Day. M T W T F S Date: __/__/__

Subject : Calculus

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Question No. (01)

The Function $g(t)$ is defined by

$$g(t) = \begin{cases} 0 & t < 0 \\ t^2 & 0 \le t \le 3 \\ 2t+3 & 3 < t \le 4 \end{cases}$$

a) State any point of discontinuity.

Solution:

a) To check possibility of the discontinuity of the Function is at $t = 0$ & 4.

First at $t = 0$

$$g(t) = t^2$$

$$g(0) = 0^2 = 0$$

For R.H.L

$$\lim g(1+h) = \lim (1+h)^2$$

$$= \lim_{h \to 0} 1 + h^2 + 2h$$

Applying limits

$= 1 + 0^2 + (2)(0)$

$= 1$

For L.H.L

$\lim_{h \to 0} g(1-h) = 2t + 3$

$= \lim_{h \to 0} 2(1-h) + 3$

$= \lim_{h \to 0} 2 - 2h + 3$

Apply limits.

$= 2 - 2(0) + 3$

$= 5$

$R.H.L \neq L.H.L = g(t) = 5$

Now at $t = 4$

$g(4) = 2(4) + 3$

$= 8 + 3$

$= 11$

For R.H.L

$\lim_{h \to 0} g(1+h) = \lim_{h \to 0} 2(1+h) + 3$

$= \lim_{h \to 0} 2 + 2h + 3$

Applying limits

$= 2 + 2(0) + 3 = 5$

For L.H.L

$\lim_{h \to 0} g(1-h) = 12$

$g(4) = R.H.L \neq L.H.L$

Point of discountinouity is at $t = 4$

b) Find, if they exist

$\lim_{t \to 3} g$

Solution

For $g(t) = t^2$

$\lim_{h \to 3} g(1+h) = \lim_{h \to 3} (1+h)^2$

$= \lim_{h \to 3} 1 + h^2 + 2h$

Applying limits

$= 1 + 3^2 2(3) \Rightarrow 16$

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L.H.L

$$\lim_{h\to 3} g(1-h) = \lim_{h\to 3} 2t+3$$

$$= \lim_{h\to 3} 2(1-h)+3$$

$$= \lim_{h\to 3} 2-2h+3$$

Applying limit.

$$= 2-2(3)+3$$

$$= 2-6+3$$

$$= -1$$

R.H.L $\neq$ L.H.L (Do not exist)

(L.H.L is -ve)

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Question NO: (02)

(i) Find the Maclaurin's series for

$Y(x) = x^2 + \sin x$

Solution:

First we find derivatives

$= Y'(x) = \frac{d}{dx}(x^2 + \sin x)$

$= Y'(x) = 2x + \cos x \rightarrow ①$

$= Y''(x) \frac{d}{dx}(2x + \cos x)$

$= Y''(x) = 2 - \sin x — ②$

$= Y'''(x) = \frac{d}{dx}(2 - \sin x)$

$= Y''(x) = -\cos x \longrightarrow ③$

Put $x = 0$ in above eq(1) eq(2) and eq (3)

$= Y(0) = (0)^2 + \sin(0)$

$Y(0) = 0$

$= Y'(0) = 2(0) + \cos(0)$

$Y'(0) = 0 + 1 = 1$

$= Y''(0) = 2 - \sin(0)$

$= Y''(0) = 2$

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$= y''' = -\cos(0)$

$y''' = -1$

By Maclaurins theorem.

$$f(x) = f(0) + x f'(0) \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} (0) + \cdots$$

$$f(x) = 0 + x(1) + \frac{x^2}{2}(2) + \frac{x^3}{6}(-1) + \cdots$$

$$f(x) = 0 + x + x^2 - \frac{x^3}{6} + \cdots$$

$$x^2 + \sin x = 0 + x + x^2 - \frac{x^3}{6} + \cdots$$

Answer.

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Question NO (03)

i) Find y'' given

$$1 + xy = x^2 + y^2$$

Solution:

$$1 + xy = x^2 + y^2$$

$$= \frac{d}{dx}(1 + xy) = \frac{d}{dx}(x^2 + y^2)$$

$$= \frac{d}{dx}(1) + \frac{d}{dx}(xy) = \frac{d}{dx}x^2 + \frac{d}{dx}y^2$$

$$= 0 + x\frac{dy}{dx} + y\frac{dx}{dx} = 2x^{2-1} + 2y^{2-1}\frac{dy}{dx}$$

$$= x\frac{dy}{dx} + y = 2x + 2y\frac{dy}{dx}$$

$$= x\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x - y$$

$$= \frac{dy}{dx}(x - 2y) = 2x - y$$

$$= \frac{dy}{dx} = \frac{2x - y}{x - 2y}$$

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$$= y' = \frac{dy}{dx} = \frac{2x-y}{x-2y}$$

$$= \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{2x-y}{x-2y}\right)$$

$$\frac{d^2y}{dx^2} = \frac{(x-2y)\frac{d}{dx}(2x-y) - (2x-y)\frac{d}{dx}(x-2y)}{(x-2y)^2}$$

$$= y'' = \frac{(x-2y)(2-y') - (2x-y)(1-2y')}{(x-2y)^2}$$

$$= y'' \frac{2x - xy - 4y + 2yy' - (2x - 4xy' - y + 2yy')}{(x-2y)^2}$$

$$= y'' \frac{\cancel{2x} - xy' - 4y + \cancel{2yy'} - \cancel{2x} + 4xy' + y - \cancel{2yy'}}{(x-2y)^2}$$

$$= y'' = \frac{4xy' - 3y - xy'}{(x-2y)^2}$$

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$$= y''' = \frac{4x(2x-y) - 3y(x-2y) - x[2x-y]}{(x-2y)(x-2y)^2}$$

$$= y'' = \frac{8x^2 - 4xy - 3xy + 6y^2 - 2x^2 + xy}{(x-2y)^3}$$

$$= y'' = \frac{6x^2 + 6y^2 - 6xy}{(x-2y)^3}$$

Answer

Question No (03)

(ii) Find y' by using logarithmic differentiation.

$$y = x^3 (1+x)^9 e^{6x}$$

Solution:

Taking ln on both sides

$$\ln y = \ln(x^3 (1+x)^9 . e^{6x})$$

$$= \ln y = \ln x^3 + \ln (1+x)^9 + \ln e^{6x}$$

$$= \ln y = 3 \ln x + 9 \ln (1+x) + 6x \ln e$$

Differentiate w.r.t (x)

$$= \frac{d}{dx} \ln y = \frac{d}{dx} 3 \ln x + \frac{d}{dx} 9 \ln(1+x) + \frac{d}{dx} 6x \ln e$$

$$= \frac{1}{y} \frac{dy}{dx} = \frac{3}{x} + \frac{9}{1+x} + \frac{1}{e^{6x}} \cdot 6$$

$$= \frac{dy}{dx} = y \left(\frac{3}{x} + \frac{9}{1+x} + \frac{6}{e^{6x}} \right)$$

$$= \frac{dy}{dx} = x^3 (1+x)^9 e^{6x} \left(\frac{3}{x} + \frac{9}{1+x} + \frac{6}{e^{6x}} \right)$$

Answer.