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Subject = Discrete Structure.

Date = 30/sep/2020

Summer - 20

Q No. 1

(a)

Solution: let a be the first term and d be the Common difference of the arithmetic Sequence. Then

$a_n = a + (n-1)d$

$a_3 = a + (3-1)d$ and

$a_8 = a + (8-1)d$

Given that $a_3 = 7$ and $a_8 = 17$ therefore

$7 = a + 2d$ .... (1) and

$17 = a + 7d$ .... (2)

Subtracting (1) from (2), we get,

$10 = 5d$

$\Rightarrow d = 2$

Substituting $d = 2$ in (1) we have

$7 = a + 2(2)$

which gives $a = 3$

Thus, $a_n = a + (n-1)d$

$a_n = 3 + (n-1)2$ (using values of a and d)

Hence the value of 36th term is

$a_{36} = 3 + (36-1)2$

$= 3 + 70 = \boxed{73}$

Q No 2

(b) Tautology:-

A Tautology is a statement form that is always true regardless of the truth values of the statement variables. A Tautology is represented by the symbol "t".

- Example: The statement form $p \vee \sim p$ is tautology

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

$$p \vee \sim p \equiv t$$

- Since we have to show that the given statement form is Tautology. So the column of the above proposition in the Truth table will have all entries as T. As clear from the table in the next slide

Truth Table

Truth Table

Hence $(p \wedge q) \vee (\sim p \vee (p \wedge \sim q)) \equiv t$

p	q	$p \wedge q$	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \vee (p \wedge \sim q)$	$(p \wedge q) \vee (\sim p \vee (p \wedge \sim q))$
T	T	T	F	F	F	F	T
T	F	F	F	T	T	T	T
F	T	F	T	F	F	T	T
F	F	F	T	T	F	T	T

Q(C)

Sol: (1). $p \rightarrow q$

(2). $\sim q \rightarrow r$

(3). $\sim p \quad \wedge \quad \sim q \rightarrow r$

Q No 3:

Solution:

Given $f(x) = 2x+3$ and $g(x) = -x^2+5$,

$(f\circ g)(x) = f(g(x))$

$= f(-x^2+5)$

$= 2(\quad)+3$ Setting up to insert the input formula

$= 2(-x^2+5)+3$

$= -2x^2+10+3$

$= -2x^2+13$

Similarly You can find $g\circ f(x)$, $f\circ f(x)$, $g\circ g(x)$

Q No. 4:

Solution:

$P(1)$ is true

for $n=1$

L.H.S of $P(1) = 1^2 = 1$

$$R.H.S \text{ of } (P(1) = \frac{1(1+1)(2(1)+1)}{6}$$

$$= \frac{(1)(2)(3)}{6} = \frac{6}{6} = 1$$

So L.H.S = R.H.S of $P(1)$. Hence $P(1)$ is true

inductive step.

Suppose $P(k)$ is true for some integer $k \geq 1$;

$$1^2 + 2^2 + 3^2 + \dots k^2 = \frac{k(k+1)(2k+1)}{6}$$

To prove $P(k+1)$ is true; i.e;

$$1^2 + 2^2 + 3^2 + \dots + (k+1)^2 = \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$$

Consider LHS of above equation (2)

$$1^2 + 2^2 + 3^2 + \dots + (k+1)^2$$

$$= 1^2 + 2^2 + 3^2 + \dots + k^2 + (k+1)^2$$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1^2)$$

$$= (k+1)\left[\frac{k(2k+1)}{6} + (k+1)\right]$$

$$= (k+1)\left[\frac{k(2k+1)+6(k+1)}{6}\right]$$

$$= (k+1)\left[\frac{2k^2+1)+6(k+1)}{6}\right]$$

$$= (k+1)\left[\frac{2k^2+k+6k+6}{6}\right]$$

$$= \frac{k+1\,(2k^2+7k+6)}{6}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

$$= \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$$

Prove by Mathematical inducation

$$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1} \quad \text{for all integers } n \geq 1$$

use the mathematical induction to prove that

$$\left(1-\frac{1}{2^2}\right)\cdot\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{n^2}\right) = \frac{n+1}{2n} \quad \text{for all integers } n \geq 2$$

Q No. 5

Ans **Types of Relation.**

Binary Relation:-

- Let A and B be sets. The binary relation R from A to B is a subset of $A \times B$

Example

- let $A = \{1,2\}$ $B = \{1,2,3\}$

Then $A \times B = \{(1,1),(1,2),(1,3),(2,2),(2,2),(2,3)\}$

let
$R1 = \{(1,1),(1,3),(2,2)\}$
$R2 = \{(1,2),(2,1),(2,2),(2,3)\}$
$R3 = \{(1,1)\}$

Domain of a Relation:-

The domain of a relation R from A to B is the set of all first elements of the ordered pairs which belong to R denoted by Dom(R).

- Symbolically $Dom(R) = \{a \in A \mid (a,b) \in R\}$

Range of a relation:-

- The range of a relation R from A to B is the set of all second elements of the ordered pairs which belong to R denoted Ran(R)

- Symbolically $Ran(R) = \{b \in B \mid (a,b) \in R\}$

Relation On a Set:-

- A relation on the Set A is a relation from A to B. In Other words, a relation on a Set A is a Subset of $A \times A$.
- Let $A = \{1,2,3,4\}$ Define a relation R on A.

Solution :-

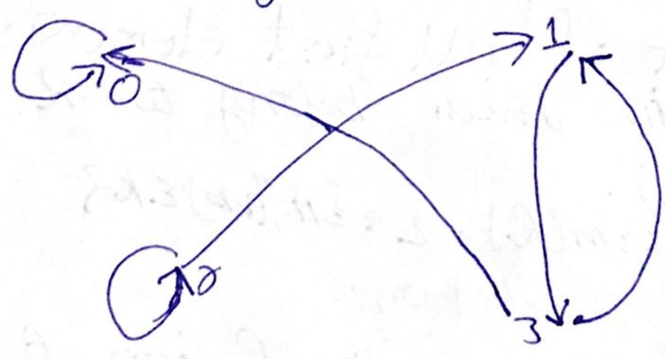
$A \times A = \{1,2,3,4\} \times \{1,2,3,4\}$

$A \times A = \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(3-4),(4,1),(4,2),(4,3),(4-4)$.

$R = \{(1,2),(1,2),(1,3),(1,4),(2,2),(2-4),(3,3),(4,4)\}$

Directed grap of a relation:.

- Let $A = \{0,1,2,3\}$
  and $R = \{(0,0),(1,3),(2,1),(2,2),(3,0),(3,1)\}$ be a binary relation on A.



Q No. 6

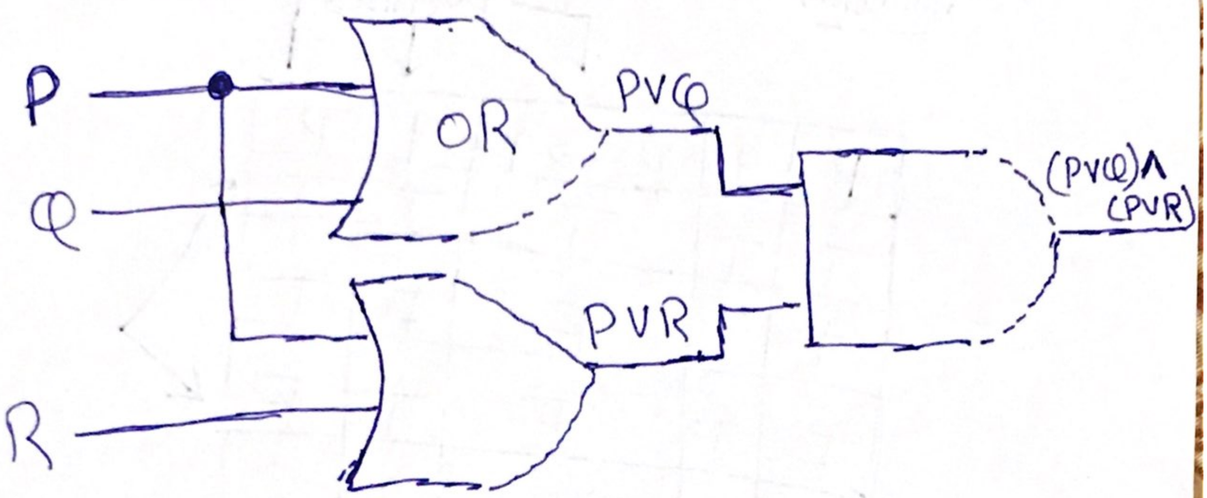
a) Solution

Premises conclusion

p	q	$p \to q$	q	p
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	F

- In the Second critical row, the Conclusion is false when the premises $p \to q$ and q are true. Therefore, the argument is invalid.

(b)



Q No: 7:

(a) $A = (a, b, c)$

$P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\},$
$\{a, b, c\}\}$

$B = \{1, 2, 3, 4\}$

$P(B) = \{\phi, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}$
$\{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\},$
$\{1, 2, 3, 4\}, \}$

Q No.7

(b) There are three forms of set as follow

Tabular form:.

• we list all the elements of a set separated by commas and enclosed within braces or curly brackets {}. For example.

• In the following examples we write the sets in Tabular form.

Examples.

• A= {1,2,3,4,5} is the Set of first five Natural Numbers

• B. {2,4,6,8,...,50} is the set of Even numbers up to 50.

C. = {1,3,5,7,9,...} is the set of positive odd numbers.

Descriptive form:.

. We state the elements of a Set in words

. Now we will write the examples dissussed in last slides in the descriptive Form.

. A = Set of first Natural Numbers. (Descriptive form)

. B = Set of Positive even integers less or equal to Fifty

· (Descripty form).
· C = Set of Positive odd integers. (Descriptive Form)

Set builder form =

· We write the Common Characteristics in Symbolic form. Shared by all the elements of the Set.

Examples =

· Now we will write the same examples which we write in tabular as well as descriptive Form, in set builder form

· $A = \{X \in N / X \leq 5\}$

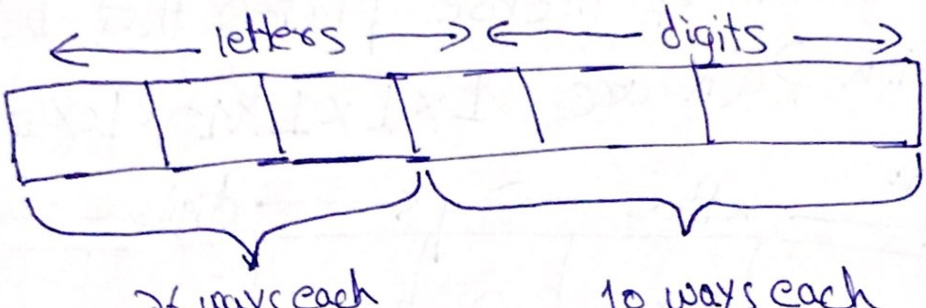
· $B = \{X \in E / 0 < X \leq 50\}$

· $C = \{X \in O / 0 < X\}$

Q No 8:

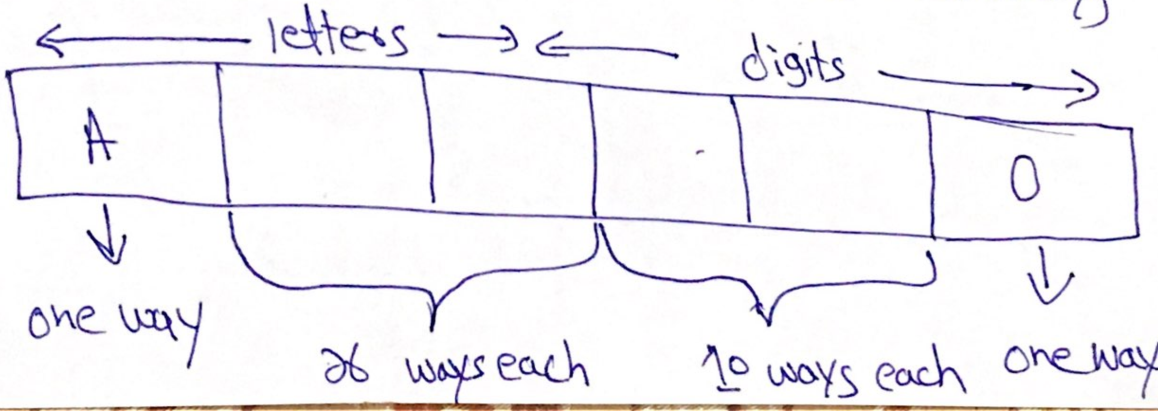
Solution

(a) Each of the three letters can be written in 26 different ways, and each of the three digits can be written in 10 different ways.



Hence, by the product rule, there is a total of $26 \times 26 \times 26 \times 10 \times 10 \times 10 = 17{,}576{,}000$ different license plates possible

(b) The first and last place can be filled in one way only, while each of second and third place can be filled in 26 ways and each of fourth and fifth place can be filled in 10 ways

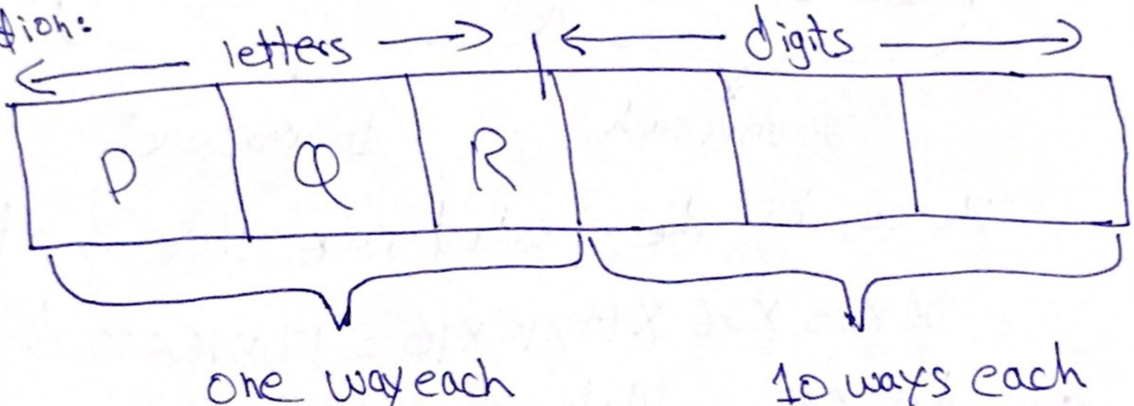


number of license plates that begin with A and end in 0 are

$$1 \times 26 \times 26 \times 10 \times 10 \times 1 = 67600$$

(C). Number of license Plates that begin with PQR are $1 \times 1 \times 1 \times 10 \times 10 \times 10 = 1000$

Solution:



number of license plates that begin with A and end in O are

$$1 \times 26 \times 26 \times 10 \times 10 \times 1 = 67600$$

(C). Number of license Plates that begin with PQR are $1 \times 1 \times 1 \times 10 \times 10 \times 10 = 1000$

Solution: