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SAAD KHAN

7300

Probability & Statistics

Q.1 Calculate the correlation coefficient b/w X and Y.

X	3	4	5	6	7	8	9	10	11	13
Y	25	24	20	20	19	17	16	13	10	8

Sol:-

correlation coefficient,  $r$

;  $n = 10$

$$r = \frac{n\sum xy - (\sum x)(\sum y)}{\sqrt{n\sum x^2 - (\sum x)^2} \sqrt{n\sum y^2 - (\sum y)^2}}$$

x	y	xy	x <sup>2</sup>	y <sup>2</sup>
3	25	75	9	625
4	24	96	16	576
5	20	100	25	400
6	20	120	36	400
7	19	133	49	361
8	17	136	64	289
9	16	144	81	256
10	13	130	100	169
11	10	110	121	100
13	8	104	169	64
$\Sigma$	76	1148	670	3240

$$r = \frac{(10)(1148) - (76)(172)}{\sqrt{(10)(670) - (76)^2} \sqrt{(10)(3240) - (172)^2}}$$

$$= \frac{-1592}{(30.39)(53.06)}$$

$$r = -0.98$$

Q16

X	20	11	15	10	17	18	21	25	28
Y	5	15	14	17	8	9	12	16	18

Sol.:-

X	Y	X <sup>2</sup>	Y <sup>2</sup>	XY
20	5	400	25	100
11	15	121	225	165
15	14	225	196	210
10	17	100	289	170
17	8	289	64	136
18	9	324	81	162
21	12	441	144	252
25	16	625	256	400
28	18	784	324	504
165	114	3309	1604	2099

Regression line for Y on X :-

$n = 9$

$$b = \frac{n\sum XY - \sum X \sum Y}{n\sum X^2 - (\sum X)^2}$$

$$= \frac{9(2099) - (165)(114)}{9(3309) - (165)^2}$$

$$b = 0.03169$$

$$a = \left(\frac{\sum Y}{n}\right) - (0.03169)\left(\frac{\sum X}{n}\right)$$

$$= \frac{114}{9} - (0.03169)\frac{165}{9}$$

$$a = 12.0856$$

$$\hat{Y} = 12.0856 - 0.03169x$$

Regression line for X on Y :-

$$b = 0.03169$$

$$a = \frac{\sum x}{n} - (0.03169) \frac{\sum y}{n}$$

$$= \frac{165}{9} - (0.03169) \left( \frac{114}{9} \right)$$

$$a = 17.9319$$

$$\hat{X} = 17.9319 - 0.03169y$$

Q.3

a) Construct the ungrouped frequency distribution

Class	frequency	Tally
0	1	
1	4	
2	6	<del>    </del>
3	12	<del>    </del> <del>    </del>
4	8	<del>    </del>
5	5	<del>    </del>
6	5	<del>    </del>
7	3	
8	2	==
9	1	
10	3	
Total	50	

b) Construct grouped frequency distribution

Class limit	frequency	Tally
0-2	11	
3-5	25	       
6-8	10	
9-11	04	
	50	

Q.2

a) Problem State ment

Sol:-

$$n = 5$$

$$P = \frac{1}{2}, q = \frac{1}{2}$$

$$\bar{x} = \text{mean} = np$$

$$S.D = \sqrt{npq}$$

$$P(x = \text{No. of heads}) = ?$$

$$P(x = x) = ?$$

$$P(x = x) = \binom{n}{x} P^x q^{n-x}$$

$$P(x=0) = \binom{5}{0} \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{5-0} = 0.03125$$

$$P(x=1) = \binom{5}{1} \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^4 = 0.15625$$

$$P(x=2) = \binom{5}{2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^3 = 0.3125$$

$$P(x=3) = \binom{5}{3} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2 = 0.3125$$

$$P(x=4) = \binom{5}{4} \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^1 = 0.15625$$

$$P(x=5) = \binom{5}{5} \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^0 = 0.03125$$

Now to find  $np$  and  $npq$

We must find  $p$  &  $q$

$$n=5, \quad p=\frac{1}{2}, \quad q=\frac{1}{2}$$

$$\bar{x} = np = 5 \times \frac{1}{2} = 2.5$$

$$S.D. = \sqrt{npq} = \sqrt{5 \times \frac{1}{2} \times \frac{1}{2}} = \sqrt{\frac{5}{4}} = 1.1180$$

2 b) Problem statement

Sol:-

Total games = 10

$$P(A \text{ will win}) = \frac{2}{3}$$

① at least 4 games  $[P(x \geq 4)]$

$$\therefore P(x \geq 4) = 1 - P(x < 4)$$

$$= 1 - [P(x=0) + P(x=1) + P(x=2) + P(x=3)]$$

$$= 1 - \left[ \binom{10}{0} \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^{10} + \binom{10}{1} \left(\frac{2}{3}\right)^1 \left(\frac{1}{3}\right)^9 + \binom{10}{2} \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^8 + \binom{10}{3} \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^7 \right]$$

$$= 1 - [0.0000169 + 0.0003387 + 0.0030483 + 0.0162576]$$

$$P(x \geq 4) = 0.9803385$$

ii) exactly 4 games :  $[P(x=4)]$

$$P(x=4) = \binom{10}{4} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^6$$

$$P(x=4) = 0.0569018$$

iii) exactly 11 games :  $[P(x=11)]$

$$P(x=11) = 0$$

iv) 6 or more games :  $[P(x \geq 6)]$

$$P(x \geq 6) = \binom{10}{6} \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^4 + \binom{10}{7} \left(\frac{2}{3}\right)^7 \left(\frac{1}{3}\right)^3 + \binom{10}{8} \left(\frac{2}{3}\right)^8 \left(\frac{1}{3}\right)^2 + \binom{10}{9} \left(\frac{2}{3}\right)^9 \left(\frac{1}{3}\right)^1 + \binom{10}{10} \left(\frac{2}{3}\right)^{10} \left(\frac{1}{3}\right)^0$$

$$= 0.2276075 + 0.2601229 + 0.1950922 + 0.0867076 + 0.0173415$$

$$P(x \geq 6) = 0.7868717$$