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Applied Calculus.

Q NO 1

Find PQ where P is the point in three dimensional space with co-ordinates (4, 1, 3) and point Q with co-ordinates (1, 2, 4). Find the distance b/w P & Q. Further find the position vector of point dividing PQ in the ratio 1:3

SOLUTION:

Co-ordinates of P = (4, 1, 3)

$$\overline{OP} = 4i + 1j + 3K$$

$$\overline{OQ} = \overline{OQ} - \overline{OP}$$

$$= (1 + 2j + 4K) - (4i + 1j + 3K)$$

$$= -3\hat{i} + 1j + 1K \quad \text{--- (a)}$$

Distance b/w P & Q $= |PQ|$

$$= \sqrt{(-3)^2 + (1)^2 + (1)^2}$$

$$\sqrt{11} \quad \text{--- (b)}$$

Let M be the mid point which divided PQ in ratio 1:3 then by ratio theorem position vector of M $= \overline{OM}$

$$= \frac{3(4i + 1j + 3k) + (1)(i + 2j + 4k)}{1 + 3}$$

$$= \frac{13i + 5j + 13k}{4} \quad \text{— (c)}$$

$\vec{OQ} = 3i + 1j + 1k$

$|PQ| = \sqrt{11}$

$\vec{OM} = \frac{13i + 5j + 13k}{4}$

Q No 2 ③

Evaluate

$$\int \frac{4x^3 + 10x + 4}{2x^2 + x}$$

SOLUTION:

$$\int \frac{4x^3 + 10x + 4}{2x^2 + x}$$

Long division of $4x^3 + 10x + 4$ by $2x^2 + x$:

Quotient: $2x - 1$

$4x^3$
$\pm 4x^3 \pm 2x^2$

$-2x^2 + 10x + 4$
$-2x^2 \mp x$

$11x + 4$

So

$$2x - 1 + \frac{11x+4}{2x^2+x} = \frac{4x^3+10x+4}{2x^2+x}$$

$$\int \frac{4x^3+10x+4}{2x^2+x} = \int 2x - 1 + \int \frac{11x+4}{2x^2+x} \quad \text{--- (a)}$$

$$= 2\int x\,dx - \int dx + \int \frac{11x+4}{2x^2+x}\,dx$$

$$= \frac{2x^2}{2} - x + \int \frac{11x+4}{x(2x+1)} \quad \text{--- (b)}$$

Now

$$\frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{(2x+1)} \quad (A)$$

$$\frac{11x+4}{x(2x+1)} = \frac{A(2x+1)+Bx}{x(2x+1)}$$

$$11x+4 = A(2x+1) + Bx$$

Put $x = 0$

$A = 4$

Now Put $x = -1/2$ in (A)

$$11(-1/2) + 4 = B(-1/2)$$

$B = 3$

Putting values of A & B in (A)

$$\frac{11x+4}{x(2x+1)} = \frac{4}{x} + \frac{3}{2x+1}$$

Integrating on b/s

$$\int \frac{11x+4}{x(2x+1)}\,dx = \int \frac{4}{x}\,dx + \int \frac{3}{2x+1}\,dx$$

$$= 4\int \frac{1}{x}\,dx + 3\int \frac{1}{2x+1}\,dx$$

$$4\ln|x| + 3/2\ \ln|2x+1|$$

Now

$$x^2 - x + 4\ln|x| + 3/2\ \ln|2x+1|$$

$$\int \frac{4x^3+10x+4}{2x^2+x}\,dx = x^2 - x + 4\ln|x| + 3/2\ \ln|2x+1| + C$$

Q No 3(a)

$$\int_0^2 x^2 e^x \, dx$$

Solution:

$$\int x^2 e^x \, dx$$

$$= x^2 \int e^x dx - \int e^x dx \, \frac{d}{dx} x^2 \, dx$$

$$= x^2 e^x - \int e^x (2x) \, dx$$

$$= x^2 e^x - 2 \int x e^x \, dx$$

$$= x^2 e^x - 2 \left[x e^x \int e^x dx \right]$$

$$= x^2 e^x - 2 \left[x e^x - \int e^x dx \right]$$

$$= x^2 e^x - 2x e^x + 2e^x$$

Applying limit

$$x^2 e^x - 2x e^x + 2e^x \Big|_0^2$$

$$2^2 e^2 - 2(2) e^2 + 2e^2 - (0 - 0 + 2e^0)$$

$(4e^2 - 4e^2 + 2e^2 - 2)$

$= 2e^2 - 2$ Answer

Q3 (b)

$$\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$$

Solution:-

$$\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}$$

By integration

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$$

let

$$y = \sqrt{x}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$2dy = \frac{1}{\sqrt{x}}\, dx$$

$$\int \sin(y)\,(2\,dy) = 2\int \sin(y)\,dy$$

$$= 2(-\cos y)$$

$$= -2\cos y$$

Put $y = \sqrt{u}$

$$= -2\cos\sqrt{u}$$

Applying limits

$$= -2\left|\cos\sqrt{u}\right|_1^2 = -2\,(\cos\sqrt{2} - \cos(1))$$

$$= -2\cos\sqrt{2} + 2\cos(1)$$

$$= \boxed{-2\cos\sqrt{2} + 2\cos(1)}$$ Answer

Q NO 4

$$u(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

laplace equation is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

$$u(x,y,z) = (x^2+y^2+z^2)^{-1/2}$$

$$\frac{\partial u}{\partial x} = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2}(2x)$$

$$\frac{\partial u}{\partial x} = -x(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial x^2} = \left[-x(-3/2)(x^2+y^2+z^2)^{-5/2}(2x)\right.$$

$$+ (x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial x^2} = 3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

→ (A)

Now

$$\frac{\partial v}{\partial y} = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2}(2y)$$

$$\frac{\partial v}{\partial y} = -y(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 v}{\partial y^2} = -\left[y(-3/2)(x^2+y^2+z^2)^{-5/2}(2y) + (x^2+y^2+z^2)^{-3/2}\right]$$

$$\frac{\partial^2 v}{\partial y^2} = 3y^2(x^2+y^2+z^2)^{-5/2} + (x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial v}{\partial z} = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2}(2z)$$

$$\frac{\partial v}{\partial z} = -(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 v}{\partial z^2} = 3z^2(x^2+y^2+z^2) - (x^2+y^2+z^2)^{-3/2}$$

P.T.O

Putting values

$$3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3z^2(x^2+y^2+z^2) - (x^2+y^2+z^2)^{-3/2}$$

$$= (x^2+y^2+z^2)^{-5/2}\left[3x^2 - (x^2+y^2+z^2) + 3y^2 - (x^2+y^2+z^2) + 3z^2 - (x^2+y^2+z^2)\right]$$

$$(x^2+y^2+z^2)^{-5/2}\left[3x^2 - x^2 - y^2 - z^2 + 3y^2 - x^2 - y^2 - z^2 + 3z^2 - x^2 - y^2 - z^2\right]$$

$$= (x^2+y^2+z^2)^{-5/2}\,[0]$$

So given $u(x,y,z)$ is solution of Laplace equation.