Name: Muhammad Ammar

Std.Id: 16602

BSSE- 2nd Semester

Department of Computer Science

Final Term Assignment

Discrete Structures

Q#1 (a)

Explain the concept of BiConditional statement.

Ans:

- A biconditional statement is defined to be true whenever both parts have the same truth value.
- The biconditional operator is denoted by a double-headed arrow.
- The biconditional pq represents "p if and only if q," where p is a hypothesis and q is a conclusion.
- The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise.

- The following is a truth table for biconditional pq.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

In the truth table above, pq is true when p and q have the same truth values, (i.e., when either both are true or both are false.)

Example

Given: x: I am breathing

y: I am alive

Problem: Write xy as a sentence.

Solution: xy represents the sentence, "I am breathing if and only if I am alive."

Q#1 (b)

Let p, q, and r represent the following statements:

p: Sam had pizza last night.

q: Chris finished her homework.

r: Pat watched the news this morning

Give a formula (using appropriate symbols) for each of these statements.

Ans:

i. Sam had pizza last night if and only if Chris finished her homework.

Ans: $\mathbf{p \leftrightarrow q}$

ii. Pat watched the news this morning iff Sam did not have pizza last night.

Ans: $\mathbf{r \leftrightarrow \neg P}$

iii. Pat watched the news this morning if and only if Chris finished her homework and Sam did not have pizza last night.

Ans: $\mathbf{r \leftrightarrow (q \land \neg P)}$

iv. In order for Pat to watch the news this morning, it is necessary and sufficient that Sam had pizza last night and Chris finished her homework

Ans: $\mathbf{r \leftrightarrow (P \land q)}$

Q#2:

Lets p, q, r represent the following statements:

p: it is hot today.

q: it is sunny

r: it is raining

Express in words the statements using Bicondtional statement represented by the following formulas:

i. $q \leftrightarrow p$

ii. $p \leftrightarrow (q \wedge r)$

iii. $p \leftrightarrow (q \vee r)$

iv. $r \leftrightarrow (p \vee q)$

Ans:

i) **It is sunny if and only if it is hot today.**

ii) **It is hot today if and only if it is sunny and it is raining.**

iii) **It is hot today if and only if it is sunny or it is raining.**

iv) **It is raining if and only if it is hot today or it is sunny.**

Q#3: Explain Argument with proper examples. Differentiate Valid and Invalid argument through proper examples, also construct a truth table showing valid and invalid arguments.

Ans:

Argument

- An argument is a set of initial statements, called premises, followed by a conclusion.
- An argument is a sequence of statements. All statements except the final one are called premises (or assumptions or hypotheses). The final statement is called the conclusion.

- An argument is considered valid if from the truth of all premises, the conclusion must also be true.
- The conclusion is said to be inferred or deduced from the truth of the premises.

For Example (1)

Here is an example:

- If I read my text, I will understand how to do my homework.
- I understand how to do my homework.
- Therefore, I read my text.

- Our first premise: is If I read my text, then I understand how to do my homework.
- Our second premise is: I understand how to do my homework.
- Our conclusion is I read my text.

Let's use t means I read my text and u means I understand how to do my homework.

Symbolically, our argument is:

$t \rightarrow u$

u

$\therefore t$

For Example (2)

Consider this argument.

- If Pat goes to the store, Pat will buy $1,000,000 worth of food.
- Pat goes to the store.
- Therefore, Pat buys $1,000,000 worth of food.

- This is a valid argument (you can test it on a truth table).
- However, even though Pat goes to the store, Pat does not buy $1,000,000 worth of food. The conclusion is false.
- How can the conclusion of a valid argument be false?

Solution

- The validity of an argument refers to its structure. Given a valid argument, the conclusion must be true if the premises are true. In this case the first premise is NOT true, and thus the conclusion does not need to be true.
- The conclusion of a valid argument can be false if one or more of the premises is false.

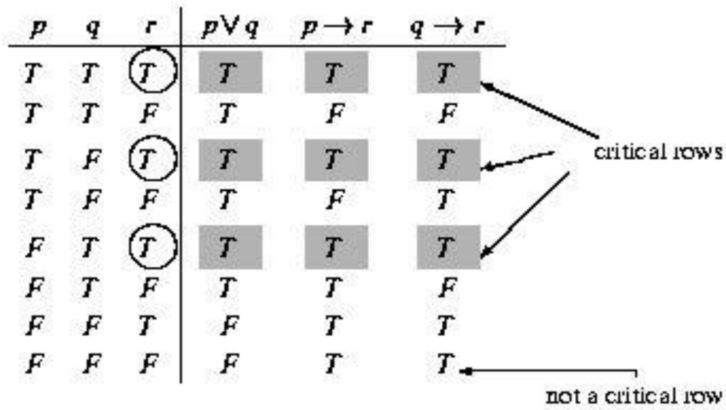
Valid argument

- The validity of an argument can be tested through the use of the truth table by checking if the critical rows, i.e. the rows in which all premises are true, will correspond to the value "true" for the conclusion.
- An argument is valid if the conclusion is true whenever all the premises are true.

For Example

Show ($p \vee q$, $p \rightarrow r$, $q \rightarrow r$, $\therefore r$) is a valid argument.

Solution:



p	q	r	$p \vee q$	$p \rightarrow r$	$q \rightarrow r$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	T	T	T
T	F	F	T	F	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	F	T	T
F	F	F	F	T	T

- We see all critical rows (in this case, those with the shaded positions all containing a T) correspond to (the circled) T(true) for r. Hence the argument is valid.

Invalid argument

- An argument that is not valid. We can test for invalidity by assuming that all the premises are true and seeing whether it is still possible for the conclusion to be false. If this is possible, the argument is invalid.

For Example

Show that the argument ($p \to q, \therefore \sim p \to \sim q$) is invalid.

Solution

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim p \to \sim q$	
T	T	F	F	T	T	
T	F	F	T	F	T	
F	T	T	F	T	F	this critical row fails
F	F	T	T	T	T	

We see that on the 3rd row, a critical row, the premise $p \to q$ is true while the conclusion $\sim$ p $\to \sim$ q is false. Hence the argument ($p \to q, \therefore \sim p \to \sim q$) is invalid.

Q#4: (a)

Explain the concept of Union, also explain membership table for union by giving proper example of truth table.

Ans:

Union

- Union of two given sets is the smallest set which contains all the elements of both the sets.
- To find the union of two given sets A and B is a set which consists of all the elements of A and all the elements of B such that no element is repeated.
- The symbol for denoting union of sets is '∪'.

For example

- Let set A = {2, 4, 5, 6}

  And set B = {4, 6, 7, 8}

- Taking every element of both the sets A and B, without repeating any element, we get a new set = {2, 4, 5, 6, 7, 8}
- This new set contains all the elements of set A and all the elements of set B with no repetition of elements and is named as union of set A and B.
- The symbol used for the union of two sets is '∪'.
- Therefore, symbolically, we write union of the two sets A and B is A ∪ B which means A union B.
- Therefore, $A \cup B = \{x : x \in A \text{ or } x \in B\}$

Membership Tables for Union

1)

$A \cup B$

A	B	$A \cup B$
1	1	1
1	0	1
0	1	1
0	0	0

2)

A	B	C	$B \cap C$	$A \cup (B \cap C)$
1	1	1	1	1
1	1	0	0	1
1	0	1	0	1
1	0	0	0	1
0	1	1	1	1
0	1	0	0	0
0	0	1	0	0
0	0	0	0	0

Q#4: (b)

Explain the concept of Intersection, also explain membership table for Intersection by using proper example of truth table.

Ans:

Intersection

- Intersection of two given sets is the largest set which contains all the elements that are common to both the sets.
- To find the intersection of two given sets A and B is a set which consists of all the elements which are common to both A and B.
- The symbol for denoting intersection of sets is '∩'.

For Example

- Let set A = {2, 3, 4, 5, 6}
  And set B = {3, 5, 7, 9}

- In this two sets, the elements 3 and 5 are common. The set containing these common elements i.e., {3, 5} is the intersection of set A and B.
- The symbol used for the intersection of two sets is '∩'.

- Therefore, symbolically, we write intersection of the two sets A and B is A ∩ B which means A intersection B.
- The intersection of two sets A and B is represented as $A \cap B = \{x : x \in A \text{ and } x \in B\}$

Membership Table for Intersection

1)

A∩B		
A	B	A∩B
1	1	1
1	0	0
0	1	0
0	0	0

Q#5 (a)

Explain the concept of Venn diagram with examples.

Ans:

Venn diagram

- A Venn diagram uses overlapping circles or other shapes to illustrate the logical relationships between two or more sets of items. Often, they serve to graphically organize things, highlighting how the items are similar and different.
- Venn diagrams, also called Set diagrams or Logic diagrams, are widely used in mathematics, statistics, logic, teaching, linguistics, computer science and business. Many people first encounter them in school as they study math or logic, since Venn diagrams

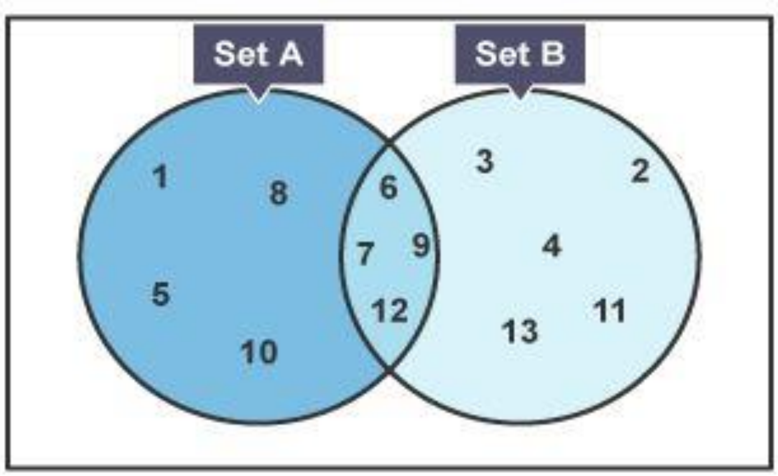
became part of "new math" curricula in the 1960s. These may be simple diagrams involving two or three sets of a few elements, or they may become quite sophisticated, including 3D presentations, as they progress to six or seven sets and beyond. They are used to think through and depict how items relate to each within a particular "universe" or segment. Venn diagrams allow users to visualize data in clear, powerful ways, and therefore are commonly used in presentations and reports. They are closely related to Euler diagrams, which differ by omitting sets if no items exist in them. Venn diagrams show relationships even if a set is empty.

Examples

1)

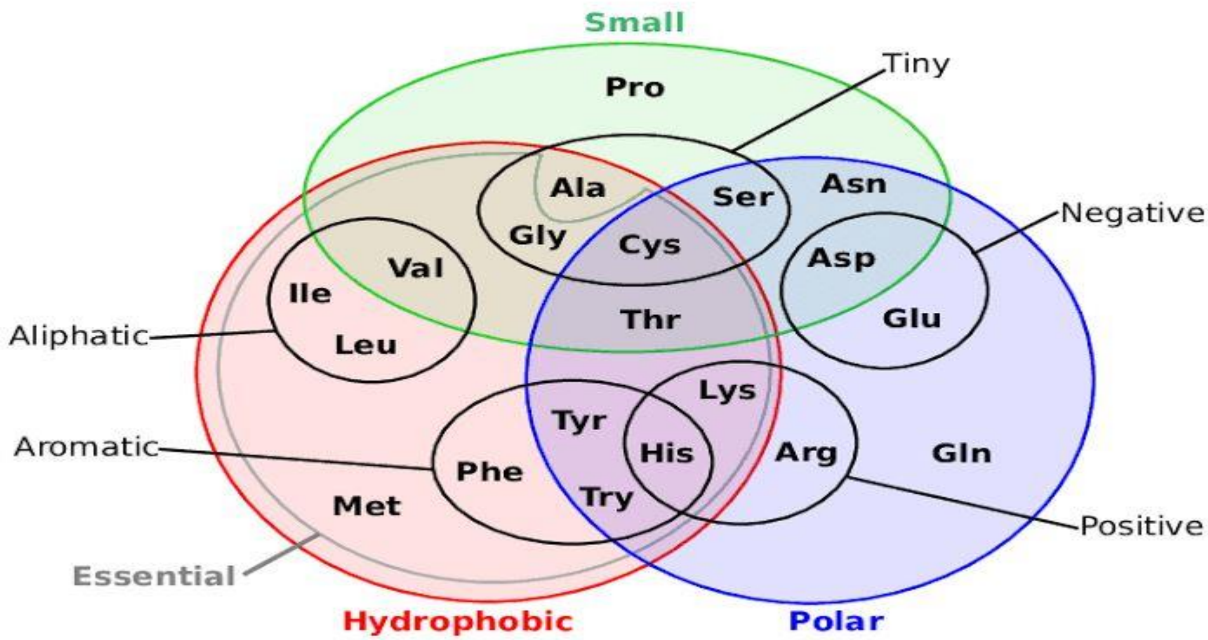
- The first Venn diagram example is in Mathematics. They are accessible when covering Sets Theory and Probability topics.

- In the diagram below, there are two sets, A = {1, 5, 6, 7, 8, 9, 10, 12} and B = {2, 3, 4, 6, 7, 9, 11, 12, 13}.
- The section where the two sets overlap has the numbers contained in both Set A and B, referred to as the intersection of A and B.
- The two sets put together, gives their union which comprises of all the objects in A, B which are {1 2 3 4 5 6 7 8 9 10 11 12 13}.



2)

- Scientist uses Venn diagrams to study human health and medicines. In the illustration below, you can see amino acids that are vital to human life.



Q#5 (b)

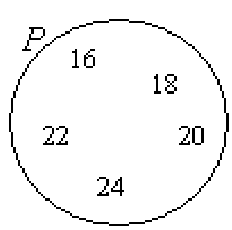
Given the set P is the set of even numbers between 15 and 25. Draw and label a Venn diagram to represent the set P and indicate all the elements of set P in the Venn diagram.

Ans:

List out the elements of P.

P = {16, 18, 20, 22, 24} ← 'between' does not include 15 and 25

Draw a circle or oval. Label it P. Put the elements in P.



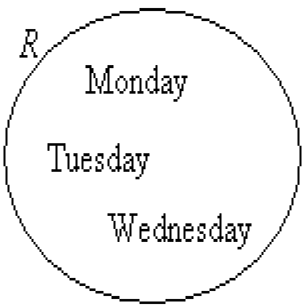
Q#5 (c)

Draw and label a Venn diagram to represent the set

R = {Monday, Tuesday, Wednesday}.

Ans:

Draw a circle or oval. Label it R . Put the elements in R.



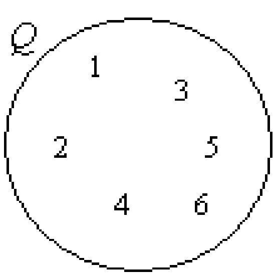
Q#5 (d)

Given the set Q = {x : 2x – 3 < 11, x is a positive integer }. Draw and label a Venn diagram to represent the set Q.

Ans:

Since an equation is given, we need to first solve for x.

$2x - 3 < 11 \Rightarrow 2x < 14 \Rightarrow x < 7$



So, Q = {1, 2, 3, 4, 5, 6}

Draw a circle or oval. Label it Q . Put the elements in Q.