ID:- 6575 | Subject:- Linear Algebra

Name:- Hadeed-Ul-Haq | Major Assignment:-

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Q1 Compute Adjoint of

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} \quad \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}$$

Cofactor of $A_{11} = (-1)^{1+1} M_{11}$

$= (-1)^2 \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}$

$= 1 [6-1]$

$= 1(5)$

$= 5$

Cofactor of $A_{12} = (-1)^{1+2} M_{12}$

$= (-1)^3 \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}$

$= (-1)[4-3]$

$= -1(1)$

$= -1$

Cofactor of $A_{13} = \cancel{(-1)^{1+3}}\ (-1)^{1+3} M_{13}$

$= (-1)^4 \begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix}$

$= 1[2-9]$

$= 1(-7)$

$= -7$

Cofactor of $A_{21} = (-1)^{2+1} M_{21}$

$= -1 |4-5|$

$= -1(-1)$

$= -1$

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Cofactor of $A_{22} = (-1)^{2+2} M_{2}$

$= (-1)^4 \begin{vmatrix} 1 & 5 \\ 3 & 2 \end{vmatrix}$

$= 1(2-15)$

$= 1(-13)$

$= -13$

Cofactor of $A_{23} = (-1)^{2+3} M_{23}$

$= (-1)^5 \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}$

$= (-1)(1-6)$

$= -1(-5)$

$= 5$

Cofactor of $A_{31} = (-1)^{3+1} M_{31}$

$= (-1)^4 \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix}$

$= 1(2-15)$

$= 1(-13)$

$= -13$

Cofactor of $A_{32} = (-1)^{3+2} M_{32}$

$= (-1)^5 \begin{bmatrix} 1 & 5 \\ 2 & 1 \end{bmatrix}$

$= (-1)\left[1 - 10\right]$

$= (-1)(-9)$

$= 9$

Cofactor of $A_{33} = (-1)^{3+3} M_{33}$

$(-1)^6 \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix}$

$= 1[3-4]$

$= 1(-1)$

$= -1$

Adjoint of A =

$$\begin{bmatrix} 5 & -1 & -7 \\ 1 & -13 & 5 \\ -13 & 9 & -1 \end{bmatrix}$$

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b $= \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 8 \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix}$

Cofactor of $B_{11} = (-1)^{1+1} M_{11}$

$$= (-1)^2 \begin{vmatrix} -1 & 8 \\ -2 & 8 \end{vmatrix}$$

$$= 1[-8-(-16)]$$

$$= 1[-8+16]$$

$$= 1[8]$$

$$= 8$$

Cofactor of $B_{12} = (-1)^{1+2} M_{12}$

$$= (-1)^3 \begin{vmatrix} 2 & 8 \\ 5 & 8 \end{vmatrix}$$

$$= (-1)[16-40]$$

$$= -1(-24)$$

$$= 24$$

Cofactor of $B_{13} = (-1)^{1+3} M_{13}$

$$= (-1)^4 \begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix}$$

$$= 1[-4-(-5)]$$

$$= 1[-4+5]$$

$$= 1[1)$$

$$= 1$$

Cofactor of $B_{21} = (-1)^{2+1} M_{21}$

$$= (-1)^3 \begin{vmatrix} 4 & 5 \\ -2 & 8 \end{vmatrix}$$

$$= -1[32-(-10)]$$

$$= -1[32+10)$$

$$= -1(42) = -42$$

Cofactor of $B_{22} = (-1)^{2+2} M_{22}$

$$= (-1)^4 \begin{vmatrix} 3 & 5 \\ 5 & 8 \end{vmatrix}$$

$$= (+1)(24-25)$$

$$= 1(-1)$$

$$= -1$$

Cofactor of $B_{23} = (-1)^{2+3} M_{23}$

$$= (-1)^5 \begin{vmatrix} 3 & 4 \\ 5 & -2 \end{vmatrix}$$

$$= (-1)(-6-20)$$

$$= -1(-26)$$

$$= 26$$

Cofactor of $B_{31} = (-1)^{3+1} M_{31}$

$$= (-1)^4 \begin{vmatrix} 4 & 5 \\ -1 & 8 \end{vmatrix}$$

$$= 1[32-(-5)]$$

$$= 1(32+5)$$

$$= 1(37)$$

$$= 37$$

Cofactor of $B_{32} = (-1)^{3+2} M_{32}$

$$= (-1)^5 \begin{vmatrix} 3 & 5 \\ 2 & 8 \end{vmatrix}$$

$$= -1[24-10]$$

$$= -1[14]$$

$$= -14$$

$$\boxed{B = \begin{bmatrix} 8 & 24 & 1 \\ -42 & -1 & 26 \\ 37 & -14 & -11 \end{bmatrix}}$$

Cofactor of $B_{33} = (-1)^{3+3} M_{33}$

$$= (-1)^6 \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix}$$

$$= +1(-3-8)$$

$$= 1(-11) \quad = -11$$

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Q2// A_{21}, A_{31}, A_{33} if :-

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$$

Cofactor of $A_{21} = (-1)^{2+1} M_{21}$

$$= (-1)^3 \begin{vmatrix} -2 & 3 \\ -3 & 2 \end{vmatrix}$$

$$= -1[-4-(-9)]$$

$$= -1[-4+9]$$

$$= -1(5)$$

$$= -5$$

Cofactor of $A31 = (-1)^{3+1} M_{31}$

$$(-1)^4 \begin{vmatrix} -2 & 3 \\ 3 & 1 \end{vmatrix}$$

$$= 1[-2-9]$$

$$= 1[-11]$$

$$= -11$$

Cofactor of $A_{33} = (-1)^{3+3} M_{33}$

$$= (-1)^6 \begin{vmatrix} 1 & -2 \\ -2 & 3 \end{vmatrix}$$

$$= 1 \; |3-4|$$

$$= 1(-1)$$

$$= -1$$

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Q3 Find Eigen Values and Eigen Vectors if,

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 1 = $(A - \lambda I)X = 0$

Step 2 = $(A - \lambda I) = 0$

Step 3 = $\lambda^3 - \begin{bmatrix} \text{Sum of diagonal Elements} \end{bmatrix} \lambda^2 + \begin{bmatrix} \text{Sum of diagonal minors} \end{bmatrix} \lambda - |A| = 0$

Step 1:-

$$(A - \lambda I)X = 0$$

SO;
$$\left(\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Step: 2

$$[A - \lambda I] = 0$$

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$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix}, \quad \lambda I \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & 1-0 & 1-0 \\ 1-0 & 3-\lambda & 2-0 \\ -1-0 & 1-0 & 2-\lambda \end{vmatrix} = 0$$

$$\begin{vmatrix} 2-\lambda & 1 & 1 \\ 1 & 3-\lambda & 2 \\ -1 & 1 & 2-\lambda \end{vmatrix} = 0$$

Step: 3

$$\lambda^3 - \left[\begin{matrix}\text{Sum of} \\ \text{diagonal}\end{matrix}\right]\lambda^2 + \left[\begin{matrix}\text{Sum of} \\ \text{minors}\end{matrix}\right]\lambda - |A| = 0$$

$$\lambda^3 - 7\lambda^2 + 14\lambda - 8 = 0$$

$$\text{Sum of diagonal} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix}$$

$$= 2 + 3 + 2$$

$$= 7$$

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Sum of minors $= \begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix}.$

$= [6-2] + [4-(-1)] + [6-1]$

$= [4] + (4+1) + (5)$

$= (4) + (5) + (5)$

$= 14$

$|A| = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix}$

$= 2\begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix} + 1\begin{vmatrix} 1 & 3 \\ -1 & 1 \end{vmatrix}$

$= 2[6-2] - 1[2+2] + 1[1+3]$

$= 2(4) - 1(4) + 1(4)$

$= 8 - 4 + 4$

$= 8$

Equation becomes;

$\lambda^3 - 7\lambda^2 + 14\lambda - 8 = 0$

$\lambda^3 - 8 - 7\lambda^2 + 14\lambda = 0$

$(\lambda^3 - 2^3) - 7\lambda(\lambda - 2) = 0$

$(\lambda - 2)(\lambda^2 + 2\lambda + 4) - 7\lambda(\lambda - 2) = 0$

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Taking $\lambda-2$ as common;

$$(\lambda-2)(\lambda^2+2\lambda+4-7\lambda]=0$$
$$(\lambda-2)[\lambda^2+4-5\lambda]=0$$

$\lambda-2=0$
$\lambda=2$

$\lambda^2+4-5\lambda=0$
$\lambda^2-4\lambda-\lambda+4=0$
$\lambda(\lambda-4)-1(\lambda-4)=0$
$(\lambda-4)(\lambda-1)=0$
$\lambda=4, \lambda=1$

The Values of Eigen are;

$\lambda = 1, 2, 4$

Eigen Vectors;

When $\lambda=1$

$$\begin{bmatrix} 2-\lambda & 1 & 1 \\ 1 & 3-\lambda & 2 \\ -1 & 1 & 2-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ -1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

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1) Finding Eigen by Grammer's Rule:

$$x_1 + x_2 + x_3 = 0 \quad \text{--- (i)}$$
$$1x_1 + 2x_2 + 2x_3 = 0 \quad \text{--- (ii)}$$

$$\frac{x_1}{|2-2|} = \frac{-x_2}{|2-1|} = \frac{x_3}{|2-1|}$$

$$\frac{x_1}{|0|} = \frac{-x_2}{|1|} = \frac{x_3}{|1|}$$

$$\boxed{X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}}$$ Eigen Vector 1

for $\lambda = 2$

$$\begin{bmatrix} 2-2 & 1 & 1 \\ 1 & 3-2 & 2 \\ -1 & 1 & 2-2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 2 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$0 + x_2 + x_3 = 0$$
$$x_1 + x_2 + 2x_3 = 0$$

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$$\frac{x_1}{1} = +\frac{x_2}{+1} = \frac{x_3}{-1}$$

$$\boxed{x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ +1 \\ -1 \end{bmatrix}}$$ Eigen vector 2

for $\lambda = 4$

$$\begin{bmatrix} 2-4 & 1 & 1 \\ 1 & 3-4 & 2 \\ -1 & 1 & 2-4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 1 & 1 \\ 1 & -1 & 2 \\ -1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Grammer's Rule

$$-2x_1 + 1x_2 + 1x_3 = 0$$
$$1x_1 - 1x_2 + 2x_3 = 0$$

$$\frac{x_1}{\begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix}} = -\frac{x_2}{\begin{vmatrix} -2 & 1 \\ 1 & 2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -2 & 1 \\ 1 & -1 \end{vmatrix}}$$

$$\frac{x_1}{|2-1|} = -\frac{x_2}{|-4-1|} = \frac{-x_3}{|-2-1|}$$

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$$\frac{x_1}{1} = \frac{+x_2}{+5} = \frac{x_3}{1}$$

$$\boxed{x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{vmatrix} 1 \\ 5 \\ 1 \end{vmatrix}}$$

Eigen Vector 3