Q1)

Ans)

$$\frac{dy}{dt} = e^{y-t} \sec(y)\,(1+t^2)$$

$y(0) = 0$ So $x = 0$, $y = 0$

$$dy = e^{y} \cdot e^{-t} \sec(y)\,(1+t^2)\,dt$$

$$\frac{1}{e^{y} \cdot \sec(y)}\,dy = (1+t^2)\,e^{-t}\,dt$$

As $\cos(y) = \frac{1}{\sec(y)}$

$$\int e^{-y} \cos y\,dy = \int (1+t^2)\,e^{-t}\,dt$$

using integration by parts

$$e^{-y} \int \cos y\,dx - \int \left(\int \cos y \cdot \frac{d}{dy} e^{-y} \right) = (1+t^2) \int e^{-t}$$

$$- \int \left(\int e^{-t} \cdot d/dt\,(1+t^2) \right) \text{ --- (i)}$$

$\Rightarrow$ L.H.S

$$e^{-y} \int \cos y\,dx - \int \left(\int \cos y \cdot \frac{d}{dy} e^{-y} \right)$$

$$e^{-y} \sin y - \int \left(\sin y \cdot e^{-y} (-1) \right)$$

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$e^{-y}\sin y + \int (\sin y \cdot e^{-y})$

$e^{-y}\sin y + \int (e^{-y}\sin y)$

Again using integration by parts

$e^{-y}\sin y + e^{-y}(-\cos y) - \int (\int \sin y \, d/dy \, e^{-y})$

$e^{-y}\sin y + e^{-y}(-\cos y) - \int (-\cos y \frac{e^{-y}}{-1})$

$e^{-y}\sin y - e^{-y}\cos y - \int (\cos y \, e^{-y})$

since $\int (\cos y \cdot e^{-y}) = L.H.S$

Since it is again same to the first one so L.H.S will become

$L.H.S = e^{-y}(\sin y - \cos y) - (L.H.S)$

$2LHS = e^{-y}(\sin y - \cos y)$

$L.H.S = \dfrac{e^{-y}(\sin y - \cos y)}{2}$

Now taking R.H.S

$$\int (1+t^2)\, e^{-t}\, dt$$

$$(1+t^2)\int e^{-t} - \int \left(\int e^{-t}\, d/dt\,(1+t^2)\right)$$

$$(1+t^2)\, e^{-t} - \int \left(-e^{-t}\,(2t)\right)$$

$$-(1+t^2)\, e^{-t} + \int (2t)\, e^{-t}$$

Again using integration by parts

$$-(1+t^2)\, e^{-t} + \left(2t \int e^{-t} - \int \left(\int e^{-t}\, d/dt\, 2t\right)\right)$$

$$-(1+t^2)\, e^{-t} + \left(-2t e^{-t} - \int (-e^{-t}\, 2)\right)$$

$$-(1+t^2)\, e^{-t} + \left(-2t e^{-t} + \int (2 e^{-t})\right)$$

$$-(1+t^2)\, e^{-t} + \left(-2t e^{-t} - 2e^{-t}\right) + C$$

$$-(1+t^2)\, e^{-t} - 2t e^{-t} - 2e^{-t} + C$$

$$-e^{-t} - e^{-t} t^2 - 2t e^{-t} - 2e^{-t} + C$$

$$-(t^2 + 2t + 3)\, e^{-t} + C = \text{R.H.S}$$

$$\frac{e^{y}(\sin y - \cos y)}{2} = -(t^2 + 2t + 3)\, e^{-t} + C$$

we know that

$t = 0, \quad y = 0$

Put it above

$$\frac{1}{2}(0-1) = -3 + c$$

$$c = 5/2$$

Put value of C

$$\boxed{\frac{e^{-y}}{2}(\sin y - \cos y) = -(u^2 + 2t + 3)e^{-t} + 5/2}$$

Q2)

Ans

$$\frac{dy}{du} = \frac{\sqrt{u+y} + \sqrt{u-y}}{\sqrt{u+y} - \sqrt{u-y}} \quad \text{—(i)}$$

This is homogenous differential eq in u and y to solve this put

$$y = vu$$

$$\Rightarrow \frac{dy}{du} = v + u\frac{dv}{du}$$

Thus eq (i) becomes

$$v + u\frac{dv}{du} = \frac{\sqrt{u+vu} + \sqrt{u-vu}}{\sqrt{u+vu} - \sqrt{u-vu}}$$

$$v + u\frac{dv}{du} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}}$$

$$v + u\frac{dv}{du} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}} \times \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} + \sqrt{1-v}}$$

$$v + u\frac{dv}{du} = \frac{1 + \cancel{v} + 1 - \cancel{v} + 2\sqrt{1-v^2}}{2v}$$

$$v + u\frac{dv}{du} = \frac{\cancel{2}\left(1 + \sqrt{1-v^2}\right)}{\cancel{2}v}$$

$$v + u\frac{dv}{du} = \frac{1 + \sqrt{1-v^2}}{v}$$

$$u\frac{dv}{du} = \frac{1+\sqrt{1-v^2}}{v} - v$$

$$u\frac{dv}{du} = \frac{1+\sqrt{1-v^2} - v^2}{v}$$

$$u\frac{dv}{du} = \frac{\sqrt{1-v^2}\ 1+(\sqrt{1+v^2})}{v}$$

$$u\frac{dv}{du} = \frac{\sqrt{1-v^2}\ (1+\sqrt{1-v^2})}{v}$$

$$\frac{v\,dv}{\sqrt{1-v^2}\,(1+\sqrt{1-v^2}} = \frac{du}{u}$$

Taking integral on b.s

$$\int \frac{v\,dv}{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})} = \int \frac{du}{u}$$

Put $1+\sqrt{1-v^2} = t$

$$\Rightarrow \tfrac{1}{2}(1-v^2)^{-1/2}(-2v)\,dv = dt$$

$$\frac{v\,dv}{\sqrt{1-v^2}} = -dt$$

$$\int -\frac{dt}{t} = \int \frac{du}{u}$$

$$-\ln t = \ln x + \ln c$$

$$-\ln\left(1+\sqrt{1-v^2}\right) = \ln cx$$

$$\ln\left(1+\sqrt{1-v^2}\right) = -\ln cx$$

$$\cancel{\ln}\left(1+\sqrt{1-v^2}\right) = \cancel{\ln}(cx)^{-1}$$

$$\left(1+\sqrt{1-v^2}\right) = 1/cx$$

$$1+\sqrt{1-\frac{y^2}{x^2}} = \frac{1}{cx}$$

$$1+\frac{\sqrt{x^2-y^2}}{x^2} = \frac{1}{cx}$$

$$x+\sqrt{x^2-y^2} = \frac{1}{c}$$

$$x+\sqrt{x^2-y^2} = c_1 \qquad \because 1/c = c_1$$

which is a required solution.

Q3)

Ans)

Sol:-

$$(D^4 + D^2) y = 3x^2 + 4\sin x - 2\cos x$$

$$\Rightarrow f(D) y = f(x)$$

As it is non-homogenous linear equation so solution will be

$$y = y_c + y_p \quad \text{---(i)}$$

complementary solution y_c

$$D^4 + D^2 = 0 \Rightarrow D^2 (D^2+1) = 0$$

$$\text{Either } D^2 = 0 \Rightarrow \boxed{D = 0}$$

$$D^2 + 1 = 0 \Rightarrow D^2 = -1$$

$$D = \sqrt{-1} \Rightarrow D = i \text{ or } D = \boxed{0 + i}$$

Roots are real complex

$$y_c = C_1 e^{0x} + e^{0x} (C_2 \cos x + C_3 \sin x)$$

$$y_c = C_1 + C_2 \cos x + C_3 \sin x$$

$$y_p = \frac{1}{f(D)} F(x)$$

$$y_p = \frac{1}{D^4+D^2}\left(3x^2 + 4\sin x - 2\cos x\right)$$

$$= \frac{3x^2}{D^4+D^2} + \frac{4\sin x}{D^4+D^2} - \frac{2\cos x}{D^4+D^2}$$

$f(D) = D^4 + D^2$

at $D=0 \rightarrow f(D) = 0$

So $f'(D) = 4D^3 + 2D$

Now also for $D=0 \Rightarrow f'(D) = 0$

again differentiating.

$f''(D) = 12D + 2$

So for $D=0$

$f''(0) = 12(0) + 2 = 2$

So replacing $\frac{1}{f(D)}$ with $\frac{x^2}{f''(D)}$

$$\Rightarrow y_p = \frac{x^2 \, 3x^2}{12D+2} + \frac{x^2}{12D+2} \cdot 4\sin x - \frac{x^2}{12D+2} \, 2\cos x$$

Putting D = 0 in all

So,

$$y_p = \frac{x^2 \cdot 3x^2}{12(0)+2} + \frac{x^2 \cdot 4\sin x}{12(0)+2} - \frac{2x^2\cos x}{12(0)+2}$$

$$y_p = \frac{3x^4}{2} + \frac{4x^2 \sin x}{2} - \frac{2x^2\cos x}{2}$$

$$= \frac{3x^4}{2} + 2x^2 \sin x - x^2 \cos x$$

So

Putting in eq (i)

$$y = C_1 + C_2 \cos x + C_3 \cos x + \frac{3}{2}x^4 + 2x^2 \sin x - x^2 \cos x$$

$$\boxed{y = C_1 + (C_2 - x^2)\cos x + (C_3 + 2x^2)\sin x + \frac{3}{2}x^4}$$