ID # 12945

Page No (1)

Question No (1)

Find the eigenvalues of A

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{bmatrix}$$

Solution No (1)

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{bmatrix}$$

$$[A - \lambda I] = 0$$

$$\begin{vmatrix} (-\lambda) & 1 & 0 \\ 0 & (-\lambda) & 1 \\ 4 & -17 & (8-\lambda) \end{vmatrix} = 0$$

$$= (-\lambda)((-\lambda) \times (8-\lambda) - 1 \times (-17)) - 1(0 \times (8-\lambda)$$
$$- 1 \times 4) + 0(0 \times (-17)$$
$$- (-\lambda) \times 4) = 0$$

$$= (-\lambda)\left((-8\lambda + \lambda^2) - (-17)\right) - 1(0-4) + 0(0-(-4\lambda))$$
$$= 0$$

$$= (-\lambda)(17 - 8\lambda + \lambda^2) - 1(-4) + 0(4\lambda) = 0$$

$$= (-17\lambda + 8\lambda^2 - \lambda^3) - (4) + 0 = 0$$

$$= (-\lambda^3 + 8\lambda^2 - 17\lambda + 4) = 0$$

ID # 12945

Page No 3

$= -(\lambda - 4)(\lambda - 0.26794919)(\lambda - 3.73205081) = 0$

$(\lambda - 4) = 0$ or $(\lambda - 0.26794919) = 0$

The eigenvalues of the

matrix A are given

$\lambda = 0.26794919, 3.73205081, 4.$

$= \times = \times = \times$

Question No # 02

Find a Matrix P that diogonal the below matrix

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

Solution

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

ID # 12945

Page No # 5

Find eigenvalues of the Matrix A

$|A - \lambda I| = 0$

$$\begin{vmatrix} (-\lambda) & 0 & -2 \\ 1 & 2-\lambda & 1 \\ 1 & 0 & 3-\lambda \end{vmatrix} = 0$$

$= (-\lambda)(2-\lambda) \times (3-\lambda) - 1 \times 0) - 0(1 \times (3-\lambda) - 1 \times 1)$
$+ (-2) 1 \times 0 - (2-\lambda) \times 1) = 0$

$= (-\lambda)((6 - 5\lambda + \lambda^2) - 0) - 0((3-\lambda) - 1) - 2(0 - (2$
$- \lambda) = 0$

ID # 12945

page No # 6

$= \cancel{(-\lambda)(6-5\lambda^2}$

$= (-\lambda)(6-5\lambda+\lambda^2) - 0(2-\lambda) - 2 + \lambda) = 0$

$\cancel{= (-6\lambda}$

$= (-6\lambda + 5\lambda^2 - \lambda^3) - 0 - (-4 + 2\lambda) = 0$

$= (-\lambda^3 + 5\lambda^2 - 8\lambda + 4) = 0$

$= -(\lambda-1)(\lambda-2)(\lambda-2) = 0$

$= (\lambda - 1) = 0$ or $(\lambda - 2) = 0$

The eignvalues of matrix

~~A~~ $\lambda = 1, 2,$

ID # 18945

Page No 7

+ Eignvalve for $\lambda = 2$

$$V_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad V_3 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

The eignvalves compose the

colum of matrix P

$$P = \begin{bmatrix} -2 & 0 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

(1) The digonal matrix D is compsed of the eignvalues

ID # 12945

Page No (8)

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

3 Now find P^{-1}

$$\begin{vmatrix} -2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix}$$

$$= -2 \times \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + 0 \times \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} - 1 \times \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}$$

$$= -2 \times (1 \times 1 - 0 \times 0) + 0 \times (1 \times 1 - 0 \times 1) - 1 \times (1 \times 0 - 1 \times 1)$$

$$= -2 \times (1+0) + 0 \times (1+0) - 1 \times (0-1)$$

$$= -2 \times (1) + 0 \times (1) - 1 \times (-1)$$

$$= -2 + 0 + 1$$

$$= -1 \longrightarrow \text{Answer}$$

Question No (3)

Determinate whether the vactor From linear dependet or independet matrix

$V1 = (1, -2, 3)$

$V_2 = (5, 6, -1)$

$V_3 = (3, 2, 1)$

Solution No (3)

Here $A = (1, -2, 3)$

$B = (5, 6, 1)$

$C = (3, 2, 1)$

ID # 12945

page No # 10

The vactor A, B, and C are linearly dependet, if their determinate is zero

i-e $|D| = 0$

$$|D| = \begin{vmatrix} 1 & -2 & 3 \\ 5 & 6 & -1 \\ 3 & 2 & 1 \end{vmatrix}$$

$$= 1 \times \begin{vmatrix} 6 & -1 \\ 2 & 1 \end{vmatrix} + 2 \times \begin{vmatrix} 5 & -1 \\ 3 & 1 \end{vmatrix} + 3 \times \begin{vmatrix} 5 & 6 \\ 3 & 2 \end{vmatrix}$$

$$= 1 \times (6 \times 1 - (-1) \times 2) + 2 \times (5 \times 1 - (-1) \times 3) + 3 \times (5 \times 2 - 6 \times 3)$$

$$= 1 \times (6 + 2) + 2 \times (5 + 3) + 3 \times (10 - 18)$$

$$= 1 \times (8) + 2 \times (8) + 3 \times (-8)$$

ID # 12945

Page No # 11

$= 8 + 16 - 24$

$= 0$

Since $|D| - 0$

So vactor A, B, C are

linearly dependet.

$= \quad x \quad = \quad x \quad = \quad x$

ID # 12945

Page (12)

Answer No (4) Part (A)

Four main things of

vactor Space:

1) Commutative law:

For all vactor u and v in V

$$u + v = v + u$$

(2) Assocative law:

For all vactors u, v, w in V, $u + (v + w) = (u + v) + w$

ID # 12945

Page 13

(3) Additive law:

The set we contains an additive identity element denoted by 0 such that for any vector v in V

$0 + v = v$ and $v + 0 = v$.

(4) Additive inverse

For each vector v in V the equations $v + x = 0$ and $x + v = 0$ have solution x in V.

ID # 12945

Page No (14)

The set $p_n(x)$ of all the polynomials over R in variables x of degree 5 forms a vector space over R.

If $P_n(x) = a_0 + a_1 x + \cdots + a_n x^n$

&

$g_n(x) = b_0 + b_1 x + \cdots + b_n x^n$, a_i, b_i

Then $f_n(x) + g_n(x) = (a_0 + b_0) + (a_1 + b_1)x + \cdots + (a_n + b_n)$

x^n is a pollynomial of $P_n(x)$

the assositive additive property is induced from the additive, addition, assostive property of R.

The zero polynomial $f_0(n) = 0$ of degree zero act as The additive identity of $P_n(n)$

$\{ \quad -f_n(n) = 0 + (-a_1)_n + \cdots + (a_n)_{n'}$

is the additive inverse of $f_n(n)$

ID # 12945

Page no (16)

Commutative properties follows from the commutative property of R Hence $P_n(n)$ is an addolitive group.

The Scaler multiplication of $a \in R$ by $f_n(n)$ is defined by

$$a \cdot f_n(n) = aq_0 + (aq_1)x + (aq_2)x_2 \cdots + (aq_n)n^n \in P_n(x)$$

If Observe proporties 3.3.3 b (1-iv) of Scaler multiple which can easy be verified So what $P_n(n)$ formes a vector space

ID # 12945
Page No # 17

Answer (4) B

Let f be a fields
a non- empty set we
togather with two binary
Operation (+) & (-) from
algebra Structure at
vector Space our the
field F. if;

v From an additive (+ve)
ablelign group.

(b) The Scaler multiplication
(·) as the function
from F x V into v Observes

ID # 12945

Page No # 18

the following properties

(1) $\forall \alpha \in F, x, y \in V, \alpha(x+y) = \alpha x + \alpha y$

2) $\forall \alpha \beta \in F, (\alpha + \beta) \cdot x = \alpha x + \beta x \cdot \forall x \in V$.

3) $\forall \alpha, \beta \in F, \forall x \in V; \alpha(\beta x) = (\alpha \beta) x$

(4) For e_F, the indentity of

F $e_f \cdot x = x, \forall x \in V$.

——— x ——— x ———