

Name Wajeehuddin

ID 7921

Subject Numerical
analysis

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(1)

Q1

$$f(x) = x^3 + 3.999 \times 10^{-4} - 0.165x^2$$

$$x_0 = 0.02$$

Solution

$$f(x) = x^3 + 3.999 \times 10^{-4} - 0.165x^2$$

$$x_0 = 0.02$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \quad i = 0, 1, 2, 3, \dots$$

$$i = 0$$

$$x_{0+1} = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$f(x) = x^3 + 3.999 \times 10^{-4} - 0.165x^2$$

$$f(x_0) = (0.02)^3 + 3.999 \times 10^{-4} - 0.165(0.02)^2$$

$$(2)$$
$$= 0.0000008 + 0.0003999 - 0.165 \times 0.0004$$
$$= 0.0000008 + 0.0003999 - 0.000066 = 0.0003419$$

$$f(x_0) = 39989.99$$

$$f'(x) = 3x^2 + 0 - 0.33x$$

$$f'(x_0) = 3(0.02)^2 - 0.33(0.02)$$
$$= 0.0012 - 0.0066$$

$$f'(x_0) = -0.0054$$

$$x_{0+1} = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 0.02 - \frac{0.0003419}{-0.0054}$$

$$= 0.02 + \frac{0.0003419}{0.0054}$$

$$x_1 = \frac{0.0004499}{0.0054}$$

$$x_1 = 0.08331$$

$$x_1 = 0.08331$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$i = 1$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$f(x_1) = x_1^3 + 3.999 \times 10^{-4} - 0.165(x_1)^2$$

$$= (0.08331)^3 + 3.999 \times 10^{-4} - 0.165(0.08331)^2$$

$$= 0.000578217 + 0.0003999 - 0.165$$

$$(0.006940556)$$

$$= 0.000578217 + 0.0003999 - 0.001145192$$

$$f(x_1) = -0.000167075$$

$$f'(x_1) = 3x_1^2 - 0.33x_1$$

$$= 3(0.08331)^2 - 0.33(0.08331)$$

$$= 0.020821668 - 0.274923$$

$$= -0.254101332$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= (0.08331) - \frac{(+0.000167074)}{+0.254101332}$$

$$= \frac{0.021002107}{0.254101332}$$

$$x_2 = 0.082652$$

$$i = 2$$

$$x_{2+1} = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$f(x) = x^3 + 3.999 \times 10^{-4} - 0.165x^2$$

$$f(x_2) = (0.082652)^3 + 0.0003999 - 0.165(0.082652)^2$$

$$= 0.000964524 - 0.01363758$$

$$= -0.000162648$$

$$f'(x_2) = 3(0.082652)^2 - 0.33(0.082652)$$
$$= 0.020494059 - 0.02727516$$
$$= -0.006781101$$

$$x_3 = 0.082652 - \frac{(+0.000162648)}{(-0.006781101)}$$

$$\frac{0.000397823}{0.006781101}$$

$$x_3 = 0.058665$$

QNO2

$x_0 = 2$

$y_0 = 0.5$

$x_1 = 2.75$

$y_1 = 0.36$

$x_2 = 4$

$y_2 = 0.25$

As we know that Lagrange
Interpolation formula

$$y = \frac{(x-x_1)(x-x_2) \dots (x-x_n)}{(x_0-x_1)(x_0-x_2) \dots (x_0-x_n)} y_0$$

$x_0 = 2$

$y_0 = 0.5$

$x_1 = 2.75$

$y_1 = 0.36$

$x_2 = 4$

$y_2 = 0.25$

$$y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2$$

7
put values

$$y = \frac{(3 - 2.75)(3 - 4)(0.5)}{(2 - 2.75)(2 - 4)}$$

$$+ \frac{(3 - 2)(3 - 4)(0.36)}{(2.75 - 2)(2.75 - 4)}$$

$$+ \frac{(3 - 2)(3 - 2.75)(0.25)}{(4 - 2)(4 - 2.75)}$$

$$y = (-0.833) + 0.384 + 0.025$$

$$y = -0.424$$

QNO3

$x = 1.0$

1.3

1.6

1.9

2.2

(8)

$y = 0.7651977$

0.6200860

0.4554022

0.2818186

0.1103623

x_i	$f(x_i)$	$f(x_{i-1}, x_i)$	$f(x_{i-2}, x_{i-1})$	$f(x_{i-3}, x_i)$	$f(x_i, x_{i+1})$
x_0 1	0.7651977	-0.4837056	-0.108734		
x_1 1.3	0.6200860	-0.5478946	-0.0494433	0.0658785	
x_2 1.6	0.4554022	-0.578612	0.006918	0.006251255	-0.0028049
x_3 1.9	0.2818186	-0.571521			
x_4 2.2	0.1103623				

(9)

$$1) f(x_0, x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$= \frac{0.6200860 - 0.7651977}{1.3 - 1}$$

$$f(x_0, x_1) = -0.4837056$$

$$2) f(x_1, x_2) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$= \frac{0.4554022 - 0.6200860}{1.6 - 1.3}$$

$$f(x_1, x_2) = -0.548946$$

$$3) f(x_2, x_3) = \frac{f(x_3) - f(x_2)}{x_3 - x_2}$$

$$= \frac{0.2818186 - 0.4554022}{1.9 - 1.6}$$

$$f(x_2, x_3) = -0.578612$$

$$4) f(x_3, x_4) = \frac{f(x_4) - f(x_3)}{x_4 - x_3}$$

$$= \frac{0.1103623 - 0.2818186}{2.2 - 1.9} = -0.571521$$

Third divided difference

$$f(x_0, x_1, x_2, x_3, x_4) = \frac{f(x_1, x_2, x_3)}{x_3 - x_0}$$

$$\frac{f(x_1, x_2, x_3) - f(x_0, x_1, x_2)}{x_3 - x_0}$$

$$= \frac{-0.0494433 - (-0.108734)}{1.9 - 1}$$

$$= 0.0658785$$

$$f(x_1, x_2, x_3, x_4) =$$

$$\frac{f(x_2, x_3, x_4) - f(x_1, x_2, x_3)}{x_4 - x_1}$$

$$\frac{0.006828 - (-0.049443)}{2.2 - 1.3}$$

$$= 0.06251255$$

$$= 0.06251255$$

4th Divided difference

$$f(x_0, x_1, x_2, x_3, x_4) = \frac{0.06251255 - 0.065878}{2.2 - 1}$$

$$= 0.0028049$$

Second divided difference

$$f(x_0, x_1, x_2) = \frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0}$$

$$= \frac{-0.548946 - (-0.4837056)}{1.6 - 1}$$

$$= -0.108734$$

$$f(x_1, x_2, x_3) = \frac{f(x_2, x_3) - f(x_1, x_2)}{x_3 - x_1}$$

$$= \frac{0.578612 - (-0.548946)}{1.9 - 1.3}$$

$$= 0.0494433$$

$$f(x_2, x_3, x_4) = \frac{f(x_3, x_4) - f(x_2, x_3)}{x_4 - x_2}$$

$$= \frac{-0.572521 - (-0.578612)}{2.2 - 1.6}$$

$$= 0.006818$$

12

$$\begin{aligned}
 f(x) = & f(x_0) + (x-x_0)f(x_0, x_1) + (x-x_0) \\
 & (x-x_1)f(x_0, x_1, x_2) + (x-x_0)(x-x_1) \\
 & (x-x_2)f(x_0, x_1, x_2, x_3) + (x-x_0)(x-x_1) \\
 & (x-x_2)(x-x_3)f(x_0, x_1, x_2, x_3, x_4)
 \end{aligned}$$

$$\begin{aligned}
 = & 0.7651977 + (x-1) - 6.4837056 + (x-1)(x-1.3) \\
 & (-0.108734) + (x-1)(x-1.3)(x-1.6) \\
 & (0.0658785) + (x-1)(x-1.3)(x-1.6) \\
 & (x-1.9)(-0.0028049)
 \end{aligned}$$