DEPARTMENT: BE (CIVIL)

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SECTION : "A"

SUBJECT : Calculus

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Question NO: 1: Find PQ where P is point in three-diamensional...

Answer:

Solution: Coordinate of $P = (4,1,3)$

$OP = 4i + 1j + 3K$

or $OQ = \overrightarrow{OQ} - \overrightarrow{OP}$

$= (i + 2j + 1K) - (4i + 1j + 3K)$

$= -3i + 1j + 1K \longrightarrow (*)$

NOW! Distance b/w P & Q $= |PQ|$

$= \sqrt{(-3)^2 + (1)^2 + (1)^2}$

$= \sqrt{9 + 1 + 1}$

$= \sqrt{11} \longrightarrow (**)$

Let M be the point which divided PQ in ratio 1:3 then by the ratio theorm position vector of $M = \overrightarrow{OM}$

$$= \frac{3(4i + 1j + 3k) + (1)(i + 2j + 4k)}{1+3}$$

$$= \frac{12i + 3j + 9k + i + 2j + 4k}{4}$$

$$= \frac{13i + 5j + 13k}{4} \rightarrow ***$$

Hence eq *, ** & *** are requesed Solution.

QNO2: Evaluate:

$$\int \frac{4x^3+10x+4}{2x^2+x}$$

Solution: $\int \frac{4x^3+10x+4}{2x^2+x}$

$= \text{let } I = \int \frac{4x^3+10x+4}{2x^2+x}\,dx$

By long Division

$$\frac{4x^3+10x+4}{2x^2+x} = 2x-1+\frac{11x+4}{2x^2+x}$$

$$= 2x-1+\frac{11x+4}{x(2x+1)}$$

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Suppose that

$$\frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{2x+1}$$

xplying both sides by $x(2x+1)$

$$11x+4 = A(2x+1) + Bx \rightarrow (i)$$

Put $x=0$ and $x=-\frac{1}{2}$ in (i)

$\boxed{A = 4}$

$$\frac{-11}{2} + 4 = -\frac{1}{2}B$$

$$= \frac{-11+8}{2} = -\frac{1}{2}B$$

$$= B = 3$$

$$\frac{11x+4}{x(2x+1)} = \frac{4}{x} + \frac{3}{2x+1}$$

$\because$

$$\int \frac{4x^3+10x+4}{2x^2+x}\,dx = \int 2x\,dx - \int dx + \int \frac{4}{x}\,dx + \int \frac{3}{2x+1}\,dx$$

$$I = 2 \cdot \frac{x^2}{2} - x + 4 \ln|x| + \frac{3}{2} \ln|2x+1| + c$$

$$I = x^2 - x + \frac{3}{2} \ln|2x+1| + 4 \ln|x| + c.$$

$$I = x^2 + x + \frac{3}{2} \ln|2x+1| + 4 \ln|x| + c$$

Q # 03: Part "A".

$$\int_0^2 x^2 e^x \, dx$$

Solution: $\int_0^2 x^2 e^x \, dx$

$$= \left| x^2 e^x \right|_0^2 - \int_0^2 e^x \, 2x \, dx$$

$$= (4e^2 - 0) - 2\int_0^2 x e^x \, dx$$

$$= 4e^2 - 2\left[x e^x \Big|_0^2 - \int_0^2 e^x \, dx \right]$$

$$= 4e^2 - 2\left[(2e^2 - 0) - \left| e^x \right|_0^2 \right]$$

$$= 4e^2 - 2\left[2e^2 - (e^2 - e^0) \right]$$

$$= 4e^2 - 4e^2 + 2(e^2 - 1)$$

$$= 2(e^2 - 1)$$

Q#3

Part B: $\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

Sol: $\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}$

let $I = \int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx \rightarrow (*)$

put $\sqrt{x} = u$

$\Rightarrow \frac{1}{2\sqrt{x}}\,dx = du$

$\Rightarrow \frac{dx}{\sqrt{x}} = 2du$

When $x \rightarrow 1$, $u \rightarrow 1$

When $x \rightarrow 2$, $u \rightarrow \sqrt{2}$

$\therefore$ Equation $*$ becomes

$I = \int_1^{\sqrt{2}} \sin u \cdot 2\,du$

$$= -2 \left| \cos u \right|_{1}^{\sqrt{2}}$$

$$= -2 \left(\cos\sqrt{2} - \cos 1 \right) \quad \text{Ans}$$

QNO 4: VERIFY THAT

$$u(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

Satisfies the Three-diamensional laplace's equation.

Solution: $u(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}} \rightarrow (1)$

We have to verify

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

NOW: Partially diff (1) w.r.t "x"

$$\frac{\partial u}{\partial x} = -\frac{1}{2}\left(x^2+y^2+z^2\right)^{-3/2}(2x)$$

$$= -x\left(x^2+y^2+z^2\right)^{-3/2}$$

Again diff w.r.t 'x' we have

$$\frac{\partial^2 u}{\partial x^2} = -\left[(x^2+y^2+z^2)^{-3/2} + x\left(-\tfrac{3}{2}\right)(x^2+y^2+z^2)^{-5/2}(2x)\right]$$

$$= -\left[(x^2+y^2+z^2)^{-3/2} - 3x^2(x^2+y^2+z^2)^{-5/2}\right]$$

⇒ In the same manner

$$\frac{\partial^2 u}{\partial y^2} = -\left[(x^2+y^2+z^2)^{-3/2} - 3y^2(x^2+y^2+z^2)^{-5/2}\right]$$

and

$$\frac{\partial^2 u}{\partial z^2} = -\left[(x^2+y^2+z^2)^{-3/2} - 3z^2(x^2+y^2+z^2)^{-5/2}\right]$$

Adding above all second order partial derivatives, we have -.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

$\Rightarrow -(x^2+y^2+z^2)^{-3/2} + 3x^2(x^2+y^2+z^2)^{-5/2}$

$= -(x^2+y^2+z^2)^{-3/2} + 3y^2(x^2+y^2+z^2)^{-5/2}$

$= -(x^2+y^2+z^2)^{-3/2} + 3z^2(x^2+y^2+z^2)^{-5/2}$

$= -3(x^2+y^2+z^2)^{-3/2} + 3(x^2+y^2+z^2)^{-5/2}(x^2+y^2+z^2)$

$= -3(x^2+y^2+z^2)^{-3/2} + 3(x^2+y^2+z^2)^{-3/2} = 0$

Hence !

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

This is laplace's equation in 3 diamension.