IQRA NATIONAL UNIVERSITY

FINAL ASSIGNMENT BS SOFTWARE ENGINEERING

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SEMESTER 2nd BS(SE)

SUBJECT: Linear Algebra

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Subject: L.A

Question 1: Determine if the following system is consistande or not.

$$\begin{aligned} x_1 - 8x_2 + x_3 &= 0 \\ 2x_2 - 8x_3 &= 8 \\ 5x_1 - 5x_3 &= 10 \end{aligned} \quad \text{Turning} \xRightarrow{A|b} \begin{bmatrix} 1 & -8 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & +5 & -5 & 10 \end{bmatrix}$$

Using Row operator:

$R_3 \leftarrow 2R_2 - R_3$

$\Rightarrow 2(0 \quad 2 \quad -8 \quad 8)$

$R_3 = (0 \quad 4 \quad -16 \quad 16) - R_3$

we get: $\begin{bmatrix} 1 & -8 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & 1 & -24 & 2 \end{bmatrix}$

Now

dividing 2 to R_2 and substracting it to R_3

$R_3 \leftarrow \frac{R_2}{2} - R_3$

$= \frac{1}{2}(0 \quad 2 \quad -8 \quad 8)$

$= (0 \quad 1 \quad -4 \quad 4)$

final Rg (2)

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Q1

weget:

$$\begin{bmatrix} 1 & -8 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & 0 & -28 & -2 \end{bmatrix}$$

As we can see perfect triangle
hence consistent.

$$\begin{aligned} x_3 \quad -8x_2 + x_3 &= 0 \\ 2x_2 - 8x_3 &= 8 \\ -28x_3 &= -2 \end{aligned}$$

Answer.

Question 2

find the inverse of

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 1 \\ 5 & -2 & 7 \end{bmatrix}$$ adjoint Metod

finding $|A|$:

$$|A| = \begin{vmatrix} 3 & 4 & 5 \\ 2 & -1 & 1 \\ 5 & -2 & 7 \end{vmatrix}$$

$= 3\times(-1)\times 7 + 4\times 1\times 5 + 5\times 2\times 2\times(-2) - 5 \times 5 - (-2)\times 1\times 3 - 7\times 2\times 4 = 7 - 46$

So the determine of $|A|$ is -46

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:Now finding Adj of A so we need to find Co-factor of normal Matrix

$$\begin{bmatrix} 3 & -4 & -5 \\ 2 & -1 & 1 \\ 5 & -2 & 7 \end{bmatrix}$$

$C_{1,1} = (-1)^{1+1} = [1 \times (-1 \times 7 - 1 \times (-2)] = -5$

$C_{1,2} = (-1)^{1+2} = [-1 \times (2 \times 7 - 1 \times 5) = -1 \times 9 = -9$

$C_{1,3} = (-1)^{1+3} = [1 \times (2 \times (-2) - (-1) \times 5] = 1$

$C_{2,1} = (-1)^{2+1} = [-1 \times (4 \times 7 - 5 \times (-2))] = -38$

$C_{2,2} = (-1)^{2+2} = [1 \times (3 \times 7 - 5 \times 5)] = -4$

$C_{2,3} = (-1)^{2+3} = [-1 \times (3 \times (-2) - 4 \times 5)] = 26$

$C_{3,1} = (-1)^{3+1} = [1 \times (4 \times 1) - 5 \times (-1)] = 9$

$C_{3,2} = (-1)^{3+2} = [-1 \times (3 \times 1 - 5 \times 2)] = -1 \times -7 = 7$

$C_{3,3} = (-1)^{3+3} = [+1 \times (3 \times (-1)) - 4 \times 2] = -11$

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Now we know to find inverse we have:

$$A^{-1} = \frac{1}{|A|} \times C^T \Rightarrow \text{formula}$$

Putting values and solving:

$$A^{-1} = \frac{1}{-46} \times \begin{bmatrix} -5 & -38 & 9 \\ -9 & +4 & 7 \\ 1 & 26 & -11 \end{bmatrix} \Rightarrow$$

$\cancel{\frac{5}{-46}} \quad \cancel{\frac{19}{-46}}$

$$\Rightarrow \begin{bmatrix} \frac{5}{46} & \frac{19}{23} & \frac{-9}{46} \\ \frac{9}{46} & \frac{2}{23} & \frac{-7}{46} \\ \frac{-1}{46} & \frac{-13}{23} & \frac{11}{46} \end{bmatrix}$$ [Answer]

Question 3: Solve the following system Guass-Jordan Method

$$2x + 2y + 4z = 18$$
$$x + 3y + 2z = 13$$
$$3x + 2y - 3z = 14$$

Q.3

$$2x + 2y + 4z = 18$$
$$x + 3y + 2z = 13$$
$$3x + 2y - 3z = 14$$

Sol

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{array}\right]; \frac{R_1}{2}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 3 & 2 & -3 & 14 \end{array}\right]; R_2 - R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 0 & -1 & -9 & -13 \end{array}\right]; R_3 - 3R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & -1 & -9 & -13 \end{array}\right]; R_2/2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & -1 & -9 & -13 \end{array}\right]; R_1 - R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -9 & -11 \end{array}\right]; R_3 + R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 11/9 \end{array}\right]; -\frac{R_3}{9}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 41/9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 11/9 \end{array}\right]; R_1 - 2R_3$$

So $x = 41/9$

$y = 2$

$z = 11/9$

Q4) Show that this matrix is Diagonisable.

$$\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

Sol :-

Matrix A is diagonisable if $A = CDC^{-1}$

$\det(A - \lambda I_3) = 0$

$$A - \lambda I_3 = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 4-\lambda & 2 & -2 \\ -5 & 3-\lambda & 2 \\ -2 & 4 & 1-\lambda \end{bmatrix}$$

$$4-\lambda \begin{vmatrix} 3-\lambda & 2 \\ 4 & 1-\lambda \end{vmatrix} - 2 \begin{vmatrix} -5 & 2 \\ -2 & 1-\lambda \end{vmatrix} - 2 \begin{vmatrix} -5 & 3-\lambda \\ -2 & 4 \end{vmatrix} = 0$$

$$4-\lambda\left[(3-\lambda)(1-\lambda)-8\right] - 2\left[(-5)(1-\lambda)+4\right] - 2\left[(-20)+2(3-\lambda)\right] = 0$$

$$4-\lambda\left[3-3\lambda-\lambda+\lambda^2-8\right] - 2\left[-5+5\lambda+4\right] - 2\left[-20+6-2\lambda\right] = 0$$

$$4-\lambda\left[\lambda^2-4\lambda-5\right]-2\left[5\lambda-1\right]-2\left[-14-2\lambda\right]=0$$

$$4\lambda^2+16\lambda-20-\lambda^3+4\lambda^2+5\lambda-10\lambda+28+4\lambda=0$$

$$-\lambda^3+8\lambda^2+15\lambda+8=0$$

$$\lambda = 9.65$$
$$\lambda = -0.82$$
$$\lambda = -0.829$$

for $\lambda = -9.65$

$$A-\lambda I_3 = \begin{bmatrix} -5.65 & 2 & -2 \\ -5 & -6.65 & 2 \\ -2 & 4 & -8.65 \end{bmatrix}$$

for $\lambda = -0.82$

$$A-\lambda I_3 \begin{bmatrix} 4.82 & 2 & -2 \\ -5 & 5.82 & 2 \\ -2 & 4 & 1.82 \end{bmatrix}$$

By solving only 2 ~~eigenspaces~~ eigenspaces or 2 basis vectors in total

So matrix A is not diagonalizable.

Q5) Determine if the following homogeneous system has a non trivial solution then describe the solution set.

$$3x_1 + 5x_2 - 4x_3 = 0 \quad \text{— (I)}$$
$$-3x_1 - 25x_2 + 4x_3 = 0 \quad \text{— (II)}$$
$$6x_1 + x_2 - 8x_3 = 0 \quad \text{— (III)}$$

Sol:-

$$A = \begin{bmatrix} 3 & 5 & -4 \\ -3 & -15 & 4 \\ 6 & 1 & -8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Add eq (I) & eq (II)

$$\begin{aligned} 3x_1 + 5x_2 - 4x_3 &= 0 \\ -3x_1 - 25x_2 + 4x_3 &= 0 \\ \hline -20x_2 &= 0 \end{aligned}$$

$$x_2 = 0$$

Add eq (I) & eq (III)

$$\begin{aligned} 3x_1 + 5x_2 - 4x_3 &= 0 \\ 6x_1 + x_2 - 8x_3 &= 0 \\ \hline 9x_1 + 6x_2 - 12x_3 &= 0 \end{aligned}$$

put $x_2 = 0$

$$9x_1 - 12x_3 = 0$$

$$9x_1 = 12x_3$$

$$x_1 = \frac{4}{3} x_3$$

So

$x_1 = 4/3\, x_3$

$x_2 = 0$

$x_3 = 0$

Q6) Reduce the matrix to Normal form & find its rank.

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Sol:-

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Reduc matrix to reduced row echelon form.

Swap matrix row $R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Cancel leading co-efficient in row R_2 by performing

$$R_2 \leftarrow R_2 - 1/3 R_1$$

$$\begin{bmatrix} 3 & 9 & 12 & 3 \\ 0 & 0 & 0 & 2 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Cancel leading co-efficient in Row R_3 by performing

$$R_3 \leftarrow R_3 - \frac{1}{3} R_1$$

$$\begin{bmatrix} 3 & 9 & 12 & 3 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Rank of a matrix is the number of all zero row

So

$$\text{Rand of } \begin{bmatrix} 1 & 3 & 4 & 0 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

$$\boxed{= 2}$$