Iqra National University, Peshawar
Department of Electrical Engineering

Final – Term Examination summer2020
Date:24/9/2020

Course Code: MTH 101 **Course Title:** Linear Algebra
Prerequisite: NA **Instructor:** HIMAYTULLAH
Module: 1 **Program:** BEE **Total Marks:** 50 **Time Allowed:**

Note: Attempt all questions.PLO: program learning outcome C:Cognitive

Q1.	(a)	. **Express** the equation of plane passing through the points A(2,-2,1) , B(-1,0,3), C(5,-3,4)	Marks 5 PLO2 C2
	(b)	**Express** a pair of planes whose intersection is the given line, $x = 2 - 3t,\ y = 3 + t\ , z = 2 - 4t$	Marks 5 PLO2 C2
Q2		. $L(x, y) = (x + 1,\ \ y,\ \ x + y)$ **illustrate** that L is linear transformation ?	Marks 10 PLO1 C3
Q3		Using the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ then **interpret** to decode the message 77 54 38 71 49 29 68 51 33 76 48 40 86 53 52	Marks 10 PLO1 C3
Q4		Find an equation of the plane passing through the point (-1, 3, 2) and perpendicular to the vector n = (0, 1, -3)	Marks 10 C3 PLO1
Q5		Find an Eigen values and Eigen vectors of matrix $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$.	Mark10 c3 plo1

FAWAD AHMAD (13204)

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Ques # 01 (A)

Sol:-

The Non Parallel Vectors

$\vec{AB} = (-3, 2, 2)$

$\vec{AC} = (3, -1, 3)$

the Perpendicular Vector is

$$n = \vec{AB} \times \vec{AC}$$

$$n = \begin{vmatrix} i & j & k \\ -3 & 2 & 2 \\ 3 & -1 & 3 \end{vmatrix}$$

$$n = i(6+2) - j(-9-6) + k(3-6)$$

$$n = 8i + 15j - 3k$$

$$n = (8, 15, -3)$$

Now $A(x, y, z) = (2, -2, 1)$

$$n(a, b, c) = (8, 15, -3)$$

So Solution Plane

$$a(x-x_0)+b(y-y_0)+c(z-z_0)$$

$$8(x-2)+15(y+2)-3(z-1)=0$$

$$8x+15y-3z-16+30+3=0$$

$$\boxed{8x+15y-3z+17=0}$$

(8)

Ques # 1 (B)

Solution :-

$$x = 2 - 3t$$

$$y = 3 + t$$

$$z = 2 - 4t$$

$\Rightarrow$

$$x - 2 = -3t$$

$$t = \frac{x-2}{-3}$$

$$y - 3 = t$$

$$t = \frac{y-3}{1}$$

$$z - 2 = -4t$$

$$t = \frac{z-2}{-4}$$

So $\frac{x-2}{-3} = \frac{y-3}{1} = \frac{z-2}{-4}$

for 1st Plane Take 1st & 2nd

$$\frac{x-2}{-3} = \frac{y-3}{1}$$

$\rightarrow \quad x-2=-y+9$

$\rightarrow \quad x+3y-11=0$

For 2nd Plane Take 1st & 3rd Eq —.

$$\frac{x-2}{-3}=\frac{z-2}{-4}$$

$$-4x+8=-3z+6$$

or

$$\boxed{4x-3z-2=0}$$

Ques # 2

Sol :-

$$L(x,y) = (x+1, y, x+y)$$

Let $u = (x_1, y_1)$ $v = (x_2, y_2)$

$$u+v = (x_1, y_1) + (x_2, y_2)$$

$$u+v = (x_1+x_2, y_1+y_2)$$

$$L(u+v) = L(x_1+x_2, y_1+y_2)$$

$$L(u+v) = (x_1+x_2+1, y_1+y_2, x_1+x_2+y_1+y_2)$$

given That $u = (x+2y_1)$

$$L(u) = L(x_1, y_1) = (x_1+1, y_1, x_1+y_1)$$

$$L(v) = L(x_2, y_2) = (x_2+1, y_2, x_2+y_2)$$

$$L(u)+L(v) = (x_1+x_2+2, y_1+y_2, x_1+x_2+y_1+y_2)$$

$\hookrightarrow$ (ii)

since $1 \neq 2$

So this is Not L.T

Ques #03

~~Sol~~ $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

77, 54, 38, 71, 49, 29
68, 51, 33, 76, 48, ~~80~~ 40
86, 53, 52

Sol:-

$$A^{-1} = \frac{1}{|A|} \text{Adj } A$$

So

$$A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix} = \begin{bmatrix} 16 \\ 8 \\ 15 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 71 \\ 49 \\ 29 \end{bmatrix} = \begin{bmatrix} 20 \\ 15 \\ 7 \end{bmatrix}$$

$$x_3 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 68 \\ 51 \\ 33 \end{bmatrix} = \begin{bmatrix} 18 \\ 1 \\ 16 \end{bmatrix}$$

$$x_4 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 76 \\ 48 \\ 40 \end{bmatrix} = \begin{bmatrix} 8 \\ 16 \\ 12 \end{bmatrix}$$

$$x_5 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 86 \\ 53 \\ 52 \end{bmatrix} = \begin{bmatrix} 1 \\ 14 \\ 19 \end{bmatrix}$$

So

16 8 15 20 15 7 18 1 16 8 16 12
P H O T O G R A P H P L

1 14 19
A N S

PHOTOGRAPH PLANS

Que # 04

Sol :-

$(-1, 3, 2) \quad n = (0, 1, -3)$

Equation of the Plane

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

Given That $P(x_0, y_0, z_0) = (-1, 3, 2)$

$n(a, b, c) = (0, 1, -3)$

Putting Values in Eq ①

$$0(x - (-1)) + 1(y - 3) + (-3)(z - 2)$$

$$= 0 + 1(y - 3) - 3z + 6$$

$$= 0 + y - 3 - 3z + 6$$

$$\boxed{= 0x + y - 3z + 3}$$

Que # 05

Sol :-

We Know That $Ax = \lambda x$

$$\begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 + x_2 \\ 2x_1 + 4x_2 \end{bmatrix} = \begin{bmatrix} \lambda x_1 \\ \lambda x_2 \end{bmatrix}$$

Then

$$x_1 + x_2 = \lambda x_1 \rightarrow (i)$$

$$-2x_1 + 4x_2 = \lambda x_2 \rightarrow (ii)$$

So $x_1 - \lambda xi + x_2 = 0$

$= (1-\lambda)x_1 + x_2 = 0$

& $-2x_1 + \overset{\lambda}{\cancel{4}}x_2 = 0$

$= -2x_1 + (4-\lambda)x_2 = 0$

$$\begin{bmatrix} 1-\lambda & 1 \\ -2 & 4-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

characteristic Eqn.

$$\begin{bmatrix} 1-\lambda & 1 \\ -2 & 4\lambda \end{bmatrix} = 0$$

$$(1-\lambda)(4-\lambda)+2=0$$

$$4-\lambda-4\lambda+\lambda^2+2=0$$

$$\lambda^2-5\lambda+6=0$$

$$\lambda^2-3\lambda-2\lambda+6=0$$

$$\lambda(\lambda-3)-2(\lambda-3)=0$$

$$(\lambda-3)(\lambda-2)=0$$

$$\lambda-3=0 \quad , \lambda-2=0$$

$\lambda=3$, $\lambda=2$ are Eigen Values

Now find Eigen Vectors of $\lambda_1=3$ Put in Eq ① & ②

Then $x_1+x_2=3x_1 \longrightarrow$ ①

$$= -2x_1+x_2=0$$

$$\Rightarrow 2x_1 - x_2 = 0$$

$$\Rightarrow -2x_1+4x_2=3x_2 \longrightarrow \text{(ii)}$$

$$\Rightarrow -2x_1+x_2=0$$

$$\Rightarrow 2x_1-x_2=0$$

$x_1 = \frac{1}{2} x_2$

let $x_2 = \gamma$ Where $\gamma \neq 0$

So $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\gamma \\ \gamma \end{bmatrix}$

Eigen Vector for $\lambda_2 = 2$ Put in (1) & (2)

$x_1 + x_2 = 2x_1 \rightarrow (i)$

$-2x_1 + 4x_2 = 2x_2 \rightarrow (ii)$

$= -x_1 + x_2 = 0 \rightarrow (i)$

$\Rightarrow x_1 - x_2 = 0$

$\Rightarrow x_1 = x_2$

$= -2x_1 + 4x_2 = 2x_2 \rightarrow (ii)$

$= -2x_1 + 2x_2 = 0$

$\Rightarrow x_1 - x_2 = 0$

$\Rightarrow x_1 = x_2$

$x_1 = \gamma$ Then $x_2 = \gamma$

So $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{pmatrix} \gamma \\ \gamma \end{pmatrix}$