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Q: A man throws two fair dice. What is the conditional probability that the sum of the two dice will be 7 given that
(1) the sum is even
(2) the sum is greater than 8
(3) The two dice had the same outcome

Ans:
The sample space s for this experiment is.

S= {(1,1) (1,2), (1,3) (1,4), (1,5), (1,6) (2,1) (2,2) (2,3) (2,4)
(2,5), (2,6), (3,1), (3,2), (3,3) (3,4) (3,5), (3,6) (4,1)
(4,2), (4,3), (4,4), (4,5) (4,6), (5,1), (5,2), (5,3) (5,4)
(5,5), (5,6), (6,1), (6,2) (6,3) (6,4), (6,5), (6,6)}

let
A= {the sum is 7}
B= {the sum is odd}
C= {the sum is greater than 8}
and D = {the two dice had the same outcomes}

A= {(1,6), (2,5), (3,4) (4,3) (5,2), (6,1)
B= {(1,2), (1,4) (1,6) (2,1) (2,3) (2,5) (3,2) (3,4) ... (6,5)
C= {(1,6), (2,5) (2,6), (3,4) (3,5), (3,6), (4,3) (4,4) ... (6,6)
D= {(1,1) (2,2), (3,3) (4,4) (5,5) (6,6)}.
A∩B= {(1,6), (2,5), (3,4) (4,3), (5,2) (6,1)}.
A∩C= {(1,6), (2,5), (3,4) (4,3), (5,2), (6,1)}
A∩D= ∅

$P(A) = \frac{6}{36}$, $P(B) = \frac{18}{36}$, $P(C) = \frac{21}{36}$

$P(D) = \frac{6}{36}$

$P(A \cap B) = \frac{6}{36}$, $P(A \cap C) = \frac{6}{36}$ and

$P(A \cap D) = 0$

Hence,

$P(A|B) = P(A \cap B) = \frac{6}{36} \times \frac{36}{18} = \frac{1}{3}$

$P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{6}{36} \times \frac{36}{21} = \frac{2}{7}$

$P(A|D) = \frac{P(A \cap D)}{P(D)} = 0 \times \frac{36}{6} = 0$

Q2 Sum of 2 has 1 way 1,1
Sum of 3 has 2 ways 1,2 and 2,1
Sum of 4 has 3 ways 1,3: 2,2: 3,1.
5 has 4 ways
6 has 5 ways
8 has 5 ways (symmetry)
9 has 4 ways
10 has 3 ways
11 has 2 ways
12 has 1 way
Those are 15/36 for each side
With a sum of 3 0/36
that leaves a 6/36 = 1/6 Probability
for a sum of 7.

Q3 A and B play a game in which A's probability of winning 2/3. In a series of 8 games what is the probability that A will win

1. Exactly 4 games
2. At least 4 games
3. From 3 to 6 games.

Sol: Given that $p = \frac{2}{3}$ $n = 8$

$q = 1 - p$

$= 1 - \frac{2}{3}$

$q = \frac{1}{3}$

Let "x" denotes the number of games won by A, then

i $P(X=4)$

$= \binom{8}{4} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^4$

$= \frac{1120}{6561}$

$= 0.1707$

(ii) $P(x \geq 4)$

$1 - P(X<4)$

$= 1 - \sum_{x=0}^{3} \binom{8}{x} \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{8-x}$

$= 1 - \left[\left(\frac{1}{3}\right)^8 + 8\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^7 + 28\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^6 + 56 \left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^5\right]$

$= 1 - \frac{1}{6561}\left[1 + 16 + 112 + 448\right]$

$= 1 - \frac{577}{6561}$

$= \frac{6561 - 577}{6561} \Rightarrow \frac{5984}{6561}$

$= 0.9121$

(iii) $P(3 \le X \le 6)$

$$\sum_{x=3}^{6} \binom{8}{x} \left(\frac{2}{3}\right)^{x} \left(\frac{1}{3}\right)^{8-x}$$

$$= \binom{8}{3}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^5 + \binom{8}{4}\left(\frac{2}{4}\right)^4\left(\frac{1}{3}\right)^4 + \binom{8}{5}\left(\frac{2}{5}\right)^5 \left(\frac{1}{3}\right)^3 + \binom{8}{6}\left(\frac{2}{3}\right)^6\left(\frac{1}{3}\right)^2$$

$$= \frac{8}{(3)^8}\left[56 + 140 + 224 + 224\right]$$

$$= \frac{8 \times 644}{6561}$$

$$= \frac{5152}{6561} = 0.7852$$

Q5 Derive the binomial distribution and find its mean and variance.

Ans 2 Derivation of binomial ~~poss~~ is distribution:

To derive the formula that gives the probability of successes in n trails for a binomial experiment, we proceed as follows.

The experiment has n trials of which may result in S or F. The sample space has 2^n possible sample points or outcome, each outcome consisting of a sentence $(a_1, a_2 \ldots a_n)$ where each a_i is either S or F. we desire to find the probability of these outcomes according to the number of successes.

First we consider the probability of zero successes i.e $P(X=0)$. In case of zero success every trial results in F and the event consists of a sequence of n F_s, i.e $(FF \ldots f)$.

$$P(FF \ldots f) = P(F)\,P(F)\,P(f) \cdots P(f) \text{ (n tim}$$

$= q^n$

Since there is only one sequence of outcome of n trials resulting in fs therefore

$P(x=0) = q^n$

Next we consider the probability of one success i.e $P(x=1)$ in this case one results in S and the remaining $(n-1)$, trials results in fs.

Mean and Variance :

let x be a random variable with bionomial distribution $b(x,n,p)$ Then its mean and variance are given by $\mu = np$ and $a^2 = npq$ respectively

Mean $\mu = E(x)$

$$= \sum_{x=0}^{n} \binom{n}{x} p^x q^{n-x}, \quad \text{where } x = 0,1,2,\ldots n$$

$$= 0 . q^n + 1 \binom{n}{1} q^{n-1} p + 2 \binom{n}{2} q^{n-2} p^2 + \cdots np^n$$

$$= np\left[q^{n-1} + \binom{n-1}{1} q^{n-2} p + \binom{n-1}{2} q^{n-3} p^2 + \cdots + p^{n-1}\right]$$

$$= np(q+p)^{n-1}$$

$= np$, because $q + p = 1$

by definition of variance

$$a^2 = E(x-\mu)^2 = E(x^2) - [E(x)]^2$$

But

$$E(x^2) = E[x(x-1)+x] = E(x(x-1)) + E(x)$$

$$= E(x(x-1)) + np$$

Now

$$E[(x(x-1)] = \sum_{x=0}^{n} x(x-1) \binom{n}{x} p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x(x-1) \cdot \frac{n(n-1)(n-2)!}{x(x-1)(x-2)(n-x)!} p^2 p^{x-2} q^{n-x}$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!\,(n-x)!} p^{x-2} q^{n-2}.$$

(x starts at 2 since $x = 0, 1$ add nothing to the sum)

The term $(n-x)$ may be written as $(n-2)-(x-2)$. Substituting $y = x-2$ and $m = n-2$ in the summation we obtain

$$E[x(x-1)] = n(n-1)p^2 \sum_{y=0}^{m} \frac{m!}{y!\,(m-y)!} p^y q^{m-y}$$

$$= n(n-1)p^2 \sum_{y=0}^{m} \binom{m}{y} p^y q^{m-y}$$

$$= n(n-1)p^2 \qquad \because \text{Summation is } 1$$

Thus $\sigma^2 = E(x^2) - [E(x)]^2$

$$= E[x(x-1)] + E(x) - [E(x)]^2$$

$$= n(n-1)p^2 + np - (np)^2$$

$$= n^2p^2 - np^2 + np - n^2p^2$$

$$= np - np^2 = np(1-p) = npq \text{ and}$$

$$\sigma = \sqrt{npq}$$

Hence the variance of the number of successes is npq, and the standard deviation is $\sqrt{npq}$.

Q6 Differentiate between Binominal frequency distribution and Binorminal distribution with the help of formula?

Ans6 The Binomial is denoted and formulated by

$$f(x) = P(X=x) = \binom{n}{x} p^x q^{n-x}$$

Where,

$$x = 0, 1, 2 \ldots n.$$

it shows only the probability of an individual.

Binominal frequency:

if the binomial probability distribution is multiplied by N, the number of experiments or sets the resulting distribution is known as the binomial frequency distribution. thus the expected frequency of x successes in N experiments is $N \binom{n}{x} p^x q^{n-x}$ I, should be noted that the the n independent trials constitute one experiment or one set.

Q7 Solution.

Measure	Data Set A	B	C	D
Co-efficient of Variation	$CV = \frac{3}{45} \times 100$	$CV = \frac{11}{60} \times 100$	$CV = \frac{5}{50} \times 100$	$CV = \frac{15}{25} \times 100$
	$CV = 6.7$	$CV = 18.3$	$CV = 10$	$CV = 60$

Q4 Proof.

Since the C_i's form a Partition of the sample space, we can apply the law of total Probability for $A \cap B$

$P(A \cap B) = \sum P(A \cap B / C_i) P(C_i)$

$P(A \cap B) = \sum P(A / C_i) P(B / C_i) P(C_i)$

(A and B are Conditionally Independent.

$P(A \cap B) = \sum P(A / C_i) P(B) P(C_i)$

∴ (B is independent of all C_i

$P(A \cap B) = P(B) \sum P(A / C_i) P(C_i)$

$P(A \cap B) = P(B) P(A)$

law of total Probability

Hence A and B are Independent.