IQRA NATIONAL UNIVERSITY PESHAWAR

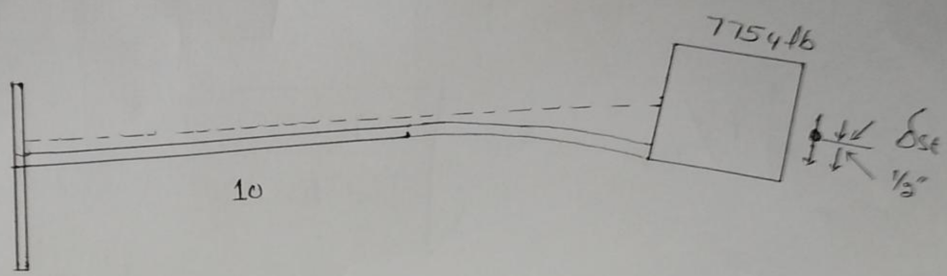
DEPARTMENT OF CIVIL ENGINEERING

SUBJECT: STRUCTURAL DYNAMICS AND EARTHQUAKE
SEMESTER: 8TH

NAME: SHAHID SHERDAD
ID: 7754
SECTION: B

Q.NO (01) ANSWER:

① Problem # 01



Sol: The general EOM For SDOF system is

$$ku + c\dot{u} + m\ddot{u} = P(t)$$

In our system is undamped $(c = 0)$ undergoing free vibration $(P(t) = 0)$

Hence EOM become $ku + m\ddot{u} = 0$ — ①

$$K = 3EI/L^3$$

$$= \frac{3 \times 29000\ k/in^2 \times 150\ in^4}{(10 \times 12)\ in^3} = 7.55\ k/in$$

In order to eliminate the chances of mistake during calculation it is more appropriate to use fundamental units like lb, ft sec or kg m, sec

(2)

$K = 7.55\ K/m = 90625\ lb/ft.$

$m = \frac{7754\ lbsec^2}{32.2ft} = \boxed{240.80\ slug}$

$\omega_n = \sqrt{K/m} = \sqrt{\frac{90625}{240.80}} = 19.39\ rad/sec$

$T_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{19.39} = \boxed{0.323\ sec}$

Substiting the corresponding values in eq (1).

$90625u + 240.80\ddot{u} = 0$

where "K" is in lb/ft and "m" is an lbsec/ft² general solution to EOM for undamped free vibration is

$u(t) = 1/24'' = 1/24\ ft$ and $\dot{u}(0) = 0$

Equivalent static force at any time (t) is

$f_s(t) = K.u(t) = \frac{90625\cos(19.39t)}{24}$

$F_s(t) = 3776.04\cos(19.39t).$

(3)

Ampitude of dynamic displacement u_0 for.

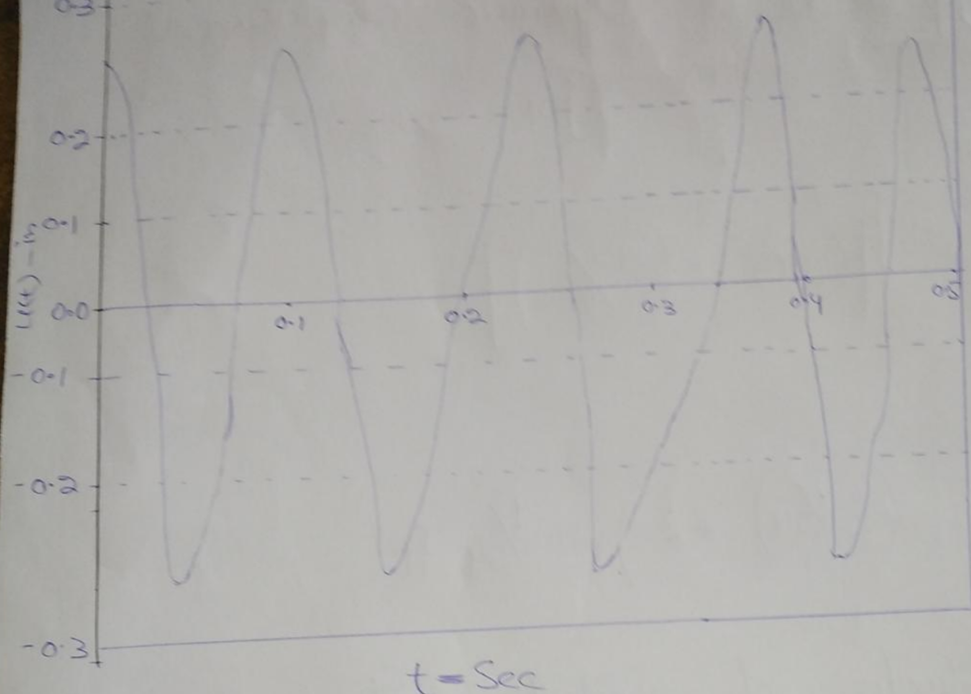
$$u_0 = \sqrt{\left[\left(u(0)^2\right) + \left(\dot{u}(0)/\omega_n\right)^2\right]} =$$

$$= \sqrt{\left(\left(1/24\right)^2 + 0\right)} = 1/24 \text{ ft}$$

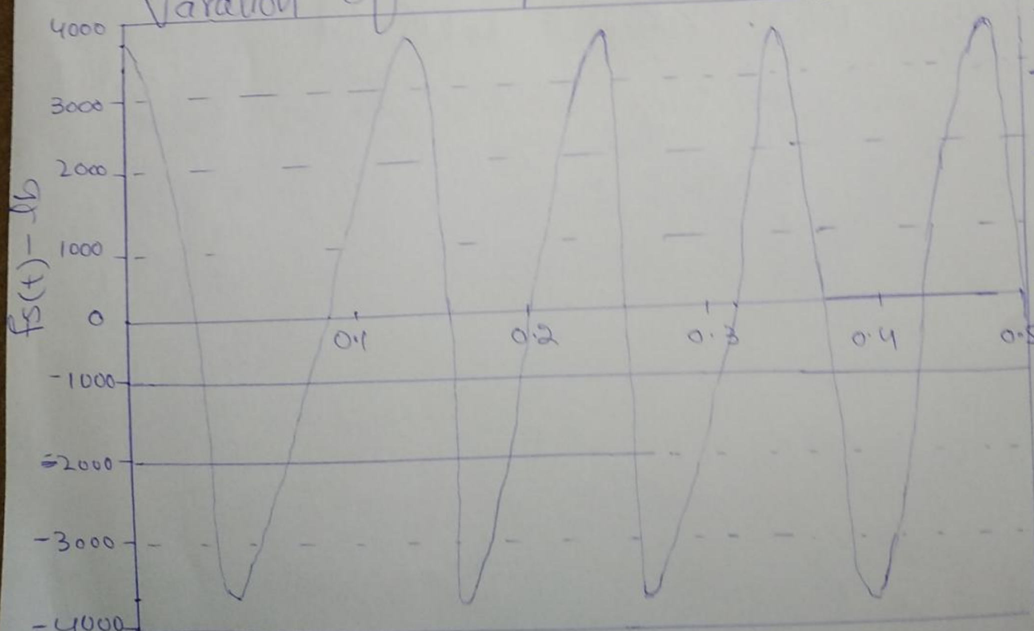
Ampitudee of equilent static force f_{s0}

$$ku_0 = 90625 \times 1/24 = 3776.04.$$

⇒



Varation of displacement with time.



Variation of Equilent static forces with time.

Q.NO (03) **ANSWER:**

Problem # 03

⇒ Given data :-

* Force = 60 Kips
* $U_1 = 7754/1000 = 7.754$ in
* After; j = 7 (cycles)
* Completed = 3.57 sec
* $U_{J+1} = 2.286$ cm = 0.9 in
* Ignore the verticle vibrating.

⇒ Req :-

(a) Damping ratios.
(b) Natural Period of Undamped Vibration.
(c) Stifness of structures.
(d) Weight of tank.
(e) Damping Co-efficient.
(f) Number of cycles to reduce the displacement emplitude to 0.5"

⇒ Solution :~

(a) ζ = Damping ratio = ?

As ;

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

By Puting values;

$$7 = \frac{1}{2(3.14)\zeta} \ln\left[\frac{7.754}{0.9}\right]$$

$$\zeta(7 \times 2 \times 3.14) = 2.15$$

$$\zeta(43.96) = 2.15$$

$$\zeta = \frac{2.15}{43.96}$$

$$\boxed{\zeta = 0.0489}$$

$$\boxed{\zeta = 4.89\,\%}$$

"b":-

$T_n = ?$

As "Seven" cycles are completed in "3.57" sec

Thus time required to complete one cycle =

$$= 7/3.57 = 1.96 \text{ sec}$$

$$\boxed{T_D = 1.96 \text{ sec}}$$

Now:-

$$\omega_D = \omega_n \sqrt{(1-\zeta^2)}$$

$$\Rightarrow \frac{2\pi}{\omega_D} = \frac{2\pi}{\omega_n} \left(\sqrt{1-\zeta^2}\right)$$

As;

$$T_D = T_n / \sqrt{1-\zeta^2}$$

$$\Rightarrow T_n = T_D \left(\sqrt{1-\zeta^2}\right)$$

$$\Rightarrow \quad = 1.96\left(\sqrt{1-(0.0489)^2}\right)$$

$$\Rightarrow \boxed{T_n = 1.957 \text{ sec}}$$

"Natural Period of Undamped Vibration"

(c) Stifness of structure, $K = ?$

As; $K = \frac{F \cdot \cos\theta}{2}$

$K = \frac{60 \cdot \cos(60^\circ)}{2}$ $\left(\begin{matrix} F = 60 \text{ kips} \\ \theta = 60^\circ \end{matrix} \right)$

$= 15 \, K/in$

(d) Weight of Tank; "W = ?"

As $W_n = \sqrt{K/m} = \sqrt{\frac{K}{(W/g)}} = \sqrt{\frac{K \cdot g}{W}}$

$\Rightarrow W_n^2 = \frac{K \cdot g}{W} \Rightarrow \left(W = \frac{K \cdot g}{W_n^2} \right)$

By Putting Values of $W_n = 2\pi / T_n$

$W = K \cdot g / \left(4\pi^2 / T_n^2\right) = K \cdot g \left(\frac{T_n^2}{4\pi^2} \right)$

$W = \frac{18000 \text{ lb}}{ft} \cdot \frac{32.2 \text{ ft}}{sec^2} \left(\frac{(1.957)^2}{4(3.14)^2} \right)$

$W = 56284.75 \text{ lb} = \boxed{56.284 \text{ k lb}}$

"e" Damping Co-efficient; "C = ?"

It is know that; $\zeta = \frac{c}{2m\omega_n}$

$\Rightarrow c = \zeta(2m\omega_n) = \zeta(2m(2\pi/T_n))$

By Puting values;

$$C = \frac{0.0489\left(2\left(\frac{56284}{32.2}\right)\left(2(3.14)\right)\right)}{1.957}$$

$$\boxed{C = 518.286 \text{ lb sec/ft}}$$

"f" No of cycles to reduce displacement altitude from "6.872 in to "0.5 in" j = ?

$$j = \frac{1}{2\pi\zeta} \ln\left(\frac{u_1}{u_{j+1}}\right)$$

$$= \frac{1}{2(3.14)(0.0489)} \ln\left[\frac{7.754}{0.9}\right]$$

$= 7.01$ OR

$$\boxed{j = 7 \text{ cycles}}$$

Q.NO (02) **ANSWER:**

QNo (02) :-

=> Given Data :-

$E = 29000 \text{ ksi}$

$I = 150 \text{ in}^4$

$\delta_{st} = 7754 \text{ lb}$

Take $\zeta = 2.5\%$

=> Solution :-

E.O.M for damped free vibration.

$Ku + Cu + mu = 0 \longrightarrow (1)$

it is known from Problem No 1 that

$K = 90625 \text{ lb/ft}$

$m = 240.80 \text{ lb} \cdot \text{sec}^2/\text{ft}$

$C_u = \xi \times 2 m W_n$

$C_u = 0.025 \times 2 (240.80)(19.39)$

$\boxed{C_u = 233.45 \text{ lb} \cdot \text{sec/ft}}$

By Subsitituting values of K, C and m (2)

ie eq (1)

$K u + C \dot{u} + m \ddot{u} = 0$

$90625 + 233.45 \dot{u} + 240.80 \ddot{u} = 0$

Solution to the E.O.M for damped free vibration is:

$$u(t) = e^{-\zeta w_n t}\left[u(0)\cos(w_D t) + \frac{1}{w_D}\left[\dot{u}(0) + u(0)(\zeta w_n)\right]\sin(w_D t)\right]$$

$$w_D = \sqrt{\frac{K}{m}} = \sqrt{\frac{90625}{240.80}}$$

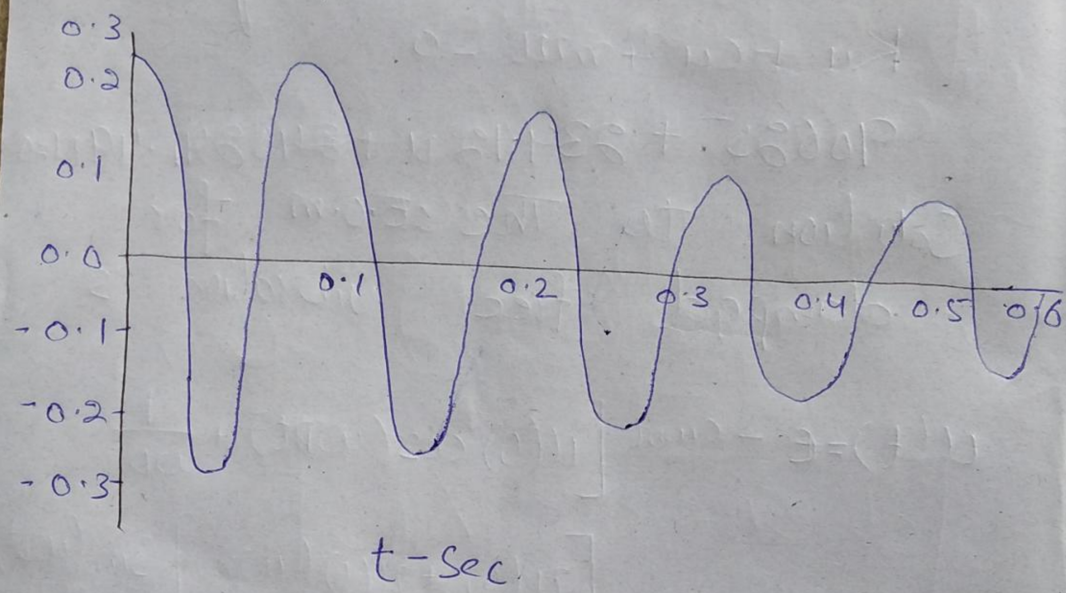
$w_D = 19.39$ rad/sec

$$u(t) = e^{-0.025 \times 19.39 t}\left[\frac{1}{24} \times \cos(19.39t) + \frac{1}{19.39} \times 0 + \frac{1}{24} \times 0.025 \times 19.39 \times \sin(19.39t)\right]$$

$$u(t) = e^{-0.48475}\left[0.04167 \times \cos(19.39t) + 0.0515 \times 0.0203 \sin 19.39t\right]$$

$$u(t) = e^{-0.48475}\left[0.04167 \times \cos(19.39t) + 0.001045 \sin 19.39t\right]$$

Damped free vibration



Damped free vibration