

Q1:- Construct a grouped distribution table for the following data and calculate Mean, Mode Median and Quartiles.

Sol:- The lowest value is 363 and the highest value is 431.

using given data

A class interval of 10.

The interval for the first class is 360 to 369 and include 363 (the lowest value).

The complete distribution table.

Observation	Tally	Freq
360-369		2
370-379		3
380-389		5
390-399		7
400-409		5
410-419		4
420-429		3
430-439		1
		Total Freq = 30

Now calculate Mean

Q1. Putting the values

$$\text{Mean} = \frac{\text{Sum of all number}}{\text{total number}}$$

$$\begin{aligned} \text{Sum of no} &= 423 + 369 + 387 + 411 + 493 + 397 + 371 + 377 + 389 \\ &+ 409 + 392 + 408 + 431 + 401 + 363 + 391 + 405 + 382 + \\ &400 + 381 + 379 + 415 + 428 + 422 + 396 + 372 + 410 \\ &+ 419 + 386 + 390 = 11917 \end{aligned}$$

$$\text{Sum} = 11917$$

Now Putting values

$$\text{Mean} = \frac{11917}{30} = 397.1$$

$$\text{Mean} = 397.1$$

Mode:

$$\frac{L + \frac{f_m - f_{m-1}}{(f_m - f_{m-1}) + (f_m - f_{m+1})} \times w$$

↳ L is Lower class boundary

↳  $f_{m-1}$  is the freq of group before modal group

↳  $f_{m+1}$  is the freq of group after modal

↳ w is the group width.

$$M = 389.5 + 8 - 3/2(8) - 3 - 5(395.5 - 389.5)$$

$$= 389.5 + 5/16 - 8(11)$$

$$= 389.5 / 1455/8$$

$$= 3131/8$$

$$= 3918$$

$$\text{Mode} = 3918$$

Q1 continueMedian:-

$$L = 389.5 \text{ (Lower class boundary of } (390-399))$$

$$n = 30$$

$$B = 2+3+5 = 10$$

$$G = 7$$

$$w = 10$$

$$= 389.5 + \frac{(30/2) - 10}{7} \times 10$$

$$= 389.5 + \frac{(15 - 10)}{7} \times 10$$

$$= 389.5 + 0.7143$$

$$= \boxed{390.21}$$

Quartile:-

$$1 + h/f(v-c)$$

$$Q = n/4 = 30/4 = 7.5$$

$$Q1 = 389.5 + 11/3 (7.5 - 7)$$

$$Q1 = 389.5 + 5.5/3$$

$$Q2 = 11535.55/3 + 5.5/3 = 1148/3 = 382.66 + 1+h / 7(v_3-c)$$

$$Q3 = 406.5 + 11/5 (22.5 - 20)$$

$$Q3 = 406.5 + 11/5 (2.5)$$

$$Q3 = 406.5 + 27.5/5$$

$$= 2032.5 + 27.5/5$$

$$= 2060/5 = \boxed{412}$$

Q2 By multiplying each of the numbers 3, 6, 2, 1, 7, 5 by 2 and then 5, we obtain 11, 17, 9, 7, 19, 15. What is the relation between the standard deviation and the means of the two sets.

Ans:-

$$\text{Mean} = \frac{3+6+2+1+7+5}{6} = \frac{24}{6} = 4$$

$$x = 4$$

$x_i$	$x_i - x$	$(x_i - x)^2$	$x_i^2$
3	$3-4=1$	1	9
6	$6-4=2$	4	36
2	$2-4=-2$	4	4
1	$1-4=-3$	9	1
7	$7-4=3$	9	49
5	$5-4=1$	1	25

$$S = \sqrt{\sum (x_i - x)^2 / n}$$

$$= \sqrt{\frac{28}{6}} = \sqrt{4.66} = 2.16$$

$$\text{Mean} = \frac{11+17+9+7+19+15}{6} = \frac{78}{6} = 13$$

$x_i$	$x_i - x$	$(x_i - x)^2$
11	$11-13 = -2$	4
17	$17-13 = 4$	16
9	$9-13 = -4$	16
7	$7-13 = -6$	36
19	$19-13 = 6$	36
15	$15-13 = 2$	4

Q2 continue

$$S = \sqrt{\sum (x_i - \bar{x})^2 / n}$$

$$= \sqrt{112/6} = \sqrt{112/6} = \sqrt{18.66}$$

$$S = 4.32$$

Q3 For the following grouped distribution table calculate the variance and standard deviation.

Class	64-84	85-104	105-124	125-144	145-164	165-184	185-204
Frequency	15	18	27	10	6	5	13

Sol<sup>n</sup>:-

Variance Formula

$$S^2 = \frac{\sum (x_i - \bar{x})^2}{n}$$

Class limits	f	x	fx	$\bar{x}$	$x - \bar{x}$	$(x - \bar{x})^2$	$f(x - \bar{x})^2$
64-84	15	74	1110	123.14	49.14	2414.7	36220.5
85-104	18	94.5	1701	123.14	105.14	11054.41	198979.38
105-124	27	114.5	3091.5	123.14	96.14	9242.8	166370.4
125-144	10	134.5	1345	123.14	113.14	12800.6	128006
145-164	6	154.5	927	123.14	117.14	13721.7	82330.2
165-184	5	174.5	872.5	123.14	118.14	13957	69785
185-204	13	194.5	2528.5	123.14	110.14	12130.8	157700.4
	<u>94</u>		<u>11575.5</u>				<u>839391.88</u>

$$\bar{x} = 11575.5 / 94$$

$$= 123.14$$

$$S^2 = 839391.88 / 93$$

$$= 9025.71$$

Variance

$$S = \sqrt{9025.71}$$

$$= 95.003$$

Standard deviation

Q4:- If two fair dice are thrown, what is the Probability of getting

1. A double six
2. A sum of 8 or more dots.

Sol:- Probability - Sample space for two dice (outcomes)

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

~~Getting a sum of 8~~

~~Let  $E_8$  = event of getting a sum of 8. The number which is a sum of 8 will be~~

~~$$E_8 = [(2,6), (3,5), (4,4), (5,3), (6,2)] = 5$$~~

~~Therefore, Probability of getting 'a sum of 8'~~

~~$$P(E/8) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$~~

~~$$P(E/8) = 5/36$$~~

~~$$P(E/8) = 0.138$$~~

Q4 continueA double Six

There are 36 possible outcomes

Let A be the event that double six occurs

$A = \{(6,6)\}$  and thus

$$P(A) = \frac{1}{36}$$

②

A sum of 8 or more dots

Let B denotes that a sum of 8 or more dots occurs.

$B = \{(2,6), (3,5), (3,6), (4,4), (4,5), (4,6), (5,3), (5,5), (5,6), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

Hence

$$P(B) = \frac{15}{36} = \frac{5}{12}$$



Q5:- Let  $C_1, C_2, \dots, C_M$  be a partition of the sample space  $S$ , and  $A$  and  $B$  be two events. Suppose we know that

\*  $A$  and  $B$  are conditionally independent given  $C_i$ , for all  $i \in \{1, 2, \dots, M\}$

\*  $B$  is independent of all  $C_i$ 's.

Prove that  $A$  and  $B$  are independent.

Sol:-

Since the  $C_i$ 's form a partition of the sample space, we can apply the law of total probability for  $A \cap B$ :

$$\begin{aligned}
 P(A \cap B) &= \sum_{i=1}^M P(A \cap B | C_i) P(C_i) \\
 &= \sum_{i=1}^M P(A | C_i) P(B | C_i) P(C_i) \quad (\text{A and B are conditionally independent}) \\
 &= \sum_{i=1}^M P(A | C_i) P(B) P(C_i) \quad (B \text{ is independent of all } C_i \text{'s}) \\
 &= P(B) \sum_{i=1}^M P(A | C_i) P(C_i) \\
 &= P(B) P(A) \quad \text{law of total Probability.}
 \end{aligned}$$