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Course Differential equation:

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a, Estimate general solution of $4y''-20y'+25y=0$

Sol: A second order linear homogeneous ODE has form of $ay''+by'+cy=0$

Now putting $y=e^{rt}$

$$4(e^{rt})''-20(e^{rt})'+25(e^{rt})=0$$

$$\Rightarrow e^{rt}(4y^2-20y'+25)=0$$

using Quadratic formula

$a=4$, $b=-20$, $c=25$

$$y=\frac{-(-20)\pm\sqrt{(-20)^2-4(4)(25)}}{2(4)}$$

$$y=\frac{20\pm\sqrt{400-400}}{8}$$

$$y=\frac{20\pm\sqrt{0}}{8}=\frac{20}{8}\ \frac{5}{2} \qquad y=5/2$$

$$\Rightarrow y=e^{rt} \Rightarrow 4r^2-20r+25=0$$

$$\Rightarrow y=c_1y_1+c_2y_2=c_1e^{5/2t}+c_2te^{5/2t}$$

$$y'=5/2\,c_1e^{5/2t}+c_2\left(e^{5/2t}+5/2\,te^{5/2t}\right).$$

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Q2 Calculate the initial value problem

$$y'' + 2y' + y = 0 \qquad y(0) = 4, \quad y'(0) = -6$$

Sol:

$$y'' + 2y' + y = 0$$

$$\frac{d^2 y(x)}{dx^2} + 2\frac{d\, y(x)}{dx} + y(x) = 0$$

Such that $y(0) = 4$, $y'(0) = -6$.

substitute $y(x) = e^{\lambda x}$ into the differential equation.

$$\frac{d^2}{dx^2}\left(e^{\lambda x}\right) + 2\,\frac{d}{dx}\left(e^{\lambda x}\right) + e^{\lambda x} = 0$$

Substitute $\frac{d^2}{dx^2}\left(e^{\lambda x}\right) = \lambda^2 e^{\lambda x}$ and $\frac{d}{dx}\left(e^{\lambda x}\right) = \lambda e^{\lambda x}$

$$\lambda^2 e^{\lambda x} + 2\lambda e^{\lambda x} + e^{\lambda x} = 0$$

factor out $e^{\lambda x}$

$$\left(\lambda^2 + 2\lambda + 1\right) e^{\lambda x} = 0$$

Since $e^{\lambda x} \neq 0$ for any finite λ, the zeros must come from the Polynomial.

$$\lambda^2 + 2\lambda + 1 = 0$$

factor $\quad (\lambda + 1)^2 = 0$

Q2 (a)

solve for λ.

$\lambda = -1$. or $\lambda = -1$

solve for the unknow constant using the intial conditions.

compute $\frac{dy(x)}{dx}$.

$$\frac{dy(x)}{dx} = \frac{d}{dx}\left(c_1 e^{-x} + c_2 e^{-x} x\right).$$

$$= -c_1 e^{-x} + c_2 e^{-x} - c_2 e^{-x} x$$

substitute $y(0) = 4$ into $y(x) = e^{-x} c_1 + e^{-x} x c_2$

substitute $y'(0) = -6$ into $\frac{dy(x)}{dx} = -e^{-x} c_1 + e^{-x} c_2 - e^{-x} x c_2$

$$-c_1 + c_2 = -6$$

solve the system

$c_1 = 4$

$c_2 = -2$.

substitute $c_1 = 4$ and $c_2 = -2$ into $y(x)$

$$y(x) = e^{-x} c_1 + e^{-x} x c_2$$

$$\boxed{y(x) = -2e^{-x}(x-2)}$$ Ans.

Q2 (B) Analyze the general Solution of

$$x^2 y'' + 3xy' + y = 0$$

sol: Put $y = x^r$

$$x^2 (x^r)'' + 3x(x^r)' + x^r = 0$$

$$\Rightarrow x^r (x^2 + 2r + 1) = 0$$

using Quadratic Formula

$a = 1$, $b = 2$, $c = 1$

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4-4}}{2}$$

$$x = -2/2 \qquad \boxed{x = -1}$$

$$y = c_1 x^r + c_2 \ln(x) x^r$$

$$\Rightarrow c_1 x^{-1} + c_2 \ln(x) x^{-1}$$

$$y = \frac{c_1}{x} + \frac{c_2 \ln(x)}{x}$$ Ans

Q3 Examine the method of undetermined Coefficient method for.

$$y'' + y' - 6y = 6x^3 - 3x^2 + 12x.$$

sol.

$$y'' + y' - 6y = 0.$$

Auxiliary eqn

$$\lambda^2 + \lambda - 6 = 0$$

$$\lambda^2 + 3\lambda - 2\lambda - 6 = 0$$

$$\lambda + 3 = 0, \quad \lambda - 2 = 0$$

$$\lambda = -3, \quad \lambda = 2$$

Roots are Real and distant.

$$y = c_1 e^{-3x} + c_2 e^{2x}$$

choice for y_p

$$y_p = k_3 x^3 + k_2 x^2 + k_1 x + k_0$$

$$y_p' = 3k_3 x^2 + 2k_2 x + k_1$$

$$y_p'' = 6k_3 x - 2k_2 \qquad \text{put in eq (i)}$$

$$6k_2 x - 2k_2 + 3k_3 x^2 + 2k_2 x + k_1 - 6k_3 x^3 - 6k_2 x^2 - 6k_1 x - 6k_0$$
$$= 6x^3 - 3x^3 + 12x$$

Comparing

$$-6k_3 = 6, \quad -6k_2 + 3k_1 = -3, \quad 6k_3 x + 2k_2 + k = n$$

Q3

$\boxed{k_3 = -1}$, $-6k_2 + 3(-1) = -3$, $6(-1) + 2(0) + k_2 = 12$

$-6k_2 - 3 = -3$, $-6 + k_1 = 12$

$-6k_2 = -3 + 3$

$k_1 = -2$

$-6k_2 = 0$

$\boxed{k_2 = 0}$

$-2k_2 + k_1 + k_0 = 0$

$-2(0) - 2 + k_0 = 0$

$\boxed{k_0 = 2}$

Q4 Examine the method of variation of Parameters for $y'' - 4y' + 4y = x^2 e^{2x}$

Sol:

$$y'' - 4y' + 4y = 0$$

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda^2 - 2\lambda - 2\lambda + 4 = 0$$

$$\lambda(\lambda - 2) - 2(\lambda - 2) = 0$$

$$(\lambda - 2)(\lambda - 2) = 0$$

$$\lambda = 2, \quad \lambda = 2$$

Root are Real and equal.

$$y = (c_1 + c_2 x)e^{2x}$$

$$y = c_1 e^{3x} + c_2 x e^{2x}$$

$$y_1 = e^{2x}, \quad y_2 = x e^{2x}$$

$$y_1' = 2e^{2x}, \quad y_1 = e^{2x} + 2xe^{2x}$$

Q5 Idetify an ODE $y'' + ay' + by = 0$

for the basis $1, e^{-3x}$

Sol: $1, e^{3x}$

$y = c_1 e^{x} + c_2 e^{-3x}$

$\lambda = 0, \quad \lambda = -3$

$\lambda - 0 = 0, \quad \lambda + 3 = 0$

$\lambda(\lambda + 3) = 0$

$\lambda^2 + 3\lambda = 0$

$a = 3, \quad b = 0$

$\lambda'' - 3y' + 0y = 0$

$y'' - 3y' = 0$

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