{dy}\,dx$$

$$= \int_0^5 \left[\int_0^x x^2\,dy + 3\int_0^x x^2\,dy\right] \cancel{dy}\,dx$$

$$= \int_0^5 \left[x^2y\Big|_0^x + 3\left\{x^2y\Big|_0^x\right.\right] \cancel{dy}\,dx.$$

$$= \int_0^5 \left[x^2(x) - x^2(0) + 3\left(x^2(x) - x^2(\right.\right.$$

$$= \int_0^5 \left[x^3 - 0 + 3x^3 - 0\right] dx.$$

$$= \int_0^5 (x^3+3x^3)\,dx.$$

$$= \int_0^5 x^3\,dx + 3\int_0^5 x^3\,dx$$

$$= \left.\frac{x^4}{4}\right|_0^5 + 3\left.\frac{x^4}{4}\right|_0^5$$

$$= \frac{1}{4}\left[(5)^4 - (0)^4\right] + \frac{3}{4}\left[(5)^4 - (0)^4\right]$$

$$= \frac{1}{4}(625-0) + \frac{3}{4}(625-0).$$

$$= \frac{625}{4} + \frac{1875}{4}$$

$$= \frac{625+1875}{4}$$

$$= \frac{2500}{4}$$

$$= 625 \text{ An}$$

Q2

Answer:

Q No 2: Evaluate.

$$\int_1^4\int_0^3 (xy+x^3y^3)\,dy\,dx$$

Solution:

$$\int_1^4\left[\int_0^3 (xy+x^3y^3)\,dx\,dy\right.$$

$$=\int_1^4\left[\int_0^3 xy\,dx+\int_0^3 x^3y^3\,dx\right]dy$$

$$=\int_1^4\left[\frac{x^2}{2}y\Big|_0^3+\frac{x^4}{4}y^3\Big|_0^3\right]dy$$

$$=\int_1^4\left[\frac{(3)^2}{2}y-\frac{(0)^2}{2}y+\frac{(3)^4}{4}y^3-\frac{(0)^3}{4}y^3\right]dy$$

$$=\int_1^4\left[\frac{9}{2}y-0+\frac{81}{4}y^3-0\right]dy$$

$$=\int_1^4\left[\frac{9}{2}y+\frac{81}{4}y^3\right]dy$$

$$=\frac{9}{2}\int_1^4 y\,dy+\frac{81}{4}\int_1^4 y^3\,dy$$

$$= \frac{9}{2} \left. \frac{y^2}{2} \right|_1^4 + \frac{81}{4} \cdot \left. \frac{y^4}{4} \right|_1^4$$

$$= \frac{9}{2\times2} \left[(4)^2 - (4)^1 \right] + \frac{81}{4\times4} \left[(4)^4 - (4)^1 \right]$$

$$= \frac{9}{4} \left[(16) - 4 \right] + \frac{81}{16} \left[256 - 4 \right]$$

$$= \frac{9}{\cancel{4}} (\cancel{12}) + \frac{81}{16} (252)$$

$$= 27 + \frac{20412}{16}$$

$$= \frac{20844}{16}$$

Q3

Answer:

QNo3: find partial derivatives w.r.t r and s.

$$f(r,s) = r.\ln(r^3+s^2)$$

Sol:

$$f(r,s) = r.\ln(r^3+s^2)$$

differente b.s w.r.t "r"

$$\frac{d}{dr} f(r,s) = \frac{d}{dr}\left(r.\ln(r^3+s^2)\right)$$

$$= r \frac{d}{dr} \ln(r^3+s^2)$$

$$+ \ln(r^3+s^2) \frac{dr}{dr}$$

$$= \frac{r(3r^2)}{r^3+s^2} + \ln(r^3+s^2)$$

$$= \frac{3r^3}{r^3+s^2} + \ln(r^3+s^2)$$

$$= \boxed{\frac{r}{r^3+s^2}(2s)} \text{ Ans}$$

Q4

Answer:

Q No 4: find Partial derivatives w.r.t "x" $f(x,y,z) = xy^2z^4 + 3yz^2$

Solution::

$$f(x,y,z) = xy^2z^4 + 3yz^2$$

taking derivatives on b/s w.r.t "x"

$$d/dx\, f(x,y,z) = \frac{d}{dx}(xy^2z^2 + 3yz^2)$$

$$= y^2z^4 + 0$$

$\boxed{= y^2z^4}$ Ans.

Q5

Answer:

Q No. 5: find the value of x & y.

$8x - y = -1, \quad 7x - y = -2.$

Solution:

$8x - y = -1 \longrightarrow (1)$

$7x - y = -2 \longrightarrow (2)$

Subtracting equation (1) & (2)

$$8x - y = -1$$

$$\underset{-}{+} 7x \underset{+}{-} y = \underset{+}{-} 2$$

$$x = 1$$

So, $\boxed{x = 1}$

$\therefore$ Putting $x = 1$ in equation (1)

$8(1) - y = -1$

$8 - y = -1$

$-y = -1 - 8$