



DEPARTMENT: BE (CIVIL)

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SECTION : "A"

SUBJECT : Calculus

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Question NO: 1: Find PQ where P is point in three-dimensional...

Answer:

Solution: Coordinate of P = (4, 1, 3)

$$OP = 4i + 1j + 3k$$

$$\begin{aligned} \text{OR } OQ &= \vec{OQ} - \vec{OP} \\ &= (i + 2j + 1k) - (4i + 1j + 3k) \\ &= -3i + 1j + 1k \longrightarrow (*) \end{aligned}$$

NOW! Distance b/w P & Q = |PQ|

$$= \sqrt{(-3)^2 + (1)^2 + (1)^2}$$

$$= \sqrt{9 + 1 + 1}$$

$$= \sqrt{11} \longrightarrow (**)$$

Let M be the point which divided PQ in ratio 1:3 then by the ratio theorem position vector of M =  $\vec{OM}$

$$= \frac{3(4i + 1j + 3k) + (1)(i + 2j + 4k)}{1 + 3}$$

$$= \frac{12i + 3j + 9k + i + 2j + 4k}{4}$$

$$= \frac{13i + 5j + 13k}{4} \rightarrow \text{****}$$

Hence eq <sup>1</sup>\*, <sup>2</sup>\*\*\*, <sup>3</sup>\*\*\*\* are required  
Solution.

QNO2: Evaluate:

$$\int \frac{4x^3 + 10x + 4}{2x^2 + x}$$

Solution:  $\int \frac{4x^3 + 10x + 4}{2x^2 + x}$

= let  $I = \int \frac{4x^3 + 10x + 4}{2x^2 + x} dx$

By long Division

$$\frac{4x^3 + 10x + 4}{2x^2 + x} = 2x - 1 + \frac{11x + 4}{2x^2 + x}$$

$$= 2x - 1 + \frac{11x + 4}{x(2x + 1)}$$

Suppose that

$$\frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{2x+1}$$

xplying both sides by  $x(2x+1)$

$$11x+4 = A(2x+1) + Bx \rightarrow \textcircled{i}$$

Put  $x=0$  and  $x=-\frac{1}{2}$  in  $\textcircled{i}$

$$\boxed{A=4}$$

$$\frac{-11}{2} + 4 = \frac{-1}{2} B$$

$$= \frac{-11+8}{2} = -\frac{1}{2} B$$

$$\boxed{B=3}$$

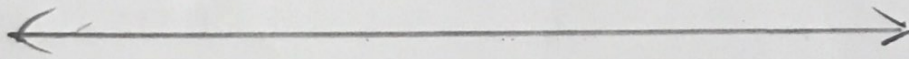
$$\frac{11x+4}{x(2x+1)} = \frac{4}{x} + \frac{3}{2x+1}$$

$$\therefore \int \frac{4x^3+10x+4}{2x^2+x} dx = \int 2x dx - \int dx + \int \frac{4}{x} dx + \int \frac{3}{2x+1} dx$$

$$I = 2 \cdot \frac{x^2}{2} - x + 4 \ln |x| + \frac{3}{2} \ln |2x+1| + C$$

$$I = x^2 - x + \frac{3}{2} \ln |2x+1| + 4 \ln |x| + C$$

$$I = x^2 + x + \frac{3}{2} \ln |2x+1| + 4 \ln |x| + C$$



Q # 03: Part "A" .

$$\int_0^2 x^2 e^x dx$$

Solution:  $\int_0^2 x^2 e^x dx$

$$= \left[ x^2 e^x \right]_0^2 - \int_0^2 e^x 2x dx$$

$$= (4e^2 - 0) - 2 \int_0^2 x e^x dx$$

$$= 4e^2 - 2 \left[ x e^x \Big|_0^2 - \int_0^2 e^x dx \right]$$

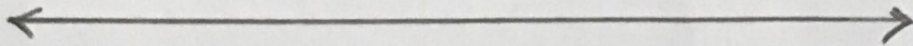
$$= 4e^2 - 2 \left[ (2e^2 - 0) - \left[ e^x \Big|_0^2 \right] \right]$$

$$= 4e^2 - 2 \left[ 2e^2 - (e^2 - e^0) \right]$$

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$$= 4e^2 - 4e^2 + 2(e^2 - 1)$$

$$= 2(e^2 - 1)$$





Q#3

Part B:  $\int_1^2 \frac{\sin \sqrt{x}}{\sqrt{x}} dx$

Sol:  $\int_1^2 \frac{\sin \sqrt{x}}{\sqrt{x}}$

Let  $I = \int_1^2 \frac{\sin \sqrt{x}}{\sqrt{x}} dx \rightarrow (*)$

Put  $\sqrt{x} = u$

$$\Rightarrow \frac{1}{2\sqrt{x}} dx = du$$

$$\Rightarrow \frac{dx}{\sqrt{x}} = 2du$$

When  $x \rightarrow 1$ ,  $u \rightarrow 1$

When  $x \rightarrow 2$ ,  $u \rightarrow \sqrt{2}$

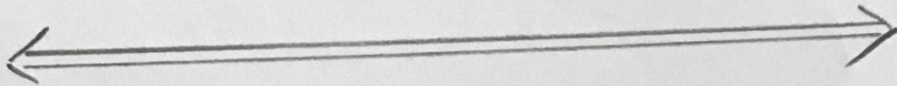
∴ Equation ~~\*~~ becomes

$$I = \int_1^{\sqrt{2}} \sin u \cdot 2 du$$

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$$= -2 \left| \cos u \right|_1^{\sqrt{2}}$$

$$= -2 (\cos \sqrt{2} - \cos 1) \quad \text{Ans}$$



QNO 4: VERIFY THAT

$$u(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$$

Satisfies the three-dimensional Laplace's equation.

Solution:  $u(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}} \rightarrow (1)$

WE have to verify

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

NOW! Partially diff (1) w.r.t "x"

$$\frac{\partial u}{\partial x} = -\frac{1}{2} (x^2 + y^2 + z^2)^{-3/2} (2x)$$

$$= -x (x^2 + y^2 + z^2)^{-3/2}$$

Again diff w.r.t 'x' we have

$$\frac{\partial^2 u}{\partial x^2} = - \left[ (x^2 + y^2 + z^2)^{-3/2} + x(-3/2)(x^2 + y^2 + z^2)^{-5/2} (2x) \right]$$

$$= - \left[ (x^2 + y^2 + z^2)^{-3/2} - 3x^2 (x^2 + y^2 + z^2)^{-5/2} \right]$$

→ In the same manner

$$\frac{\partial^2 u}{\partial y^2} = - \left[ (x^2 + y^2 + z^2)^{-3/2} - 3y^2 (x^2 + y^2 + z^2)^{-5/2} \right]$$

and

$$\frac{\partial^2 u}{\partial z^2} = - \left[ (x^2 + y^2 + z^2)^{-3/2} - 3z^2 (x^2 + y^2 + z^2)^{-5/2} \right]$$

Adding above all second order partial derivatives, we have -

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

$$\Rightarrow -(x^2 + y^2 + z^2)^{-3/2} + 3x^2(x^2 + y^2 + z^2)^{-5/2}$$

$$= -(x^2 + y^2 + z^2)^{-3/2} + 3y^2(x^2 + y^2 + z^2)^{-5/2}$$

$$= -(x^2 + y^2 + z^2)^{-3/2} + 3z^2(x^2 + y^2 + z^2)^{-5/2}$$

$$= -3(x^2 + y^2 + z^2)^{-3/2} + 3(x^2 + y^2 + z^2)^{-5/2}$$

$(x^2 + y^2 + z^2)$

$$= -3(x^2 + y^2 + z^2)^{-3/2} + 3(x^2 + y^2 + z^2)^{-3/2} = 0$$

Hence!

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

This is Laplace's equation  
in 3 dimensions.

